

Acoustic imaging with conventional frequency domain beamforming and generalized cross correlation: a comparison study

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Abstract

Nowadays, acoustic cameras have become a standard tool for visualizing sound sources. The main post-processing techniques for generating acoustic images are associated with beamforming in the time or frequency domain. While many advanced techniques have been developed for the latter in order to achieve high resolution, such as deconvolution or inverse methods, the former have been less investigated. Recent studies have shown that time-domain beamforming can provide a narrow main lobe with low side lobe levels in different situations. However, a comparison of both techniques in terms of acoustic imaging does not exist. This paper provides a detailed comparison of conventional frequency-domain beamforming (CBF) and time-domain beamforming based on the generalized cross-correlation technique (GCC). First, numerical data are used for assessing both techniques. Then, experimental tests are carried out in a hemi-anechoic room using loudspeakers or power tools. As expected, acoustic images provided by CBF and GCC are found to be very similar but with different computation times. In order to improve the acoustic image, advanced techniques are considered (Clean-SC and the GCC based on geometric mean). When dealing with low frequency, Clean-SC is not able to detect both sources while the GCC based on geometric mean provides an accurate source separation.

Keywords: Acoustic imaging, beamforming, time domain, frequency domain, generalized cross-correlation, deconvolution technique

1. Introduction

Nowadays, many acoustic manufacturers propose acoustic cameras for visualizing sound sources. These tools include a microphone array, a camera and

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4 a data acquisition system. The digitized microphone signals are combined in
5 order to generate an acoustic image while the camera captures an image of the
6 environment under study. Then, both images are overlaid. The result is usually
7 a black and white image of the environment with filled colored contours. The
8 peaks of the contours exhibit the source positions and the colors provide the
9 sound levels. Acoustic imaging techniques have been used for a wide range of
10 applications from cars [1, 2, 3] to aircraft [4, 5, 6] or even snowmobiles [7]. If
11 the microphone array is set close to the object under study, typically less than
12 a wavelength away, the common technique is the near-field acoustic hologra-
13 phy [8, 9]. However, if the source is far from the microphone array, the classic
14 technique used is beamforming [10, 11]. In this work, the focus is made on the
15 comparison of the frequency and time domain beamforming.

16 The standard beamforming technique for generating an acoustic image, also
17 know as conventional beamforming (CBF) [12] is applied in the frequency-
18 domain. First, the Cross Spectral Matrix (CSM), obtained from a Fast Fourier
19 Transform of the microphone signals, is computed. The self noise of the micro-
20 phones may be removed by setting to 0 the diagonal elements of the CSM [13].
21 Then, a virtual focusing plane containing the source is considered which allows
22 for the computation of the steering vector [14, 15]. Finally, the CSM and the
23 steering vector are combined in order to generate the acoustic image. The two
24 main characteristics of an acoustic image are the main lobe, which indicates
25 the source position, and the side lobes which represent spurious sources posi-
26 tions [16]. The CBF is known as a fast and robust technique, but presents some
27 drawbacks when dealing with low or high frequencies. Indeed, the main lobe
28 width enlarges when frequency decreases, while the side lobe amplitude increases
29 with the frequency [17]. In order to tackle theses issues, improvements have been
30 proposed such as optimized geometries [18, 19, 20, 21, 22, 23] or advanced post-

31 processing techniques [24, 25, 26]. In terms of geometry, the common tradeoff
32 seems to be now the spiral arrangements for planar array [27], while for the
33 post-processing, deconvolution techniques or inverse problems become standard
34 techniques. The growing interest in acoustic imaging with CBF has been re-
35 cently highlighted by the publication of three review papers dealing with ad-
36 vanced post-processing techniques, array geometry or applications [24, 25, 26].
37 As mentioned by the Ref. [26], the major steps forward in acoustic imaging have
38 been performed in the frequency-domain although some time-domain beamform-
39 ing techniques exist.

40 The time-domain beamforming consists in delaying and summing the micro-
41 phone signals (called D&S) [12, 28, 29]. Unlike the CBF which is a narrow-band
42 technique (an acoustic image is obtained for each frequency bin), the D&S is a
43 broadband technique. Indeed, the whole microphone signals are delayed, there-
44 fore no frequency band is selected. In 2004, Dougherty has investigated the
45 influence of a pre-filter operation on microphone signals in order to select the
46 n^{th} octave band [30]. He also introduced a formulation for diagonal removal
47 and cross-shaped array for improving the acoustic image. However, the acous-
48 tic images provided by the time D&S and CBF were not compared. In 2006,
49 Jaeckel has discussed the strength and weakness of D&S beamforming [31].
50 The strength of D&S beamforming were the applicability to non-stationary and
51 strongly transient signals [32] as well as good resolution of the broadband sig-
52 nals. The weakness were the need for high sampling rate and the poor resolution
53 at low frequency.

