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Joint economic design of production, continuous sampling inspection and preventive maintenance of a deteriorating production system

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Abstract – Standard continuous sampling procedures and tables are conventionally applicable only to continuous production processes that are statistically 'in-control'. Consequently, these standards cannot be used to control quality in deteriorating production processes. Moreover, existing continuous sampling models do not consider interactions with production, inventory and maintenance aspects. In this paper, we attempt to fill these gaps in the literature. We investigate the joint design and optimization of a type-1 continuous sampling plan (CSP-1), make-to-stock production and preventive maintenance of a stochastic production system subject to both quality and reliability deteriorations. Two models of CSP-1 are considered and compared: the classical CSP-1 as in the standard procedures, and a CSP-1 plan with a stopping rule that is combined with condition-based maintenance. For both models, the optimization problem is to minimize the total incurred cost under a constraint on the outgoing quality. A combination of mathematical formulation, simulation and optimization techniques is used to solve such stochastic and constrained problems. Numerical examples are given to illustrate the resolution approach and to highlight some interesting aspects in the interactions between production, inventory, quality, maintenance and reliability. The results obtained demonstrate that sampling inspection plans realize significant cost savings compared to the 100% inspection which is commonly used in the literature of integrated models, and that using the CSP-1 with an inspection stopping rule for deteriorating processes is more cost-effective than the classical CSP-1.

Keywords – Deteriorating production process, production/inventory control, continuous sampling plan, inspection stopping rule, preventive maintenance, simulation optimization

1. Introduction

The integration of production, maintenance and quality control has attracted much attention in the past three decades. Many integrated models have been proposed in the literature to study various interactions and intersections between the three fundamental functions. Examples of such models include the integration of production and preventive maintenance (PM) planning (see for example the literature review by Budai et al., 2008), the integration of production and quality control policies (see the literature review by Inman et al., 2013) and the combination of PM and Statistical Process Control (SPC) techniques (e.g., Ben-Daya and Rahim, 2000; Yeung et al., 2007; Panagiotidou and Tagaras, 2010; Xiang, 2013; Yin et al., 2015). However, despite production, maintenance and quality control being strongly interrelated, the simultaneous integration of the three functions has received relatively very little attention in the literature (Hadidi et al., 2012).

Moreover, the quality control policies used in the existing integrated models are either 100% inspection of all items produced or control charts. Nevertheless, sampling inspection techniques have not yet been integrated simultaneously with production and PM policies. Acceptance sampling plans have been widely used in the industry for a long time to reduce the cost and time of quality inspection and to statistically control the outgoing quality (Montgomery, 2008a). In recent years, some authors have investigated the integration of acceptance sampling plans with production policies. For example, Bouslah et al. (2013, 2015) studied the interactions between the design of the lot-by-lot single sampling plan and the production-inventory settings for *batch production* systems. Also, Cao and Subramanian (2013) proposed an integrated quantity and quality model for performance analysis of manufacturing systems with continuous sampling plans.

Continuous sampling plans, which consist of alternating sequences of sampling inspection and 100% inspection, were initially introduced by Dodge (1943) to control the outgoing quality for *continuous production systems*. A continuous production system is a system that is dedicated to the production of a very narrow range of standardized products with high-volume sales (Blackstone, 2010). Thus, the setups are seldom changed, contrary to the batch production systems where setups are frequent (for these systems, lot-by-lot acceptance sampling plans are more suitable for quality control rather than the continuous sampling plans). To achieve standardization and low cost, the productive equipments use automation and complex technologies and they are organized and sequenced according to the routing of the jobs, which makes the material flows continuously during the production process as in transfer and assembly lines (Kim and Lee, 1993; Blackstone, 2010). In practice, continuous sampling plans have been popularly employed in various industrial sectors where continuous production systems are used such as in electronics, automobile, military and food industries (see Anthony, 2004; Antila et al., 2008; Oprime and Ganga, 2013). The design of the first generation of continuous sampling plans as in Dodge (1943) and in the military standard MIL-STD-1235 series are purely based on quality criteria such as the Average Outgoing Quality Limit (AOQL), and completely neglect the economic impact of such designs.

The economic design of the type-I continuous sampling plan (CSP-1), which is the most popular continuous sampling plan used in industry, has attracted many researchers over the past two decades. Vander Wiel and Vardeman (1994) and Cassady et al. (2000) have formulated CSP-1 cost models to prove that, for a steady production process with a constant defective rate, the optimal inspection policy is either no inspection or 100% inspection. Haji and Haji (2004) have shown that the economic CSP-1 generally leads to either 100% inspection or random partial inspection depending on the quality costs and the fraction of defectives produced. Those models are merely based on economic considerations. Chen and Chou (2002, 2003) and Eleftheriou and Farmakis (2011) suggested various extensions of Cassady et al.'s model considering an Average

Outgoing Quality Limit (AOQL) constraint. In the presence of the AOQL constraint, it is found that the CSP-1 is economically optimal.

All the aforementioned CSP-1 design models are commonly based on the assumption of an 'in-control' production process which is capable to yield a stable product quality. This assumption is absolutely unrealistic for a wide range of manufacturing systems where the production process is subject to quality deterioration (Rivera-Gomez et al., 2013a). In addition, while several studies have shown the strong interdependencies between quality, maintenance and productivity (Ben-Daya and Duffuaa, 1995; Lee et al., 2007; Colledani and Tolio, 2009; Rotab Khan and Darrab, 2010), almost all of the existing CSP-1 models do not consider any interactions with production, inventory and maintenance aspects. An exception is Cao and Subramanian (2013), who provided an analytical framework to evaluate the effects of the CSP-1 design parameters on the Work-In-Progress (WIP) inventory and manufacturing throughput.

Furthermore, the continuous sampling plans provide a lot of useful quality information that could be exploited for process condition monitoring and maintenance-decision making. For example, according to the CSP-1 procedure, one can easily recognize that an excessive long sequence of 100% inspection exhibits a significant increase in the proportion of defectives produced (Schilling and Neubauer, 2009). Surprisingly, an interesting study related to this topic has not attracted much attention so far: Murphy (1959) suggested some criteria based on quality inspection data to determine when the manufacturing process must be stopped to correct the process condition. These criteria have been called the inspection stopping rules for CSP-1 plans. In the literature, much effort has been devoted in recent years to integrating quality information from either 100% inspection or control chart techniques in the PM policies (e.g., Radhoui et al., 2010; Panagiotidou and Tagaras, 2010; Pan et al., 2012; Zhang et al., 2015). Nevertheless, unlike 100% inspection and control charts, the integration of the CSP-1 stopping rules in maintenance decision-making has not yet been studied in the literature.

This paper has four main objectives. The first is to develop a new joint economic design approach of production, continuous sampling inspection and PM policies for *continuous-flow* production systems subject to both quality and reliability deteriorations. This aims to jointly optimize the control parameters of those interrelated policies, in such a way to minimize the total operating cost while satisfying a predefined restriction on the AOQL. The second objective is to investigate how the proposed approach can properly extend the use of the continuous sampling plans to control quality of unstable and even deteriorating production processes (as the application of those plans are currently limited to stable processes). The third objective is to show how using CSP-1 plans rather than the 100% inspection policy, which is usually used in the literature to deal with deteriorating processes, can generate significant economic savings. Finally, the fourth objective is to demonstrate how additional cost savings can be achieved by using the CSP-1 with inspection stopping rules for deteriorating processes rather than the classical CSP-1. Advanced simulation techniques have been used to model, simulate, optimize and compare three integrated models associated with the three aforementioned inspection policies: 100% inspection, classical CSP-1 and CSP-1 with stopping rules. An extensive sensitivity analysis is also conducted to explore the effects of the system parameters on the optimal solutions and to illustrate the effectiveness of the proposed models.

This paper is organized as follows. Section 2 presents the notations used and the description of the problem being studied. In Section 3, we formulate the three integrated models. In Section 4, we present the resolution approach used to solve the three optimization problems. An illustrative numerical example is provided in Section 5. Sensitivity and comparative analyses are given in Section 6. Finally, Section 7 concludes the paper.

2. Problem statement

2.1. Notations

The notations used in this paper are defined as follows:

Decision variables:

s	Surplus inventory
m	PM period
i	Clearance number
f	Fraction of sampling
r	Inspection stopping threshold

Model parameters:

u_{max}	Maximum production rate
d	Demand rate
AOQL	Average Outgoing Quality Limit
τ_{pm}	Random variable denoting the preventive maintenance duration
τ_{cm}	Random variable denoting the corrective maintenance duration
τ_{insp}	Unit inspection duration
τ_{rect}	Unit rectification duration
C_h	Unit inventory holding cost per unit time
C_b	Unit backlog cost per unit time ($C_b >> C_h$)
C_{pm}	Preventive maintenance cost
C_{cm}	Corrective maintenance cost
C_{insp}	Unit inspection cost
C_{rect}	Unit rectification cost of a defective item
C_{def}	Unit cost of accepting/selling a defective item
$p(.)$	Proportion of defective items (function of cumulative production)
$F(.)$	Probability distribution of failure (function of cumulative production)

Other notations will be introduced where they are needed.

2.2. Problem description and assumptions

We consider a single-unit, *continuous production system* subject to aging which leads to an increasing failure rate and an increasing proportion of defectives produced. Both reliability and quality deteriorations are operation-dependent. Failures are instantaneously detected and they are removed by corrective maintenance (CM) interventions with a random duration τ_{cm}. The productive unit is preventively maintained through time-based preventive maintenance (TBPM) actions of random duration τ_{pm}. Both stochastic durations τ_{cm} and τ_{pm} follow general distributions. The cost and duration of the PM activities are smaller than those of the CM, i.e., $C_{pm} < C_{cm}$ and

$E[\tau_{pm}] < E[\tau_{cm}]$. We assume that both CM and PM restore the production unit to an 'as good as new' state. This assumption is reasonable in real-life for situations where maintenance interventions may include the replacement of key, failed and deteriorating components in the production unit.

The production unit supplies a serviceable downstream stock to meet a continuous and constant market demand d, as shown in Figure 1. As the quality of delivered products is a key factor in sustaining the market share, a quality inspection of the final products is necessary to meet the Average Outgoing Quality Limit (AOQL) requirement. Depending on the quality control policy used (100% inspection or sampling), an item produced may or may not be subject to quality inspection by attributes before being added to the serviceable stock. The defective items sorted during quality inspection are perfectly rectified before they are transmitted to the final stock. We consider that the inspection and rectification delays are not negligible. Hence, the serviceable stock is affected by two sources of uncertainties: the stochastic maintenance durations, and the variability of quality control delay which mainly depends on the variation of the inspection frequency (in the case of continuous sampling plans).