54 Advanced signal processing algorithms are mainly implemented in the fre-
55 quency domain, few techniques exist in the time-domain [33, 34] and are inspired
56 by the well known frequency-domain technique Clean-SC [35]. Recently, stud-
57 ies have taken the advantages of the generalized cross-correlation formulation

58 (GCC) to improve D&S beamforming [36, 37]. The GCC has been introduced by
59 Knapp in 1976 [38]; this technique allows a pre-filtering operation by a weight-
60 ing function before computing the cross-correlation. The most known weighting
61 function is the PHase Transform (PHAT) which divides the cross spectrum of a
62 microphone pair by its absolute value [38]. When a broadband signal is consid-
63 ered, this weighting function narrows the main lobe of the cross-correlation and
64 therefore improves the acoustic image. Other improvement techniques based on
65 GCC have been proposed such as a spatial weighting [39], the use of the gen-
66 eralized mean [40] or enhanced weighting functions [41]. Each technique allows
67 for narrowing the main lobe and decreasing the amplitude of the side lobes. In-
68 verse method based on sparsity constraints have also been proposed to achieve
69 acoustic imaging with high resolution [42, 43, 44].

70 In 2014, Hamid *et al.* have compared the performance of the time and fre-
71 quency domain beamforming in terms of computation time [45]. However, from
72 the author’s knowledge no study has provided a detailed comparison of acous-
73 tic images given by both techniques. The CBF is considered as the standard
74 technique and advanced techniques are mainly based on its formulation. The
75 D&S beamforming is rarely used but seems to be more appropriated for some
76 cases. Although, the CBF and D&S beamforming should provide similar results,
77 their formulations are different as well as the advanced techniques. Therefore,
78 a detailed comparison of both techniques would allow for selecting the more
79 appropriated one.

80 The objective of this work is to propose a detailed comparison of both tech-
81 niques in terms of acoustic images. First, the theoretical background is intro-
82 duced in Section 2. Then, numerical data are used to compare the acoustic
83 images provided by both techniques (Section 3). Finally, experimental data are
84 considered in Section 4. The acoustic images are discussed in the simple case

85 of two loudspeakers. Then power tools are considered, a leaf blower (aeroa-
 86 coustic source) and a nail gun (impulsive source). A comparison of advanced
 87 post-processing is also discussed.

88 2. Theoretical background

89 2.1. Microphone signals

90 A source signal $s(\mathbf{r}_s, t)$ is generated by an omnidirectional acoustic point
 91 source located at \mathbf{r}_s and recorded by a set of M microphones at locations \mathbf{r}_m .
 92 The signal x_m recorded by the microphone m is given by

$$x_m(t) = s(\mathbf{r}_s, t - \Delta t_{ms}) + v_m(t), \quad (1)$$

93 where t represents time and $v_m(t)$ is an uncorrelated additive noise due to
 94 background or sensor noise. The term Δt_{ms} corresponds to the time of flight
 95 (ToF) between the source and the microphone and is defined by the Euclidean
 96 distance

$$\Delta t_{ms} = \frac{\mathbf{r}_{ms}}{c_0} = \frac{\|\mathbf{r}_m - \mathbf{r}_s\|_2}{c_0}, \quad (2)$$

97 where c_0 is the sound speed and $\|\cdot\|_p$ is the l_p -norm of a vector or a matrix.

98 The cross spectrum $C_{x_m x_n}(f)$ between the microphone signals x_m and x_n is
 99 given by

$$C_{x_m x_n}(f) = X_m(f)X_n(f)^H, \quad (3)$$

100 where X_m and X_n are the frequency domain microphone signals obtained with
 101 the Fast Fourier Transform (FFT) and the superscript $(\cdot)^H$ represents the Her-
 102 mitian transpose.

103 With the CBF, the cross and auto spectra are gathered into the so-called
 104 Cross Spectral Matrix (CSM), denoted \mathbf{C} , and are usually averaged using the
 105 Welch's periodogram [46] for removing background noise.

106 With the GCC, the cross spectra are used to recover the cross-correlation
 107 function using the Inverse Fast Fourier Transform (iFFT).

108 For each technique, a scan zone where the source positions is sought has to
 109 be defined. In the case of a planar microphone array, the scan zone is usually a
 110 plane regularly discretized with L points at position r_l .

111 2.2. Frequency-domain beamforming: Conventional Beamforming (CBF)

112 In the frequency domain, the beamformer output power $Z_l(f)$ for a frequency
 113 bin is given by

$$Z_l(f) = \mathbf{h}_l(f)^H \mathbf{C}(f) \mathbf{h}_l(f), \quad (4)$$

114 where $\mathbf{h}_l(f)$ is known as the steering vector and is a normalized Green's function
 115 between the microphone located at \mathbf{r}_m and the scan point at \mathbf{r}_l . In order to
 116 reduce the self-noise of the microphones, the diagonal elements of the CSM are
 117 set to 0.

118 There are several formulations of the steering vector in the literature [14],
 119 here one element of the steering-vector is

$$h_l(\mathbf{r}_m, f) = \frac{g_l(\mathbf{r}_m, f)}{\sqrt{M} \sqrt{\mathbf{g}_l(f)^H \mathbf{g}_l(f)}}, \quad (5)$$

120 which is referred as formulation IV in reference [14].

121 Note that the vector $\mathbf{g}_l(f)$ gathers elements from matrix $g_l(\mathbf{r}_m, f)$ that cor-
 122 respond to each microphone location \mathbf{r}_m . The vector $g_l(\mathbf{r}_m, f)$ is based on the
 123 free-field Green's function, thus frequency-domain beamforming makes the as-
 124 sumption of monopolar propagation,

$$g_l(\mathbf{r}_m, f) = \frac{r_{0l}}{r_{ml}} \exp(-jk(r_{ml} - r_{0l})), \quad (6)$$

125 where r_{0l} is the same as r_{ml} but using a reference microphone, usually the center
 126 microphone of the array and $k = 2\pi f/c_0$ represents the wavenumber. In the

127 following, the acoustic images provided by the frequency domain beamforming
 128 are referred as CBF.

129 *2.3. Time-domain beamforming: Generalized Cross-Correlation (GCC)*

130 When the speed of sound is known, the ToF between a microphone and a scan
 131 point can be computed. Each microphone signal is delayed by the corresponding
 132 ToF, which corresponds to steer the microphone array into the direction of the
 133 scan point. When the sum of the delayed microphone signals is maximized the
 134 source is localized. This technique is known as delay-and-sum beamformer and
 135 its output $y_l(t)$ can be expressed for the scan point l as

$$y_l(t) = \sum_{m=1}^M x_m(t + \Delta t_{ml}), \quad (7)$$

136 where Δt_{ml} is the Tof between microphone m and scan point l .

137 The beamformer output power $Y_l(t)$ is given by

$$Y_l(t) = E\{y_l^2(t)\} = \sum_{m=1}^M \sum_{n=1}^M R_{x_m, x_n}(\Delta t_{ml} - \Delta t_{nl}), \quad (8)$$

138 where $E\{\cdot\}$ is the mathematical expectation and R_{x_m, x_n} the cross-correlation
 139 function between the microphone signals x_m and x_n defined by

$$R_{x_m, x_n}(\tau) = E\{x_m(t)x_n(t + \tau)\}, \quad (9)$$

140 where τ is a time lag.

141 The cross-correlation function R_{x_m, x_n} is typically recovered from the iFFT
 142 of the cross spectrum $C_{x_m x_n}$ (Eq. 3) between microphone signals

$$R_{x_m, x_n}(\tau) = \sum_{f=0}^{N_f-1} W(f)C_{x_m x_n}(f) \exp\left(j2\pi \frac{f}{N_f} \tau\right), \quad (10)$$

143 where f is the frequency bin, N_f the number of frequency bins and $j = \sqrt{-1}$.
 144 A weighting function $W(f)$ can be used for improving the cross-correlation
 145 estimation which leads to the GCC [38, 47].

146 As the scan zone has a finite size, a peak time delay τ_{max} can be defined
 147 from all scan points which allows for selecting the values of the cross-correlation
 148 between $[-\tau_{max} : \tau_{max}]$. The peak time delay is given by

$$\tau_{max} = \max \left(\frac{1}{c_0} |\mathbf{r}_{ml} - \mathbf{r}_{nl}| \right), \quad (11)$$

149 with $|\cdot|$ the absolute value.