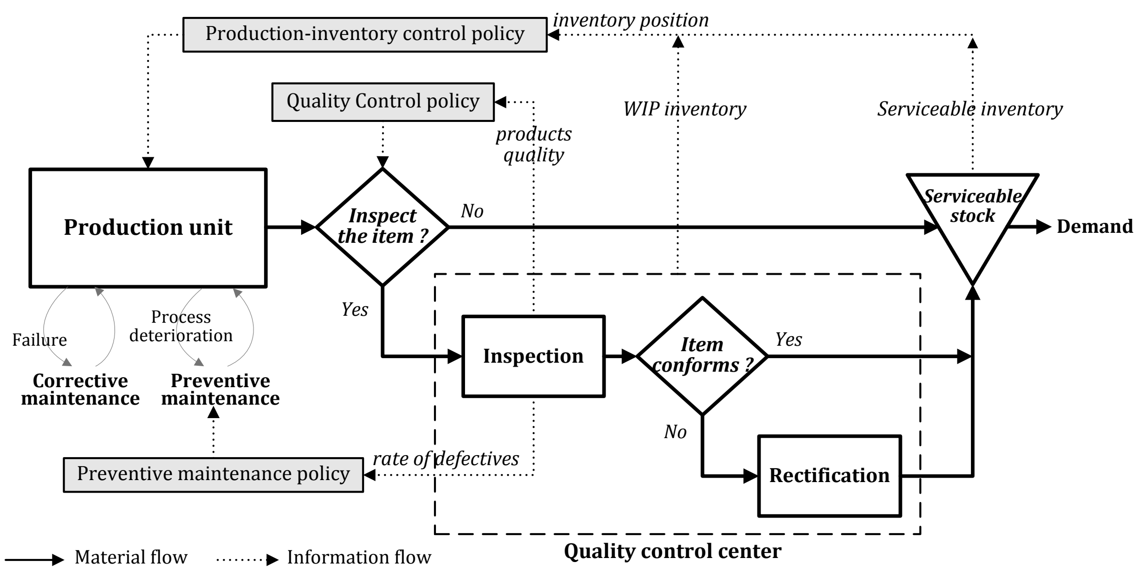

Fig. 1. Manufacturing system under study

The production rate $u(.)$ is flexible and can be set at any time at a value between 0 and a maximum level u_{max} ($u_{max} > d$). A make-to-stock production policy is used in order to avoid shortages during maintenance actions and to mitigate uncertainties in quality control. Herein, the well-known hedging point policy (HPP) is employed to control the production rate over time (Berthaut et al., 2010). The HPP consists in building a safety stock s after each production interruption by setting the production rate $u(.)$ at its maximum level u_{max}. Once built, the safety stock s shall be maintained by setting $u(.)$ at the level of the demand rate. Our choice of the hedging point method for production-inventory control is motivated by its optimality, simplicity and ease of implementation (Sarimveis et al., 2008).

Several models integrating the HPP and PM policies have been proposed in the literature. For example, Berthaut et al. (2010) studied the joint optimization of the HPP and periodic TBPM. Radhoui et al. (2009, 2010) integrated HPP with a PM policy that is based on the proportion of defectives found in lots produced. Rivera-Gomez et al. (2013a, 2013b) developed models integrating the HPP and age-based preventive maintenance for deteriorating production systems. In this study, the periodic TBPM policy is adopted because its ease of implementation in practice as it does not require keeping records on unit usage and age (Wang, 2002).

As a matter of fact, production, inventory, quality, maintenance and reliability closely interact and interrelate with each other, which influence the overall operational performance. Thus, these interactions and interdependencies should be taken into consideration when designing integrated models. For example, the acceleration of production during periods of safety stock build-up, which depends on the level of the safety stock setting, increases both quality and reliability deteriorations. Consequently, maintenance and quality inspection activities are increasingly required to cope with the effects of the production speed-up. However, excessive maintenance actions could have negative effects on the availability of the productive unit. A condition-based predictive maintenance (CBPM) strategy could be more appropriate for deteriorating processes to enhance the planning and efficiency of the PM activities (Mann et al., 1995; Jardine et al., 2006). In a context of integrated operations management, a closed-loop maintenance policy can be employed based on the feedback of quality information such as monitoring the observable proportion of defectives captured in quality inspection (Colledani and Tolio, 2012). In the literature, maintenance based on feedback quality information is generally coupled with 100% inspection policy, as in Hsu and Kuo (1995) and Radhoui et al. (2009, 2010). Excessive quality control, such as the 100% inspection, increases the WIP inventory and the manufacturing lead time. In continuous-flow production systems, continuous sampling inspection plans represent an alternative quality control strategy to the 100% inspection. Continuous sampling plans can be used to significantly reduce the quality inspection efforts while satisfying the outgoing quality requirement. In order to study those complex interactions, and to compare the derived scenarios of integrated production, maintenance and quality control policies, we consider the following integrated models:

- Model A, integrating the 100% inspection plan, the HPP and a periodic TBPM policy;
- Model B, integrating the classical CSP-1 plan, the HPP and the TBPM policy used in Model A;
- Model C, integrating the CSP-1 plan with a stopping rule, the HPP and a combined TBPM and CBPM.

In this research, we aim to develop an optimization approach to find the optimal solution for each of the three integrated models, to appraise the performance of those models when optimal solutions are applied and to conduct a comparative analysis. The first purpose of the comparative analysis is to show how the sampling inspection policy could significantly improve the performance of the manufacturing system (i.e., models B and C versus Model A). The second purpose consists of investigating how the CSP-1 stopping rules can be used for condition monitoring of deteriorating processes in order to improve the overall operational performance (i.e., Model C versus Model B).

3. Problem formulation

The state of the production unit can be described at each instant t by two continuous-time components, including:

- A discrete-state stochastic process $\{\alpha(t), t\geq 0\}$ taking values $\{0,1,2\}$ such that: $\alpha(t) = 0$, if the production unit is under CM at time t; $\alpha(t) = 1$, if it is available for production, and $\alpha(t) = 2$, if it is under PM.
- A piecewise continuous variable $a(t)$ which represents the age of the productive unit at time t. This age is measured by the cumulative number of items produced at time t since the last maintenance (CM or PM, whichever occurs last). It is calculated using the following differential equation:

$$\frac{\partial a(t)}{\partial t} = u\left(t, \alpha(t)\right), \forall\, t \geq T, a(T) = 0 \qquad (1)$$

where $u(t,\alpha(t))$ is the production rate at time t, also denoted $u(t)$. T is the completion time of the last maintenance.

We consider that both the probability of failure $F(.)$ and the proportion of defectives produced $p(.)$ are continuous increasing functions of the age $a(.)$. In practice, these functions can be determined from real data using mathematical, numerical and statistical techniques, as shown by Meeker and Escobar (1998) and Lai and Xie (2006).

3.1. Model A (integrated 100% inspection, HPP and TBPM)

Quality control policy

Quality control consists of 100% inspection of all items produced, so that all defective items are sorted and rectified before being transmitted to the serviceable stock. This policy is widely used in the literature of integrated models for simplicity, as there is no quality control variable to be optimized herein. The 100% inspection policy ensures the delivery of defect-free products to consumers. However, it increases the WIP inventory and the costs of quality inspection and rectification.

Preventive maintenance policy

The productive unit is preventively maintained at fixed time intervals with a period m, irrespective of the unit's age.

Production-inventory control policy

The so-called hedging point policy is used to instantly control the production rate $u(.)$ as follows:

$$u\left(t, \alpha(t)\right) = \begin{cases} u_{\max} & \text{if } \{x(t) < s\} \text{ and } \{\alpha(t) = 1\} \\ d & \text{if } \{x(t) = s\} \text{ and } \{\alpha(t) = 1\} \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

where, s is the safety stock, also called the hedging level, and $x(t)$ is the instantaneous inventory position, which is the sum of the serviceable inventory level $x_s(t)$ (inventory stock if positive and backlog if negative) and the WIP inventory in the quality center $x_q(t)$ (sum of items under inspection and rectification, see Figure 1). Under this production control policy, the dynamics of the inventories $x_s(t)$ and $x_q(t)$ can be described by the following equations:

$$\frac{\partial x_s(t)}{\partial t} = u(t) - \frac{\partial x_q(t)}{\partial t} - d \qquad (3)$$

$$\frac{\partial x_q(t)}{\partial t} = u(t) - u\left(t - \tau_{insp}\right) \cdot \left(1 - p\left(a\left(t - \tau_{insp}\right)\right)\right) - u\left(t - \tau_{insp} - \tau_{rect}\right) \cdot p\left(a\left(t - \tau_{insp} - \tau_{rect}\right)\right) \qquad (4)$$

where, $u(t)\cdot(1-p(a(t)))$ represents the effective production rate at time *t*, i.e., the number of conforming items produced in the time unit, and $u(t)\cdot p(a(t))$ represents the defective production rate at time *t*, i.e., number of defective items produced in the time unit. Thus, $x_q(t)$ is the difference between the cumulative production and the cumulative quantities of products fully inspected and rectified up to time *t*. From Eq. (4), one can see the impact of quality control delays on inventory dynamic.

Optimization problem

The optimization problem of Model A consists of finding the optimal values of the TBPM period *m* and the hedging level *s*, which minimize the expected total incurred cost per unit time (ETC). This cost includes the total quality cost, the inventory holding/backlog cost and the total maintenance cost.

The average quality cost per unit time $Q(t)$ during the interval [0, *t*] includes the costs of 100% inspection and rectification of defectives produced. It is given by:

$$Q(t)=\frac{1}{t}\cdot\left(C_{insp}\int_0^t u(z).dz+C_{rect}\int_0^t p(a(z))\cdot u(z).dz\right) \tag{5}$$

The average cost per unit time of inventory holding and backlog $G(t)$ during [0, *t*] is given by:

$$G(t)=\frac{1}{t}\cdot\int_0^t\left(C_h\left(x_q(z)+x_s^+(z)\right)+C_b x_s^-(z)\right)\cdot dz \tag{6}$$

where $x_s^+(t)=\max\left(x_s(t),0\right)$ and $x_s^-(t)=\max\left(-x_s(t),0\right)$.

The average maintenance cost per unit time $M(t)$ during [0, *t*] includes the costs of both CM and PM actions as follows:

$$M(t)=\frac{C_{cm}\ N_{cm}(t)+C_{pm}\ N_{pm}(t)}{t} \tag{7}$$

where $N_{cm}(t)$ and $N_{pm}(t)$ are respectively the numbers of CM and PM during [0, *t*].

Therefore, the optimization problem is to solve the following non-linear and stochastic model:

$$\begin{cases}\text{Minimize} & ETC(s,m)=\lim_{t\to\infty}\left(Q(t)+G(t)+M(t)\right)\\ \text{Subject to} & \text{Eqs. (1)-(4)}\\ & s,\ m>0\end{cases}$$

3.2. Model B (integrated CSP-1, HPP, and TBPM)

Quality control policy

A CSP-1 plan is used for quality control rather than the 100% inspection policy. The procedure of the CSP-1 plan is as follows (Dodge, 1943):

Step 1: Inspect 100% of the items consecutively as produced and continue such inspection until i items in succession are found clear of defects. i is called the clearance number or clearing interval.

Step 2: When i successive items are found clear of defects, discontinue 100% inspection, and randomly inspect a fraction f of the products ($0 \le f \le 1$).

Step 3: If a sample item is found defective, revert immediately to the 100% inspection (Step 1).

All defective items found are rectified before they are added to the serviceable stock. However, the defectives that have been accepted during sampling inspection will be transmitted to consumers. The Average Outgoing Quality (AOQ) for given CSP-1 parameters i and f, and a given proportion of defectives produced p can be estimated as follows (Schilling and Neubauer, 2009):

$$AOQ(i, f, p) = p\frac{(1-f)(1-p)^i}{f+(1-f)(1-p)^i} \tag{8}$$

The AOQL is the maximum level of AOQ over all possible values of p. Using Dodge's (1943) results, the AOQL is given by:

$$AOQL = \frac{(i+1)\,p_M - 1}{i} \tag{9}$$

where p_M is the proportion of defectives at which the AOQL occurs.

The manufacturer must select the clearance number i and the sampling fraction f such that the long-run AOQ, denoted AOQ_∞, does not exceed a specified AOQL. Then, from Eqs. (8) and (9), for given values of i and AOQL, the sampling fraction f must be greater than or equal to a minimum fraction f_{min} calculated as follows:

$$f_{\min}(i, AOQL) = \frac{\left(1-\frac{1+i\cdot AOQL}{i+1}\right)^{i+1}}{i\cdot AOQL + \left(1-\frac{1+i\cdot AOQL}{i+1}\right)^{i+1}} \tag{10}$$

Preventive maintenance policy

The PM policy is the same as in Policy A.