150 The projection of the cross-correlation function for a single microphone pair
 151 over the scan zone is called Spatial Likelihood Function [48] (SLF)

$$SLF = R_{x_m, x_n}(-\tau_{max} : \tau_{max}). \quad (12)$$

152 Commonly, the computation of the beamformer output power Eq. 8 is per-
 153 formed over the $M_p = M(M - 1)/2$ microphone pairs due to the symmetry of
 154 the cross-correlation matrix (redundant information) [49]. Moreover, the auto
 155 correlation terms (auto correlation of microphone signals) are removed because
 156 they do not bring any information to the acoustic image. These terms are known
 157 as "DC" component or self-noise [49]. Finally, the acoustic image is provided by
 158 the arithmetic mean of the SLF. In the following, the acoustic images provided
 159 by the time domain beamforming are referred as the GCC.

160 2.4. Detailed computation of CBF and GCC

161 For both acoustic imaging techniques, the first step is to gather the micro-
 162 phone signals into a matrix. Then, the scan zone is defined. With the CBF, the
 163 cross spectral matrix and the steering vector are computed. Then, the acoustic
 164 images are given by Eq. 4 or are averaged over the frequency band if required.

165 With the GCC, the microphone signals are filtered in the frequency band re-
166 quired. The SLF are computed for each microphone pair and averaged in order
167 to get the acoustic image. In the following, both techniques (CBF and GCC)
168 are compared with numerical and experimental data. The Figure 1 provides the
169 detailed computation of both techniques.

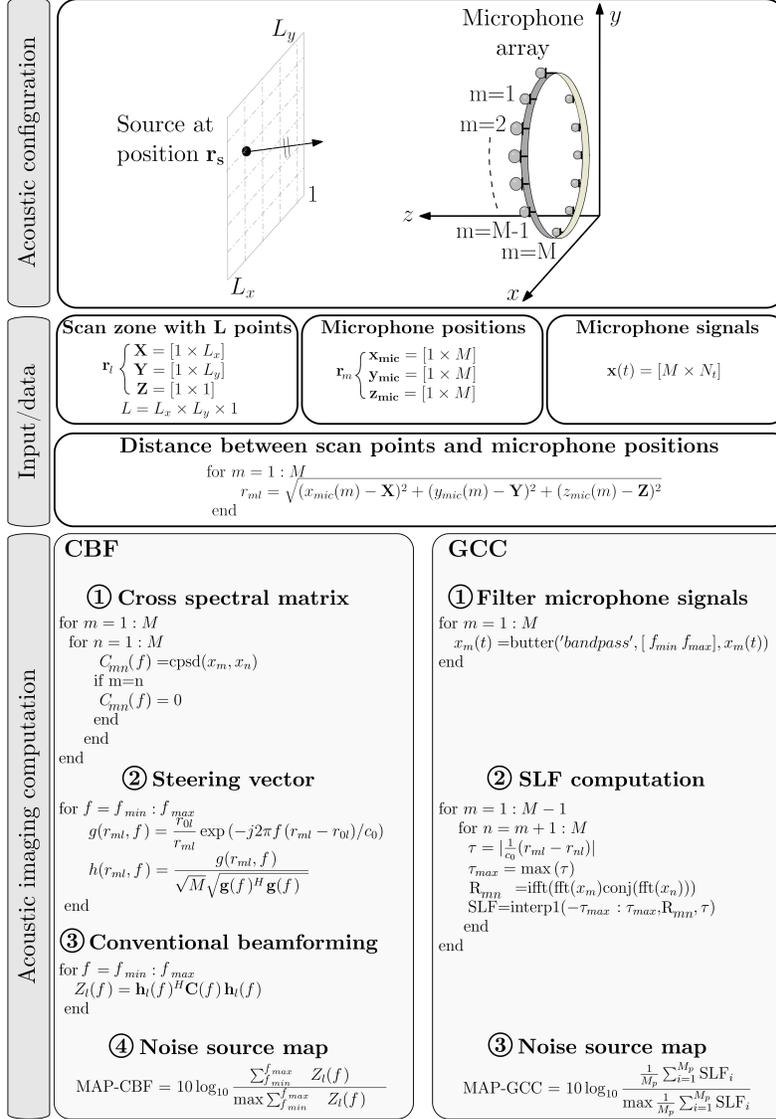


Figure 1: Detailed computation of the CBF and GCC.