Production-inventory control policy

The production-inventory control policy is the same as in Policy A. Consequently, the dynamic of the serviceable inventory x_s is also described by Eq. (3). However, the dynamic of the WIP inventory x_q is affected by the alternation of sampling and 100% inspection as follows:

$$\begin{aligned}\frac{\partial x_q(t)}{\partial t} &= \left(ind\{\Gamma(t)=1\}\cdot f + ind\{\Gamma(t)=2\}\right)\cdot u(t) \\ &\quad - u\left(t-\tau_{insp}\right)\left(1-Y(t)\right) - u\left(t-\tau_{insp}-\tau_{rect}\right)\cdot Y\left(t-\tau_{rect}\right)\end{aligned} \tag{11}$$

where $\Gamma(t)$ describes the actual CSP-1 inspection mode at time t, as follows: $\Gamma(t)=1$, if the CSP-1 is in the sampling inspection mode, and $\Gamma(t)=2$, if the CSP-1 is in the 100% inspection mode. $Ind\{.\}$ is an indicator function defined as follows: $Ind\{\Theta(.)\}=1$ if $\Theta(.)$ is true, and $Ind\{\Theta(.)\}=0$ if $\Theta(.)$ is false. Thus, this function is used in Eq. (11) to indicate whether the CSP-1 is in the sampling inspection mode (i.e., $\Gamma(t)=1$) or not (i.e, $\Gamma(t)=2$). $Y(t)$ is the proportion of defectives found in quality inspection (random sample of fraction f or 100% inspection) at time t. If $\Gamma\left(t-\tau_{insp}\right)=1$, then $Y(t)$ is a random number described by the conditional distribution $P\left\{Y(t) \mid p\left(a\left(t-\tau_{insp}\right)\right)\right\}$ with an expected mean equal to $f \cdot p\left(a\left(t-\tau_{insp}\right)\right)$. Otherwise, if $\Gamma\left(t-\tau_{insp}\right)=2$, $Y(t)$ is exactly equal to $p\left(a\left(t-\tau_{insp}\right)\right)$ as in Eq. (4).

Optimization problem

The decision variables of Model B are the CSP-1 parameters i and f, the hedging level s and the TBPM period m. The objective is to minimize the excepted total incurred cost while meeting the AOQL requirement. The average inventory holding/backlog cost $G(t)$ and the average total maintenance cost $M(t)$ during $[0, t]$ are calculated respectively using Eqs. (6) and (7) as in Model A. Herein, the average quality cost $Q(t)$ in the period $[0, t]$ includes the cost of 100% inspection, the cost of rectification and the cost of accepting/selling defective items. $Q(t)$ is given by:

$$Q(t)=\frac{1}{t}\cdot\left(\begin{array}{l} C_{insp}\int_0^t\left(ind\left\{\Gamma(z)=1\right\}\cdot f+ind\left\{\Gamma(z)=2\right\}\right)\cdot u(z).dz+ \\ C_{rect}\int_0^t Y\left(z+\tau_{insp}\right)\cdot u(z).dz+C_{def}\int_0^t\left(p\left(a(z)\right)-Y\left(z+\tau_{insp}\right)\right)\cdot u(z).dz \end{array}\right) \tag{12}$$

Hence, the optimization problem is to solve the following mixed-integer, non-linear and stochastic model:

$$\begin{cases} \text{Minimize} & ETC\left(s,m,i,f\right)=\lim_{t\to\infty}\left(Q(t)+G(t)+M(t)\right) \\ \text{Subject to} & \text{Eqs. (1)-(3), (11)} \\ & f_{\min}\left(i, AOQL\right)\leq f\leq 1 \\ & s,\ m,\ i\ \geq 0\ ;\quad i: \text{integer} \end{cases}$$

3.3. Model C (integrated CSP-1 with a stopping rule, HPP and combined TBPM/CBPM)

Quality control policy

A CSP-1 plan is basically employed for quality control as in Model B. However, a stopping rule is incorporated into the CSP-1 procedure in order to avoid wasted labour and resources in situations of excessive, long 100% inspection sequences. This involves shutting down production in order to restore the process condition, as soon as the proportion of defectives in any one 100% inspection sequence reaches or exceeds a given threshold r ($0 < r < 1$). Note that the problem of long 100% inspection sequences is often observed in deteriorating processes, overmuch occurs than in stable processes. This is because the probability that the CSP-1 will shift again from 100% inspection to the sampling inspection decreases as quality deteriorates with production unit usage.

Preventive maintenance policy

The feedback information from the quality inspection that has been incorporated into the CSP-1 plan as an inspection stopping rule should be also integrated in the PM strategy. Thus, we suggest combining the periodic TBPM with a CBPM as follows: the productive unit is preventively maintained after a period m of time since the last PM, or when the proportion of defectives sorted during the 100% inspection, $\beta(.)$, reaches or exceeds a given threshold r, whichever occurs first. Let $\Omega_k(t)$ denote a binary function with 1 if a kth PM has to be performed at time t, and 0 if not. Then, the PM control policy can be described by the following equation:

$$\Omega_{k+1}(t) = \begin{cases} 1 & \text{if } \beta(t) \geq r \text{ or } t - T_k \geq m \\ 0 & \text{otherwise} \end{cases}, \quad k=1,..,\infty \tag{13}$$

where T_k is the completion time of the last kth PM.

The relevance of the proportion of defectives sorted during the 100% inspection sequences on recognizing the real condition of the production process, even this information is partially observable (i.e., not available during sampling sequences), lies in the fact that the dynamic of the CSP-1 over time reflects in itself the degree of quality deterioration. Indeed, while the CSP-1 remains in the sampling inspection mode, the process quality can expect to be considered acceptable as no defective item is found in the random samples. In such situations, the length of the sampling sequence is in itself an inference on the healthiness of the production process, so that there is no need to investigate the proportion of defectives produced. However, the fact that the CSP-1 shifts to the 100% inspection mode indicates that the process quality has moved above the acceptable level. Starting from this point (i.e., switching from sampling inspection to 100% inspection), the CSP-1 provides a complete information about the defective items produced. Based on a continuous monitoring of the observed production quality during the 100% inspection periods, a CBPM is performed as soon as the proportion of defectives surpasses the threshold r. The TBPM is more useful in situations where the random sampling inspection fails to capture any defective product while the process deterioration condition is already critical.

Production-inventory control policy

The production-inventory control policy is the same as in Policies A and B. The dynamics of the final inventory x_s and the WIP inventory x_q are, respectively, described by Eqs. (3) and (11).

Optimization problem

The decision variables of Model C are the clearance number i, the sampling inspection f, the stopping inspection rule r, the TBPM period m, and the hedging level s. The objective is to minimize the excepted total incurred cost while meeting the AOQL constraint. The average inventory holding/backlog cost $G(t)$ and the average quality cost $Q(t)$ during a period $[0, t]$ are calculated, respectively, using Eqs. (6) and (12). Also, the average maintenance cost $M(t)$ during $[0, t]$ is calculated using the general Eq. (7), given that $N_{pm}(t)$ is the total number of both TBPM and CBPM actions during $[0, t]$. Hence, the optimization problem is to solve the following mixed-integer, non-linear and stochastic model:

$$\begin{cases} \text{Minimize} \quad ETC(s,m,i,f,r) = \lim\limits_{t\to\infty}\left(Q(t)+G(t)+M(t)\right) \\ \text{Subject to} \quad \text{Eqs. (1)-(3), (11), (13)} \\ \qquad\qquad\quad f_{\min}\left(i, AOQL\right) \leq f \leq 1 \\ \qquad\qquad\quad 0 < r < 1 \\ \qquad\qquad\quad s,\, m,\, i \ \geq 0 \ ; \quad i: \text{integer} \end{cases}$$

4. Resolution approach

The three above-formulated optimization problems are non-linear, constrained and highly stochastic. The stochastic events are mainly the random occurrence of failures, the CM and PM actions which are following general distributions and the uncertainty in the dynamic of the CSP-1 which is based on the products quality and random samples. In addition, models B and C are mixed-integer problems because the discreteness constraint on the clearance number *i*. An explicit analytical expression of the average maintenance cost for models A and B can be derived from Eq. (7) based on previous findings in the literature as in Barlow and Proschan (1965). However, deriving a closed-form expression of the average maintenance cost for Model C is challenging as the CBPM intimately depends on the dynamic of the CSP-1 with a stopping rule. Moreover, computation of the total inventory/backlog and quality costs either analytically or numerically is very challenging too. This is because the complexity of the inventories' dynamics as in Eqs. (3), (4) and (11), the stochastic behaviour of maintenance actions and the complexity dynamic of the CSP-1. Thus, classical mathematical programming methods cannot be used to solve the three complex stochastic models under study, as there is no way to derive the closed-form analytical expressions for the objective functions. Rather, we used a combination of simulation, statistical and optimization techniques to estimate the objective function and to find the optimal solution for each integrated model.

4.1. Simulation-optimization approach

Simulation-optimization approaches consist in combining computer simulation with optimization techniques to heuristically solve problems that are analytically and numerically intractable (Gosavi, 2014). Discrete-event simulation has been increasingly used in the literature to imitate stochastic and complex manufacturing systems and to solve a wide range of operations management problems (see the review by Negahban and Smith, 2014). In this study, we use a combined discrete-continuous simulation to accurately model both discrete events and continuous variables. Thus, the resolution approach consists of the following step-by-step simulation-optimization methodology (Figure 2):

- *Step* 1 - *Mathematical model*: Analytically formulate each optimization problem, as shown in Section 3. This provides a rigorous modelling of the system dynamic as a function of its state, the definition of the decision variables, the objective function to be minimized and the problem constraints.
- *Step* 2 - *Simulation model*: Transform each mathematical model into a discrete-continuous simulation model according to the following logic: the continuous-time equations (e.g., unit age

$a(.)$, probability of failure $F(.)$, proportion of defectives produced $p(.)$ and inventory-consumption rate d) are modelled and calculated instantly with C++ subroutines, while the discrete events (e.g., failures occurrence, CM and PM actions, production rate change, CSP-1 inspection mode change, etc.) are modelled with the SIMAN language in *Arena Simulation* environment. Hence, for each model, the expected total incurred cost for given values of the decision variables is obtained from simulation.

- *Step* 3 - *Cost function estimation and Optimization*: Use Design of Experiments (DOE) and Response Surface Methodology (RSM) to fit the total incurred costs calculated from experimental data by second-order regression models (Myers et al., 2009). The regression model for each integrated control policy must include the main effects and interactions between its decision variables. Those interaction effects play an important role to obtain an optimal trade-off solution for each integrated production, quality and maintenance control policy. Then, each optimization problem can be solved using non-linear constrained optimization techniques such as the penalty and barrier methods (Bazaraa et al., 2006). The optimal solution is determined within the feasible region defined by the problem constraints and the region of the DOE. This sequential procedure is iteratively repeated in order to fully explore the admissible experimentation region and to therefore bring out a global optimal solution.

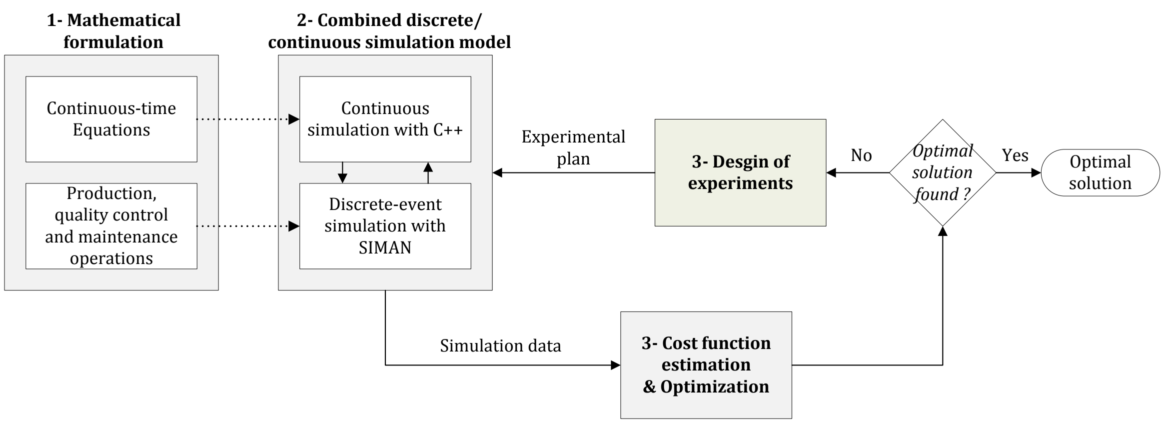

Fig. 2. Simulation-optimization approach

4.2. Simulation models

A simulation model has been developed for each integrated model A, B and C, and executed with *Arena Simulation* software. The differential equation (1) is continuously integrated in C++ using the Runge–Kutta–Fehlberg method (Pegden et al., 1995). The given functions describing the probability of failure $F(.)$ and the proportion of defectives $p(.)$ are calculated instantly using the C++ mathematical functions and operators. Discrete events are used to model the discrete-material-flow as shown on Figure 1. Thus, the dynamic of the serviceable stock $x_s(.)$ is modeled by combining discrete events (inflow of finished products) and continuous modeling of the demand as in Eq. (3). Then, the surplus inventory $x_s^+(.)$ and the backlog $x_s^-(.)$ are instantly derived in C++ from calculation of the final inventory $x_s(.)$. Additional discrete events have also been used to model the production control policy as in Eq. (2), the CSP-1 procedure, the planned PM actions, the stochastic occurrence of breakdowns and the restoration of the production unit to the 'as-good-as-new' state after of each maintenance action. The stochastic durations of CM and PM actions are randomly generated following predefined probability distributions. The duration of simulation runs t_∞ is set in such a way to ensure that the steady-state is reached. Both discrete and continuous parts of the simulation model work synchronously to calculate the performances of the three integrated models (see Bouslah et al., 2013). At the end of each simulation run, the average total inventory/backlog cost $G(t_\infty)$, the average maintenance cost $M(t_\infty)$ and the average quality cost $Q(t_\infty)$ are calculated using the corresponding formulas as in Section 3.

To check the accuracy of the simulation models, we used a set of verification and validation techniques such as tracing the models' operation, testing for reasonableness, testing the models' structure and data and using the animation and debug features of *Arena* software (Pegden et al., 1995). For example, Figure 3 represents a simulation sample of the dynamics of operations of Model C over time. Figures 3.(a), 3.(b) and 3.(c) show that the production-inventory control policy performs correctly with respect to the inventory position $x(.)$ and the production unit state $\alpha(.)$ as in Eq. (2). They also show the effects of the CM and PM interventions on depleting the serviceable stock, resulting sometimes in shortage situations. Figures 3.(d) and 3.(e) depict, respectively, the impact of the production unit usage on the reliability and quality deteriorations. These Figures also show the effects of maintenance actions on the restoration of the process quality and the production system reliability to the initial conditions. Figure 3.(f) shows the dynamic of the CSP-1 plan (i.e., alternation between sampling and 100% inspection sequences). Finally, from Figures 3.(a) and 3.(e), we verify that the PM control policy operates properly as in Eq.(13): a PM action is triggered either when the observable rate of defectives exceeds the inspection stopping threshold r or after a period of m since the last PM.

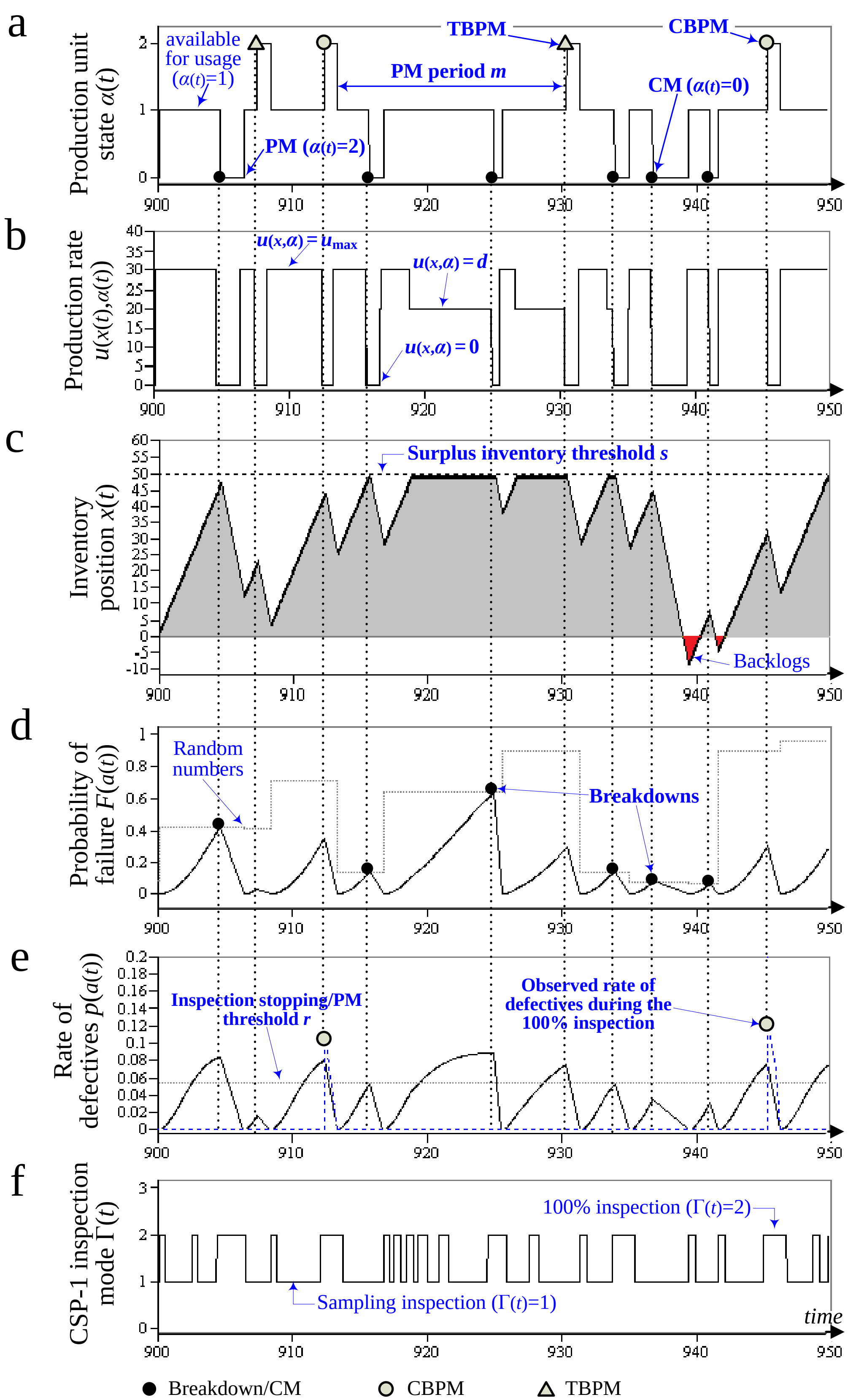

Fig. 3. Dynamics of operations during the simulation run

5. Numerical Example

A basic hypothetical example is used to illustrate the resolution approach and to compare the models presented. We consider that the probability of failure $F(.)$ follows a two-parameter Weibull distribution given by:

$$F\left(a(t)\right) = 1 - \exp\left(-\lambda_f \; a(t)^{\gamma_f}\right) \tag{14}$$

where λ_f and γ_f are given positive constants. Similarly, the proportion of defective items produced $p(.)$ increases with age $a(.)$ as follows:

$$p\left(a(t)\right) = p_0 + \eta\left(1 - \exp\left(-\lambda_q \; a(t)^{\gamma_q}\right)\right) \tag{15}$$

where p_0 is a very small proportion of defectives produced at the initial condition, λ_q and γ_q are given positive constants and η is the boundary considered in the quality deterioration. The parameters of Eqs.(14) and (15) can be derived from historical data records using techniques such as the maximum likelihood estimation and the median-rank regression methods (Soliman et al., 2006; Olteanu and Freeman, 2010).

We consider the following values of parameters in the appropriate units for the illustrative example: u_{max}=30, d=20, C_h=1.2, C_b=18, C_{pm}=700, C_{cm}=1800, C_{ins}=6, C_{rect}=35, C_{def}=50, $\tau_{ins}=5\times10^{-3}$, $\tau_{rect}=15\times10^{-3}$, τ_{pm}~Log-Normal(1,0.1), τ_{cm}~Gamma(0.5,2.5), $AOQL$=2.0%., $\lambda_r=3\times10^{-5}$, γ_r=2.0, $\lambda_q=4\times10^{-4}$, γ_q=1.8, p_0=0.01% and η= 0.09.

5.1. Experimentation and results

Simulation runs are conducted according to a complete 3^2 design of experiments for Model A (as there are only two independent variables s and m), while Box-Behnken experimental plans are used for both models B (four independent variables s, m, i and f) and C (five independent variables s, m, i, f and r). The Box-Behnken design is suitable for plans with more than two factors because of its rotatable feature and its efficiency in terms of number of runs required (Montgomery, 2008b). For each combination of independent factors, simulation is replicated four times. The simulation horizon t_∞ of each replication is set to 500000 units of time to ensure that the steady state is achieved (it takes on average 55 seconds for each replication on a computer with a 2.80 GHz CPU).

For each integrated model, collected simulation data are used to fit the dependent variable (i.e., average total incurred cost) by a continuous, convex, second-order regression function. To check the fitness of the regression models, we used a set of validation techniques (Myers et al., 2009). First, the model's overall performance is evaluated. This is referred to as the coefficient of multiple determination R-squared and the adjusted R-squared, which represent the proportion of total variation explained by the regression model. The values of these two coefficients should be close to 1. Second, a complete residual analysis is conducted to check the homogeneity of variances and the normality assumption of residuals. Third, once the optimization is performed, each optimal solution is cross-checked to ensure the validity.