170 2.5. Criteria for assessing an acoustic image

171 The result provided by both techniques is an acoustic image where a main
172 lobe should be located at the source position. Unfortunately, in addition to

173 the main lobe, the acoustic image also contains several side lobes (Figure 2.a).
174 Moreover, the size of the main lobe and the amplitude of the side lobes are
175 directly linked to the microphone array geometry, the source-array distance and
176 the source characteristics [16, 17].

177 In the past, criteria have been developed to assess the acoustic image. The
178 first criterion is the Main Lobe Width (MLW) at -3 dB which assesses the
179 ability to separate two sources [16]. The MLW can be expressed in centimeters
180 or degrees (Figure 2.b). In the case of a 2D acoustic image, this criterion is
181 obtained by slicing the map along the main directions. If the distance between
182 two sources is smaller than the MWL, the main lobes will be merged and the
183 two sources can not be separated. Therefore, the smaller the MWL, the better
184 the source localization. This criterion is also known as spatial resolution.

185 The second criterion is the Maximum Side lobe Level (MSL) which is given
186 by the amplitude difference between the main lobe and the nearest side lobe.
187 The MSL is expressed in decibel (dB). Again, in the case of a 2D acoustic image,
188 this criterion is obtained by slicing the map along the main directions. If the
189 side lobes amplitude is high, they could be interpreted as sources with lower
190 amplitude. Therefore, the higher the MSL, the better the source localization.
191 This criterion is also known as dynamic resolution.

192 An example of 2D acoustic image (for an arbitrary configuration with a single
193 source) is given in Figure 2.a where the main lobe indicates the source position.
194 This main lobe is surrounded by a side lobe. Secondary side lobes are also
195 present. To make the difference with the main side lobe, these secondary side
196 lobes are called spurious lobes in the following. To assess the acoustic image,
197 slices along the lines $x = 0$, $y = 0$ and $x = y$ are shown in Figure 2.b. In this
198 case, the MLW and MSL are independent of the slicing directions. However,
199 both criteria do not take into account the spurious lobe which have a higher

200 amplitude and are different for each direction. Therefore, although both criteria
 201 provide a useful information, they do not completely characterize the acoustic
 202 image especially the 2D aspect.

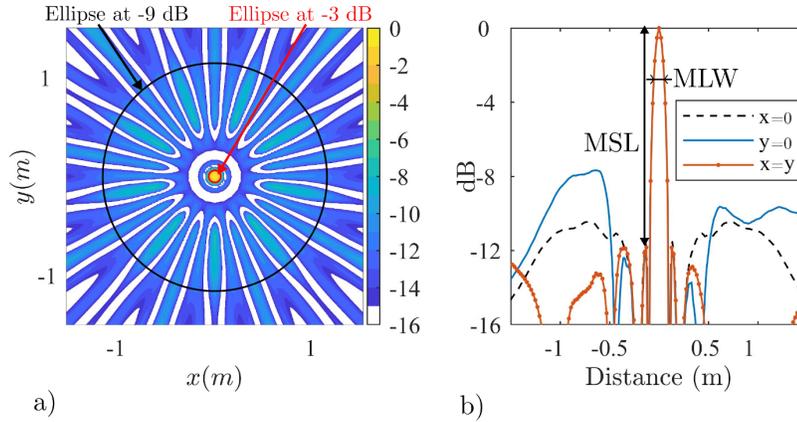


Figure 2: a) Example of acoustic image with ellipses at -3 dB (red) and -9 dB (black). b) Slices of the acoustic image along the lines $x = 0$, $y = 0$ and $x = y$. (color online)

203 To fully characterize the acoustic image, a criterion based on the surface of
 204 an ellipse is proposed. The goal is to surround the acoustic image values higher
 205 than a threshold. For instance, if the goal is to determine the spatial resolution,
 206 the threshold is set to -3 dB and the ellipse which surrounds all the higher values
 207 is searched. The main advantage is that the 2D aspect is taken into account.
 208 Again, the smaller the ellipse's surface, the better the source localization. If
 209 the threshold value is decreased, for instance up to -9 dB, the ellipse's surface
 210 will increase because the side lobes will be taken into account too. Finally, the
 211 ellipse's surface is also divided by the microphone array surface which provides
 212 a dimensionless criterion, called Ellipse Array Ratio (EAR). Low value of this
 213 criterion means a small surface ellipse and is expected for an efficient source
 214 localization. The EAR criterion has the advantage to provide an unique value
 215 to characterize either the main lobe or the side lobes no matter which direction

216 is considered.