The simulation results are handled using the statistical software STATISTICA in order to produce the analysis of variance (ANOVA), and to seek and validate the regression models fitting the total incurred costs. ANOVA analyses are carried out as presented in Tables 1, 2 and 3. All factors and quadratic effects and most of interactions are statistically significant for the response variables

(P-Value ≤ 5%). Moreover, the three ANOVA tables indicate that the F-ratio test for 'lack of fit' is not significant. The adjusted R-squared values for models A, B and C are, respectively, 0.9818, 0.9777 and 0.9728. This states that the second-order regressions models explain more than 97.0% of the variability observed in the excepted total incurred costs. Let $\psi_A(.)$, $\psi_B(.)$ and $\psi_C(.)$ be, respectively, the regression functions for models A, B and C. From STATISTICA, the corresponding cost functions are given as follows:

$$\psi_A(s,m) = 784.64 - 7.12\, s + 49.02\times10^{-3}\; s^2 - 16.27\, m + 317.74\times10^{-3}\; m^2 + 106.09\times10^{-3}\; s\cdot m \qquad (16)$$

$$\begin{aligned}\psi_B(s,m,i,f) &= 921.67 - 10.37\, s + 77.13\times10^{-3} s^2 - 37.52\, m + 110.21\times10^{-2} m^2 - 505.60\times10^{-3} i\\ &+ 5.57\times10^{-3} i^2 - 57.67\times10^{-3}(1/f) + 545.82\times10^{-6}(1/f)^2 + 197.69\times10^{-3} s\cdot m + 20.97\times10^{-4} s\cdot i\\ &+ 434.85\times10^{-3} s\cdot(1/f) + 18.85\times10^{-3} m\cdot i - 98.19\times10^{-4}\; m\cdot(1/f) - 158.26\times10^{-6} i\cdot(1/f)\end{aligned} \qquad (17)$$

$$\begin{aligned}\psi_C(s,m,i,f,r) &= 753.93 - 8.26\, s + 698.01\times10^{-4} s^2 - 11.87\, m + 245.19\times10^{-3} m^2 + 335.88\times10^{-3} i\\ &+ 47.76\times10^{-4} i^2 - 128.11\times10^{-3}(1/f) + 464.85\times10^{-6}(1/f)^2 - 193.87\times10^{-2}\; r + 20.92\times10^{-3} r^2\\ &+ 81.51\times10^{-3} s\cdot m + 834.62\times10^{-6} s\cdot i + 127.11\times10^{-6} s\cdot(1/f) - 12.71\times10^{-4} s\cdot r + 11.43\times10^{-4} m\cdot i\\ &- 65.07\times10^{-4}\; m(1/f) - 99.44\times10^{-4} m\cdot r - 16.63\times10^{-4} i(1/f) - 72.83\times10^{-4} i\cdot r + 24.34\times10^{-4}(1/f) r\end{aligned} \qquad (18)$$

Table 1. ANOVA table for the Model A

Effect	SS	d.f.	MS	F-ratio	P-Value	Significant
$s + s^2$	8363.54	2	4181.768	321.397	0.000317	Yes
$m + m^2$	12658.50	2	6329.248	486.445	0.000170	Yes
$s \cdot m$	1119.90	1	1119.901	86.072	0.002650	Yes
Lack of Fit	184.69	3	61.562	4.731	0.117030	No
Pure Error	39.03	3	13.011			
Total SS	23980.40	11		R^2 =0.9901; R^2_Adjusted= 0.9818		

Table 2. ANOVA table for the Model B

Effect	SS	d.f.	MS	F-ratio	P-Value	Significant
$s + s^2$	41262.92	2	20631.46	3083.527	0.000324	Yes
$m + m^2$	39122.79	2	19561.39	2923.598	0.000342	Yes
$i + i^2$	9401.16	2	4700.58	702.537	0.001421	Yes
$1/f + 1/f^2$	1648.51	2	824.25	123.191	0.008052	Yes
$s \cdot m$	2251.02	1	2251.02	336.432	0.002959	Yes
$s \cdot i$	21.29	1	21.29	3.182	0.216399	No
$s \cdot (1/f)$	3.03	1	3.03	0.452	0.570575	No
$m \cdot i$	154.78	1	154.78	23.133	0.040614	Yes
$m \cdot (1/f)$	138.83	1	138.83	20.749	0.044968	Yes
$i \cdot (1/f)$	303.05	1	303.05	45.293	0.021373	Yes
Lack of Fit	925.03	10	92.5	13.825	0.069295	No
Pure Error	13.38	2	6.69			
Total SS	91326.24	26		R^2 =0.9897; R^2_Adjusted= 0.9777		

Table 3. ANOVA table for the Model C

Effect	SS	d.f.	MS	F-ratio	P-Value	Significant
$s + s^2$	33601.66	2	16800.83	1762.547	0.000000	Yes
$m + m^2$	7874.98	2	3937.49	413.075	0.000003	Yes
$i + i^2$	44378.09	2	22189.04	2327.815	0.000000	Yes
$1/f + 1/f^2$	5141.50	2	2570.75	269.693	0.000008	Yes
$r + r^2$	1991.87	2	995.94	104.482	0.000083	Yes
$s \cdot m$	602.04	1	602.04	63.159	0.000508	Yes
$s \cdot i$	3.92	1	3.92	0.411	0.549493	No
$s \cdot (1/f)$	16.38	1	16.38	1.718	0.246911	No
$s \cdot r$	55.59	1	55.59	5.832	0.060497	No
$m \cdot i$	95.20	1	95.20	9.987	0.025088	Yes
$m \cdot (1/f)$	108.14	1	108.14	11.345	0.019928	Yes
$m \cdot r$	9.41	1	9.41	0.987	0.366164	No
$i \cdot (1/f)$	286.23	1	286.23	30.027	0.002759	Yes
$i \cdot r$	673.42	1	673.42	70.648	0.000391	Yes
$(1/f) \cdot r$	84.85	1	84.85	8.901	0.030675	Yes
Lack of Fit	423.51	20	21.18	2.222	0.191761	No
Pure Error	47.66	5	9.53			
Total SS	91208.75	45		R^2 =0.9849; R^2_Adjusted= 0.9728		

Figures 4.(a), 4.(b) and 4.(c) present the projection of the cost response surfaces on different two-dimensional spaces. In Figure 4.(b), the AOQL constraint described by Eq.(10) separates the space (i,1/f) into two regions: the region with gray-shaded contours represents the infeasible solutions (i.e., the AOQL constraint is not satisfied), while the remaining space represents the region of feasible solutions. The optimal solutions of the three policies are presented in Table 4. From 20 replications of simulation, we validated the optimal solutions by verifying that the corresponding estimated optimal costs ψ_A^* =\$455.7, ψ_B^* =\$421.2 and ψ_C^* =\$398.2 fall, respectively, within the confidence intervals [\$455.23, \$457.21], [\$419.43, \$421.62] and [\$396.14, \$399.17].

[Insert Table 4]

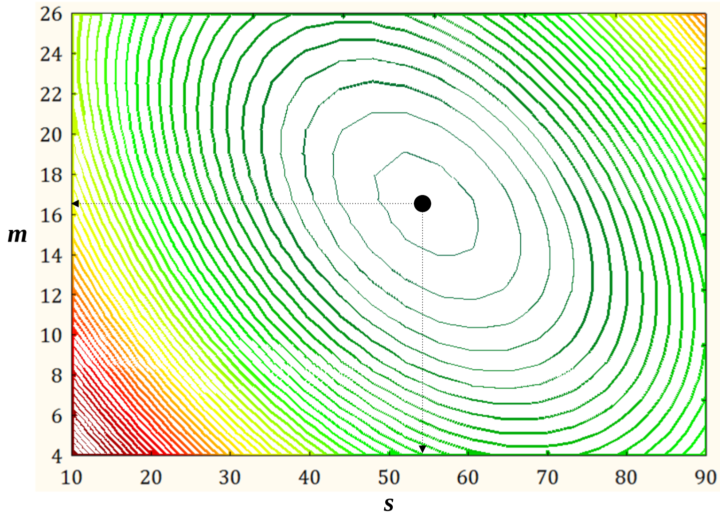

Fig. 4.(a). Cost response surface (s, m) for Model A

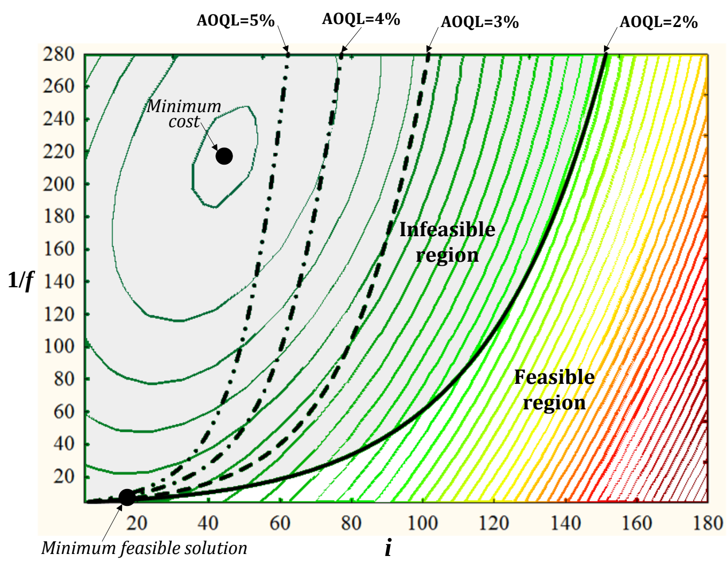

Fig. 4.(b). Cost response surface (i, $1/f$) for Model B

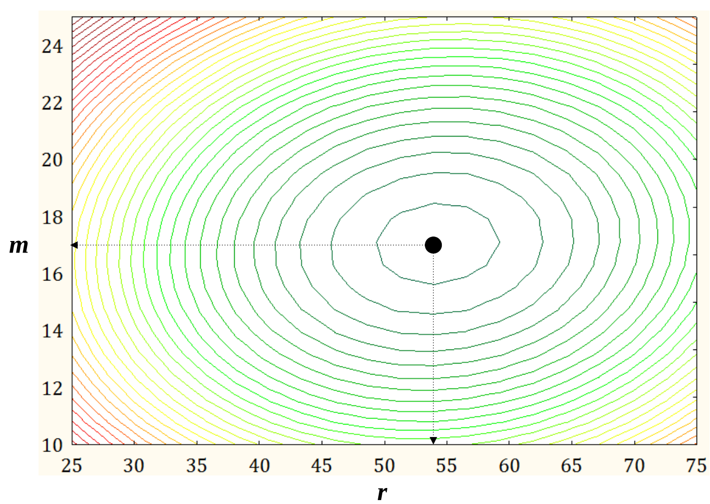

Fig. 4.(c). Cost response surface (r, m) for Model C

5.2. Comparison of the performances of the three integrated models

Table 4 contains complementary performance indices obtained from simulation when the optimal solutions are applied. Quality performance indices include the long-run Average Fraction of Defectives Produced denoted by ADP_∞, the long-run Average Fraction of production Inspected denoted by AFI_∞ and the long-run Average Outgoing Quality denoted by AOQ_∞. The reliability/maintenance indices are the long-run Failure Rate denoted by FR_∞, the long-run Frequency of PM denoted by FPM_∞ and the long-run availability of the production unit denoted by AV_∞. The inventory indices are the average WIP inventory per unit time denoted by $E[x_q]$, the average serviceable inventory per unit time denoted by $E[x_s^+]$ and the average backlog per unit time denoted by $E[x_s^-]$. Note that the WIP inventory $E[x_q]$ intimately depends on the AFI_∞, so that $E[x_q]$ increases as AFI_∞ increases and vice versa. Finally, let Δ-B/A, Δ-C/A and Δ-C/B be the cost differences between the three models calculated as follows:

$$\Delta\text{-i/j}(\%) = \frac{\psi_i^*(.) - \psi_j^*(.)}{\psi_j^*(.)} \times 100,\ (i,j) \in \{(B,A),(C,A),(C,B)\} \qquad (19)$$

The incurred cost under models B and C are significantly better than that under Model A, i.e., Δ-B/A = -7.57% and Δ-C/A = -12.62%. These significant economic savings are mainly due to the fact that the sampling inspection considerably reduces the inspection efforts, the total lead time and the WIP inventory. In fact, under policies B and C, only 60.8% to 62.2% of the production should be inspected in the long run to meet the AOQL requirement (AOQ_∞ = 1.53% under Model B and AOQ_∞ = 1.45% under Model C, both less than the predefined $AOQL$ = 2.0%). Moreover, the optimal safety stock s^* dropped from 54.8 under Model A to 51.2 and 49.6 under models B and C, respectively. Consequently, the average WIP inventory per unit time $E[x_q]$ dropped from 0.23 to 0.13-0.14 under models B and C. Similarly, the average serviceable stock $E[x_s^+]$ dropped from 41.5 under Model A to 37.0 and 35.4 under models B and C. This led to, respectively, 10.84% and 14.69% of reductions in the total inventory. However, the average backlog per unit time $E[x_s^-]$ increases from 1.3 under policy A to 1.8 and 1.5 under policies B and C, respectively. The increase of the risk of shortage under the last two policies is due to the increase in the uncertainty in the quality control delay as explained in Section 2.2. In addition, we see that, when the CSP-1 plan is used, the PM is further frequent (i.e., FPM_∞ increases from 0.0568 under Model A to 0.0744 and 0.1529 under models B and C) in order to improve the process quality (i.e., ADP_∞ dropped from 5.62% under Model A to 5.45% and 4.37% under models B and C, respectively). Thus, the reduction of the quality control activities (i.e., using the CSP-1 plan rather the 100% inspection) is compensated with an increase in the PM actions. This is an interesting observation as it shows how the PM and the CSP-1 plan interact with each other to control the level of the outgoing quality. As the FPM_∞ increases, the production unit reliability is improved so that the long-run failure rate FR_∞ dropped from 0.1049 under Model A to 0.0992 and 0.0662 under models B and C, respectively.

Furthermore, from Table 4, we find that Model C is more profitable in comparison with Model B, i.e., Δ-C/B = -5.46%. In fact, the incorporation of the predictive maintenance coupled with the CSP-1 dynamic into the PM strategy provides the data required to recognize the actual process condition (as explained in Section 3.3). Based on that data, CBPM actions are performed on an as-needed basis. This reduces the occurrence of breakdowns and avoids unnecessary TBPM actions. Table 4 illustrates that the optimal TBPM period m^* increased from 12.30 under Model B to 17.05

under Model C, and that the frequency of all PM actions has been concurrently doubled from 0.0744 to 0.1529. These results show that less TBPM are performed compared with Model B and that most of the PM actions are triggered by the CBPM. As a result, both quality and reliability have been significantly improved. Indeed, only 4.37% of defectives are produced in the long-run under Model C compared to 5.45% defective production under Model B. Moreover, the long-run failure rate FR_∞ dropped from 0.0992 under Model B to 0.0662 under Model C. Furthermore, note that the average backlog $E[x_s^-]$ consequently decreases from 1.8 to 1.5 despite the fact that the optimal hedging level s^* decreased from 51.2 to 49.6.

6. Sensitivity analysis and comparative study

Another set of experiments was conducted to measure and analyze the sensitivity of the proposed integrated models with respect to the ranges of system parameters. The objective is to study the behaviour of the three integrated models and to compare their incurred costs for different system conditions derived from the basic case.

6.1. Impact of cost and deterioration parameters

Table 5 presents eighteen configurations of cost and deterioration parameters derived from the basic case by varying their values above and below one at a time by 50%. The variations of the optimal solutions of the three integrated models compared to the basic case make sense and can be explained as follows:

• *Variation of the holding inventory cost*: When the holding cost C_h increases (case 1), the three integrated models react by decreasing the optimal surplus inventory s^*. In models B and C, the optimal clearance number i^* increases and the optimal sampling fraction f^* decreases in such a way to reduce the fraction of production inspected AFI_∞ and to consequently reduce the WIP inventory. As the decrease of the AFI_∞ deteriorates the outgoing quality (i.e., AOQ_∞ increases), the optimal period m^* decreases in order to more frequently reinstate the process quality to initial conditions through TBPM actions. The optimal CBPM threshold r^* in model C becomes less restricted due to the decrease of the PM period m^*. In model A, the PM period m^* increases because the decrease of the surplus inventory s^* which slows down the process deterioration. Note that a lower holding cost produces the opposite effects (case 2).

• *Variation of the backlog cost*: When the backlog cost C_b increases (case 3), the optimal surplus inventory s^* increases in order to provide better protection to the serviceable stock against shortages. The optimal TBPM period m^* decreases in order to improve the reliability of the production equipment and to reduce the effects of failures. Because increasing PM activities also enhances the process quality, the optimal sampling fraction f^* decreases and the clearance number i^* increases such that the severity of the CSP-1 plan is reduced (i.e., AFI_∞ decreases). In model C, the optimal threshold r^* increases (so less CBPM actions are performed) because the decrease of the TBPM period m^*. The decrease of the backlog cost has the opposite effects (case 4).

• *Variation of the corrective maintenance cost*: When the CM cost C_{cm} increases (case 5), the PM should be performed more frequently in order to reduce the occurrence of failures, so that the optimal TBPM period m^* decreases. This also implies improving the process quality, which explains the fact that the optimal CSP-1 plan becomes reduced (i.e., f^* decreases and i^* increases such that the AFI_∞ is reduced). The optimal threshold r^* becomes less restricted because the TBPM actions become more frequent as the optimal period m^* decreases. Note that a decrease in the CM cost produces the opposite effects (case 6).

• *Variation of the preventive maintenance cost*: When the PM cost C_{pm} increases (case 7), the optimal period m^* increases in order to reduce the number and cost of periodic TBPM actions. The optimal surplus inventory s^* decreases to reduce the intensity of process deterioration due to the production speed-up during periods of safety stock build-up. Moreover, in models B and C, the CSP-1 plan becomes tighter in order to maintain the outgoing quality lower than the allowable AOQL level. So, the optimal sampling fraction f^* increases and the clearance number i^* decreases such that the AFI_∞ increases. In model C, the optimal threshold r^* becomes more restricted in order to increase the frequency of CBPM actions and to compensate for the decrease in TBPM actions. Note that the decrease in the PM cost produces the opposite effects (case 8).

• *Variation of the inspection cost*: In models B and C, when the inspection cost C_{insp} increases (case 9), the optimal sampling fraction f^* decreases to reduce the inspection efforts during sampling inspection periods, while the clearance number i^* increases to satisfy the AOQL requirement. The optimal TBPM period m^* decreases in order to improve the process quality more frequently. In Model C, the optimal CBPM threshold r^* increases due to the decrease of m^*. In Model A, the optimal basic solution remains unchanged as a 100% inspection policy is used. Note that a lower inspection cost produces the opposite effects (case 10).

• *Variation of the rectification cost*: When the rectification cost C_{rect} increases (case 11), the three models react by increasing the frequency of the periodic TBPM (i.e., m^* decreases) in order to improve the process quality. In models B and C, the CSP-1 plan becomes reduced so that more defectives are accepted (i.e., less rectification efforts). Thus, the optimal sampling fraction f^* decreases and the optimal clearance number i^* increases in order to lower the AFI_∞. In model C, the optimal CBPM threshold r^* increases due to the decrease of m^*. Note that the decrease in the rectification cost has the opposite effects (case 12).

• *Variation of the cost of accepting a defective item*: In models B and C, when the cost of selling a defective item C_{def} increases (case 13), the optimal sampling fraction f^* increases and the optimal clearance number i^* decreases so that the CSP-1 plan becomes tighter (as the AFI_∞ increases). This implies that quality inspection should be intensified in order to reduce the AOQ_∞. The frequency of TBPM actions slightly decreases (i.e., m^* increases) because the increase of quality control activities. In Model C, the optimal threshold r^* decreases to carry out the CBPM actions more frequently and to improve the outgoing quality. In Model A, the optimal basic solution remains unchanged as the 100% inspection involves defect-free products. Note that a lower cost of a defective item sold has the opposite effects (case 14).

• *Variation of the reliability deterioration rate*: When the deterioration of the production unit reliability increases (case 15), failure occurrence becomes more frequent. As a result, the three integrated models react by increasing the surplus inventory s^* to mitigate the higher risk of shortage and decreasing the optimal period m^* to perform the TBPM actions more frequently. Because more frequent TBPM improves the production quality, the optimal sampling fraction f^* decreases and the optimal clearance number i^* increases so that the optimal CSP-1 plan in both models B and C becomes reduced (i.e., the AFI_∞ decreases). In Model C, similar to the variation of the TBPM period m^*, the optimal threshold r^* decreases to carry out the CBPM actions more frequently. In addition, since the threshold r^* is basically used as a CSP-1 stooping rule and to assess the process quality, its significant sensitivity to the reliability deterioration shows that it also reflects the reliability of the production unit. This is because both quality and reliability deteriorations are operation-dependent. Finally, note that the decrease in the reliability deterioration rate produces the opposite effects (case 16).

• *Variation of the quality deterioration rate*: When the deterioration of the process quality increases (case 17), the three integrated models react by increasing the optimal sampling fraction f^* and decreasing the optimal clearing number i^* in order to tighten the CSP-1 plan (as the AFI_∞ increases) and to improve the outgoing quality. In addition, the optimal period m^* decreases to perform the TBPM actions more frequently and to enhance the process quality. Likewise, the optimal threshold r^* in Model C decreases to intensify the frequency of CBPM. In the three models, the surplus inventory s^* increases as a result of the increase of PM activities. The decrease in the quality deterioration rate has the opposite effects (case 18).

[Insert Table 5]

6.2. Influence of the AOQL constraint

Additional experiments have been conducted to analyze the influence of the AOQL constraint on models B and C. We should recall that Model A is insensitive to the AOQL constraint. Table 6 presents the optimal solutions of models B and C for different levels of the AOQL. The first observation from Table 6 is that, as expected, the optimal costs of both models B and C increase in response to the decrease in the AOQL and vice versa. When the AOQL is restricted (i.e., $AOQL <$ 2.0%), the optimal sampling fraction f^* increases and the clearance number i^* decreases such that the CSP-1 plan becomes tighter (i.e., AFI_∞ increases), and in order to improve the outgoing quality (i.e., AOQ_∞ decreases taking values less than the AOQL). In Model C, the optimal fraction r^* decreases in order to increase the frequency of CBPM actions and to improve the production quality. For a highly restricted AOQL, the optimal inspection policy of both models B and C leads to a near-100%-inspection policy (e.g., $AFI_\infty \geq 97.7\%$ for $AOQL \leq 0.1\%$).

When the AOQL is oppositely varied (i.e., increasing AOQL above 2.0%), the optimal sampling fraction f^* decreases in order to reduce the severity of the CSP-1 plan (i.e., AFI_∞ decreases). The optimal clearance number i^* firstly increases to compensate for the decrease in the sampling fraction f^* as the AOQL constraint is still active, and then it diminishes as the AOQL constraint becomes less and less restricting. From Table 6, the switch in the variation of the optimal clearance number i^* occurs at AOQL = 5.0% in Model B, and at AOQL = 4.5% in Model C. The optimal period m^* is first maintained at the same level while the AOQL constraint is active (m^* =12.3 in Model B, and m^* =17.1 in Model C), and, once the AOQL is less constrained, it climbs to a higher level in order to reduce the frequency of TBPM actions (m^* rises to 13.2 in Model B and to 19.4 in Model C). In Model C, similar to the reaction of i^* when the AOQL increases, the optimal threshold r^* first increases to reduce the CBPM actions while the frequency of the TBPM is maintained at the same level, and then it perversely decreases in order to perform more CBPM actions when the TBPM actions are less frequently performed (r^* rises up to 5.672% and then it starts decreasing to 4.668% for $AOQL \geq$ 8.0%). In both models, the optimal hedging level s^* decreases due to the reduction of the AFI_∞.

When the AOQL constraint becomes completely inactive for Model B (i.e., $AOQL \geq 7.0\%$), the optimal quality control policy leads to a near-no-inspection policy (0.46% of production inspected during sampling periods and only 3.1% of products are inspected in the long-run). This result is aligned with previous findings in the literature showing that the optimal CSP-1 plan with no AOQL constraint leads to either no-inspection or 100% inspection (Vander Wiel and Vardeman, 1994; Cassady et al., 2000). However, when the AOQL constraint becomes completely inactive for Model C (i.e., $AOQL \geq 8.0\%$), the CSP-1 plan is still relevant so that 5.0% of production is randomly inspected during sampling periods and more than 10.0% of production is inspected in the long run. This also means than about 10.0% of production should be at least inspected to monitor the products quality and to maintain the visibility of the process condition. In addition, we notice that r^* takes its smallest value 4.668% when the AOQL is greater than 8.0%. This shows the important role of the CBPM in determining the economic level of process quality and in improving the production unit reliability even when the AOQL constraint is inactive. All these results demonstrate the relevance of the strategy combining continuous sampling plans with stopping rules, CBPM and TBPM to optimally control quality inspection and maintenance activities.

[Insert Table 6]

6.3. Concluding remarks and comparison of the integrated models

From the preceding analyses (Sections 5.2, 6.1 and 6.2) and the experimental results in tables 5 and 6, we can draw the follow conclusions. **Firstly**, using the CSP-1 plan in integrated models is always more-cost effective than the 100% inspection. In fact, by applying models B and C, it is possible to economically determine the optimal level of quality inspection which is a combination of safety stock, PM and CSP-1 settings. This avoids the waste of excessive quality control (in the case of 100% inspection). For example, from Table 5, the inspection of product quality (i.e., AFI_{∞}) can be reduced by 25% to 50% while the AOQL is properly satisfied. **Secondly**, the parameters that mostly influence the amount of economic savings when the classical CSP-1 is employed rather the 100% inspection, Δ-B/A, are the AOQL, the PM cost, the quality related costs and the process deterioration functions. The economic savings Δ-B/A significantly increase as the AOQL constraint is less and less restrained (more than 25% of cost savings as in Figures 5.(a) and 5.(b)). Then, Δ-B/A reaches its maximum level once the AOQL constraint becomes completely inactive. **Thirdly**, additional economic savings, Δ-C/B, are achieved by using the CSP-1 with the stopping rule (r) rather the classical CSP-1. In fact, such a rule involves the incorporation of the CBPM into the PM policy, which reduces the waste of unnecessary TBPM actions. For example, from tables 5 and 6, the TBPM actions in Model C are on average 30% less frequent than those in Model B. Δ-C/B is mostly impacted by the AOQL and the costs of backlog, CM, PM and quality inspection. Figure 5 depicts the impact of different combinations of those parameters on Δ-C/B. We observe that significant cost savings (more than 5.0% as in Figures 5.(a) and 5.(b) and up to 10% as in Figures 5.(c) and 5.(d)) are particularly realized when the AOQL takes intermediate values (i.e., $0.5\% \leq AOQL \leq 4.5\%$). Δ-C/B is less important for highly restricted AOQL (i.e., $AOQL < 0.5\%$) as the CSP-1 leads to a near-100%-inspection plan, and also for reduced AOQL restriction (i.e., $AOQL > 4.5\%$) as the CSP-1 trends to a near-no-inspection policy.

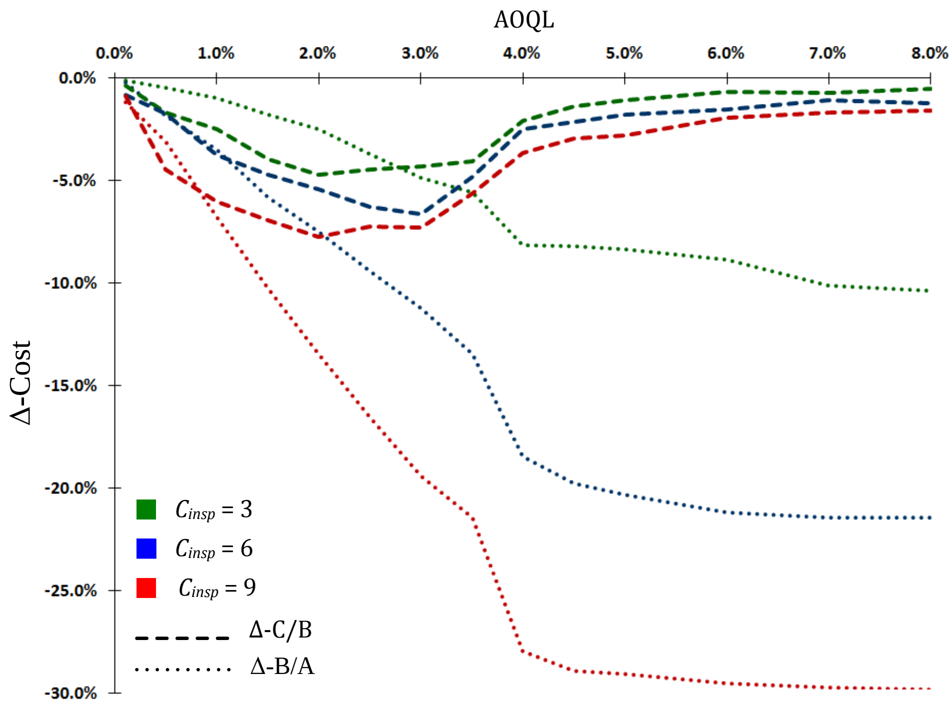

Fig. 5.(a). Cost comparison with different C_{insp} and *AOQL*

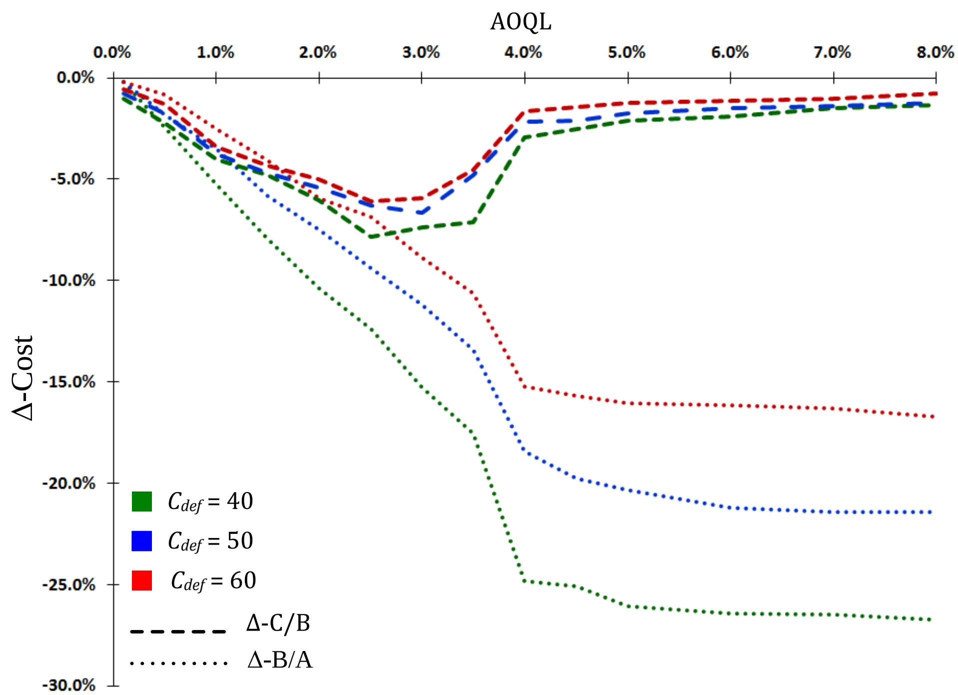

Fig.5.(b). Cost comparison with different C_{def} and *AOQL*

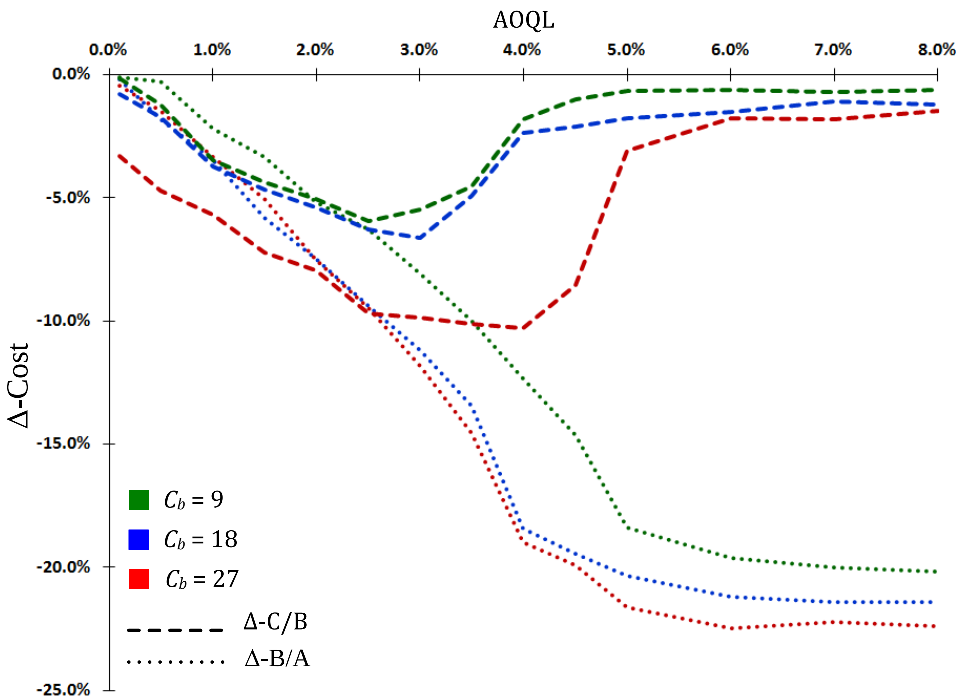

Fig. 5.(c). Cost comparison with different C_b and *AOQL*

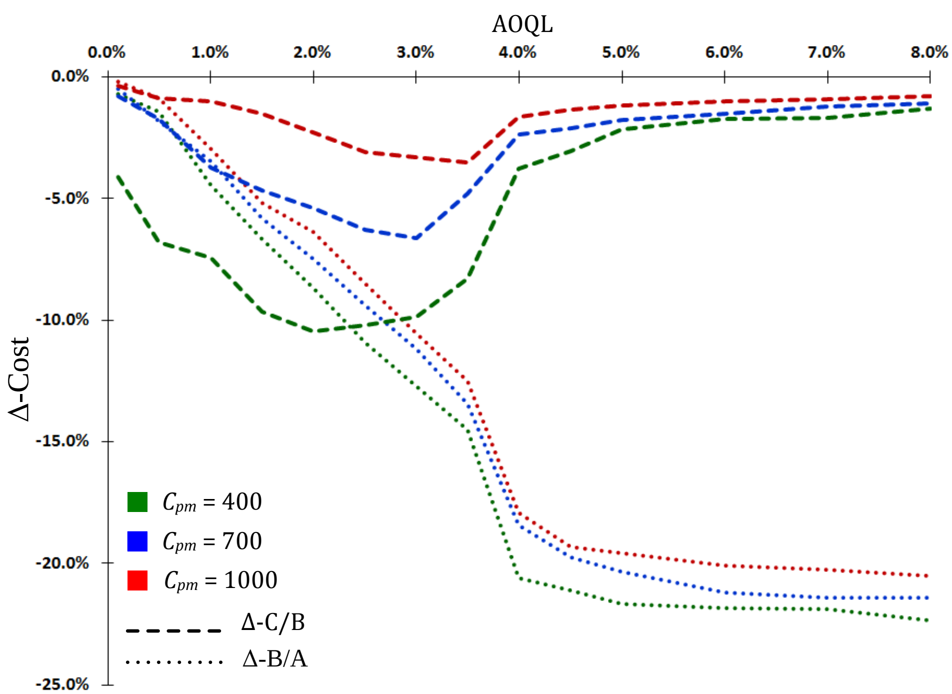

Fig.5.(d). Cost comparison with different C_{pm} and *AOQL*

7. Conclusion

In the literature, the design of continuous sampling plans has considerably evolved over the past three decades from design purely for quality requirements with no economic consideration to the economic design under quality constraints. Nevertheless, the existing continuous sampling plans models do not consider interactions with production, inventory and maintenance aspects. In this paper, we have developed new models to the joint economic design of type-1 continuous sampling plan, production, inventory and preventive maintenance for deteriorating production processes subject to an AOQL constraint. The proposed models contribute to research on integrated production, quality and maintenance in four ways. First, we have shown that continuous sampling plans can be used for deteriorating processes, provided that the interrelations with production, maintenance and process quality are fully considered in the design process of those plans. In practice, this finding should extend the application of continuous sampling plans to new industrial areas, as they are presently limited to stable production processes. Second, we demonstrated through arguments and experiments that using continuous sampling plans rather than the 100% inspection policy increases the overall operational performance and can realize important cost savings. Third, we have found that the CSP-1 with stopping rules is more effective for deteriorating processes, than the classical CSP-1. In fact, when a CSP-1 stopping rule is coupled with the CBPM, unnecessary TBPM actions are avoided and therefore additional cost savings are achievable. One advantage of this strategy lies in the fact that the CBPM based on quality information feedback does not require costly and advanced technology for data acquisition and analysis such as vibration, corrosion and acoustics analysis techniques. Quality information can easily be collected from the CSP-1 and interpreted to assess the process condition. Finally, another important contribution of this study lies in the effectiveness of the proposed modeling and optimization framework to tackle complex and highly stochastic optimization problems in integrated operations management.

The integrated production, CSP-1 and maintenance models proposed in this paper can be applied for continuous production systems subject to reliability and quality deteriorations, whose inspection is only performed at the end of production, and where both closed-loop production-inventory control and sampling plans are effective such as in the electronics and semiconductor industries (see Antila et al., 2008; Cao and Subramaniam, 2013; Mok, 2009). Managerial implications for implementing those integrated models require a real-time visibility and control of operations, WIP, finished products inventory, products quality and inspection rate. In addition, historical data related to the products quality should be properly recorded to manage the CSP-1 procedure, to monitor the production process and to schedule the CBPM actions. This is can be easily supported by modern computer software such as the Manufacturing Execution Systems (Kletti, 2007).

One limitation of our study is to assume that only finished products are inspected at the end of manufacturing operations. Nevertheless, inspection of intermediate products could reduce the total cost of poor quality and improve the outgoing quality. Possible extensions of this paper could be carried out to develop integrated production, sampling inspection and maintenance models for multistage manufacturing systems. Those models should address important design problems in multistage systems such as optimal inspection location, sampling plan optimization at each inspection point and optimal quality control of complex products with many attributes. Further research could be conducted to study more sophisticated continuous sampling plans such as the Dodge-Torrey's (1951) improvements of the CSP-1 plan (i.e., CSP-2 and CSP-3) and the multilevel continuous sampling plans as suggested by Lieberman and Solomon (1955). The main advantage of those plans is their ability to meet the AOQL requirement with less inspection effort than the CSP-1.

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Table 4. Comparison of the three optimal solutions

Model	Optimal solution					Optimal cost		Quality			Reliability/Maintenance			Inventory		
	s^*	m^*	i^*	f^*	r^*	$Cost^*$	Confidence Interval	ADP_∞	AFI_∞	AOQ_∞	FR_∞	FPM_∞	AV_∞	$E[x_q]$	$E[x_s^+]$	$E[x_s^-]$
Model A	54.8	16.46	-	-	-	455.7	[455.23, 457.21]	5.62%	100.0%	0.00%	0.1049	0.0568	0.852	0.23	41.5	1.3
Model B	51.2	12.30	17	0.4222	-	421.2	[419.43, 421.62]	5.45%	62.2%	1.53%	0.0992	0.0744	0.841	0.14	37.0	1.8
Model C	49.6	17.05	14	0.4838	5.421%	398.2	[396.14, 399.17]	4.37%	60.8%	1.45%	0.0662	0.1529	0.823	0.13	35.4	1.5

Table 5. Sensitivity analysis for cost and deterioration parameters

Case Number	Parameter	Variation	Model A			Model B							Model C								Cost differences	
			*s**	*m**	*Cost*	*s**	*m**	*i**	*f**	*Cost**	AOQ_∞	AFI_∞	*s**	*m**	*i**	*f**	*r**	*Cost**	AOQ_∞	AFI_∞	Δ-B/A	Δ-C/B
basic	**-**	**-**	**54.8**	**16.46**	**455.7**	**51.2**	**12.30**	**17**	**0.4222**	**421.2**	**1.53%**	**62.2%**	**49.6**	**17.05**	**14**	**0.4838**	**5.421%**	**398.2**	**1.45%**	**60.8%**	**-7.57%**	**-5.45%**
1	C_h	+50%	48.1	17.49	479.2	47.3	11.99	20	0.3699	442.4	1.59%	60.1%	45.4	16.63	19	0.3864	5.818%	411.6	1.73%	55.5%	-7.67%	-6.96%
2		-50%	61.6	15.57	429.4	55.2	12.63	14	0.4838	397.3	1.45%	64.7%	54.0	17.77	11	0.5566	4.854%	373.8	1.26%	65.2%	-7.49%	-5.91%
3	C_b	+50%	59.5	15.62	466.2	54.1	12.03	24	0.3118	429.7	1.67%	58.0%	53.0	16.69	15	0.4621	5.632%	404.0	1.51%	59.5%	-7.84%	-5.97%
4		-50%	40.9	19.29	438.6	43.2	13.13	7	0.6762	408.5	1.11%	74.6%	40.6	18.16	12	0.5309	5.052%	386.8	1.32%	63.6%	-6.86%	-5.30%
5	C_{cm}	+50%	55.8	15.41	502.9	52.1	11.53	18	0.4038	461.9	1.55%	60.2%	50.4	15.46	20	0.3699	5.634%	423.8	1.79%	54.6%	-8.16%	-8.26%
6		-50%	54.3	17.03	409.4	50.6	12.89	15	0.4621	379.8	1.49%	64.8%	49.0	18.18	7	0.6762	5.137%	375.1	0.99%	73.1%	-7.23%	-1.26%
7	C_{pm}	+50%	54.2	17.13	466.3	50.5	12.91	16	0.4416	434.0	1.50%	64.0%	49.0	18.28	10	0.5839	5.214%	424.5	1.19%	66.9%	-6.94%	-2.18%
8		-50%	56.2	15.20	445.5	52.1	11.65	18	0.4038	403.8	1.54%	60.3%	50.6	15.23	18	0.4038	5.594%	370.6	1.68%	56.3%	-9.35%	-8.24%
9	C_{insp}	+50%	54.8	16.46	516.2	51.5	12.00	31	0.2343	445.2	1.84%	55.1%	49.8	16.91	43	0.1481	6.161%	412.0	1.94%	47.9%	-13.75%	-7.46%
10		-50%	54.8	16.46	396.7	50.6	12.70	11	0.5566	386.7	1.34%	68.1%	48.9	17.48	9	0.6128	4.951%	368.6	1.13%	68.7%	-2.54%	-4.67%
11	C_{rect}	+50%	54.9	16.14	473.8	51.3	12.12	19	0.3864	431.1	1.57%	60.8%	49.8	16.88	16	0.4416	5.602%	401.4	1.58%	58.4%	-9.00%	-6.89%
12		-50%	54.7	16.76	439.5	51.0	12.49	15	0.4621	411.5	1.47%	63.7%	49.4	17.25	13	0.5067	5.259%	392.9	1.40%	62.1%	-6.38%	-4.53%
13	C_{def}	+50%	54.8	16.46	455.7	50.8	12.62	14	0.4838	430.9	1.45%	64.7%	49.1	17.59	11	0.5566	5.192%	410.0	1.26%	65.2%	-5.74%	-4.86%
14		-50%	54.8	16.46	455.7	51.6	11.97	21	0.3543	409.8	1.62%	59.4%	50.1	16.55	19	0.3864	5.676%	383.8	1.75%	55.5%	-10.37%	-6.35%
15	γ_r	+50%	66.1	14.51	528.9	63.1	10.18	22	0.3394	502.4	1.56%	55.9%	60.3	15.26	17	0.4222	3.626%	465.1	1.54%	56.4%	-5.01%	-7.44%
16		-50%	45.3	17.32	380.8	42.6	13.21	12	0.5309	348.4	1.51%	68.9%	40.7	22.26	11	0.5566	6.248%	337.7	1.34%	65.8%	-8.51%	-3.08%
17	γ_q	+50%	55.4	15.70	467.7	53.5	11.58	10	0.5839	436.2	1.61%	70.8%	51.7	16.44	8	0.6435	3.449%	421.7	1.48%	71.6%	-6.73%	-3.33%
18		-50%	54.3	17.14	443.4	50.8	12.96	25	0.2990	397.1	1.46%	50.8%	49.3	20.01	19	0.3864	8.025%	367.7	1.41%	52.4%	-10.43%	-7.41%

Table 6. Sensitivity analysis for the AOQL constraint

AOQL	Model B							Model C								Cost differences	
	s^*	m^*	i^*	f^*	$Cost^*$	AOQ_∞	AFI_∞	s^*	m^*	i^*	f^*	r^*	$Cost^*$	AOQ_∞	AFI_∞	Δ-B/A	Δ-C/B
0.1%	51.2	12.3	15	0.9590	454.2	0.08%	97.8%	49.6	17.0	12	0.9667	5.392%	450.7	0.07%	97.7%	-0.20%	-0.79%
0.5%	51.2	12.3	15	0.8142	446.9	0.39%	89.5%	49.6	17.0	12	0.8465	5.396%	439.1	0.35%	89.0%	-1.81%	-1.74%
1.0%	51.2	12.3	16	0.6527	439.3	0.78%	79.5%	49.6	17.0	13	0.7031	5.408%	422.8	0.72%	78.5%	-3.49%	-3.75%
1.5%	51.2	12.3	16	0.5348	428.6	1.16%	70.8%	49.6	17.0	13	0.5952	5.413%	408.5	1.07%	69.8%	-5.84%	-4.69%
2% (basic)	**51.2**	**12.3**	**17**	**0.4222**	**421.0**	**1.53%**	**62.1%**	**49.6**	**17.1**	**14**	**0.4838**	**5.421%**	**398.3**	**1.45%**	**60.8%**	-7.50%	-5.40%
2.5%	51.2	12.3	18	0.3296	412.5	1.90%	53.9%	49.6	17.1	16	0.3671	5.453%	386.6	1.92%	51.1%	-9.38%	-6.27%
3.0%	51.2	12.3	20	0.2399	404.2	2.29%	45.7%	49.6	17.1	17	0.2883	5.466%	377.4	2.34%	43.6%	-11.20%	-6.63%
3.5%	51.1	12.3	22	0.1707	394.0	2.70%	37.8%	49.6	17.1	21	0.2259	5.531%	375.0	2.63%	39.3%	-13.44%	-4.81%
4.0%	50.6	12.3	52	0.0197	371.3	3.98%	14.5%	49.4	17.2	31	0.0964	5.672%	366.2	3.60%	25.9%	-18.42%	-2.29%
4.5%	50.6	12.3	58	0.0092	365.3	4.43%	8.5%	48.0	19.3	33	0.0590	5.001%	357.6	4.25%	15.7%	-19.74%	-2.11%
5.0%	48.6	13.2	71	0.0039	362.7	4.59%	6.1%	48.0	19.3	29	0.0508	4.925%	356.2	4.27%	14.1%	-20.32%	-1.77%
6.0%	48.9	13.2	52	0.0044	358.7	4.69%	3.7%	48.0	19.4	24	0.0506	4.810%	354.1	4.38%	12.2%	-21.19%	-1.29%
7.0%	48.9	13.2	45	0.0046	357.7	4.73%	3.1%	48.1	19.4	21	0.0500	4.729%	353.3	4.38%	11.1%	-21.40%	-1.24%
≥8.0%	48.9	13.2	45	0.0046	357.7	4.73%	3.1%	48.2	19.4	18	0.0500	4.668%	353.9	4.43%	10.2%	-21.40%	-1.08%