217 Many methods exist to define an ellipse surrounding a set of data. In this
218 work the confidence ellipse is considered: first a threshold value has to be de-
219 fined, then the acoustic image values below this threshold are discarded. The
220 covariance matrix of this new data set is computed and the eigenvalues and
221 vectors are searched. The minor and major axes of the ellipse are given by the
222 eigenvalues and the orientation by the eigenvectors [50]. The 90% confidence
223 ellipses are shown in Figure 2.a with thresholds equal to -3 dB and -9 dB. The
224 value of 90% has been chosen to avoid influence of outliers. The first threshold
225 allows for surrounding the main lobe only while the second takes also into ac-
226 count the side lobes. This value has been chosen because beamforming is not
227 able to localize two sources with an amplitude difference larger than 9 dB (it is
228 typically the side lobe level). Finally, the EAR criterion is given by

$$EAR = \frac{S_{ellipse}}{S_{array}} \quad (13)$$

229 where $S_{ellipse}$ and S_{array} are the ellipse and microphone array surfaces, respec-
230 tively.

231 **3. Numerical comparison of the CBF and GCC**

232 *3.1. Single source*

233 The EAR criterion is now used to compare both techniques the CBF and
234 GCC in the case of a single source in front of a 16-microphones circular array.
235 The microphone array radius and the source-array distance range from 0.4 m
236 to 1.4 m and from 0.2 m to 1.6 m respectively. The source signal is a white
237 noise and the microphone signals are filtered in the 1000 Hz octave band. The
238 frequency sampling is 44,100 Hz and 1 s of source signal is used. The scan zone
239 is a square with 3 m sides and 40,401 points (201×201 grid). The EAR criterion

240 values at -3 dB and -9 dB obtained with the CBF and GCC are shown in Figure 3
 241 with respect to source-array distance and array radius. Both techniques provide
 242 similar trends. For instance, if the array radius is small, increasing the source-
 243 array distance increases the EAR criterion. Both techniques yield similar EAR
 244 criterion values which means that both techniques provide similar main lobe
 245 and side lobe levels.

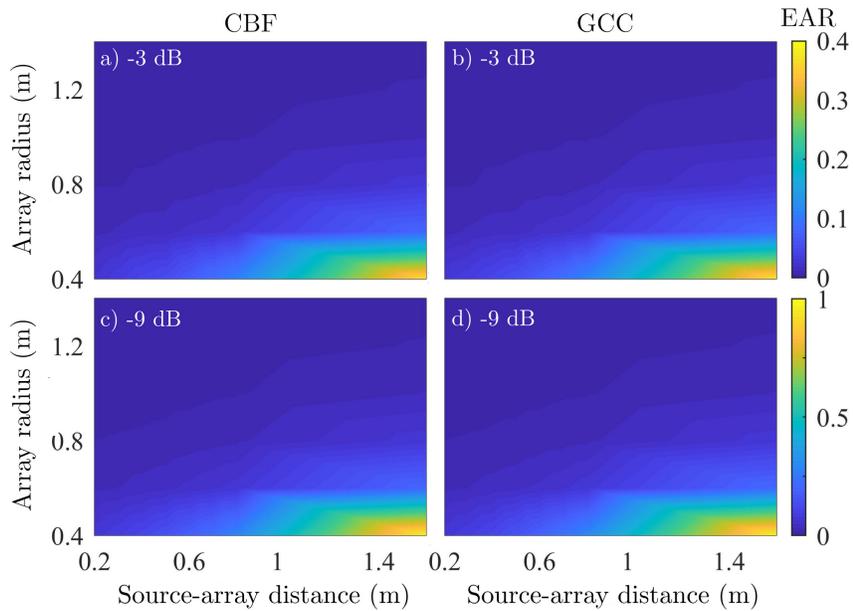


Figure 3: EAR criterion at -3 dB (a-b) and -9 dB (c-d) obtained with the CBF (left) and GCC (right). A 16-microphones circular array is used. The source signal is a white noise and the microphone signals are filtered in the 1000 Hz octave band. (color online)

246 As the EAR criterion is similar for both techniques at the 1000 Hz octave
 247 band, the acoustic images for octave bands ranging from 250 Hz to 4000 Hz are
 248 displayed in Figure 4 in order to visually assess the likely differences. Again, the
 249 acoustic images are very similar for both techniques, which provide the same
 250 EAR criterion for all the octave bands. The only difference is the spurious
 251 lobes below -13 dB which appear due to spatial aliasing [16]. For example, the

252 GCC acoustic image obtained for the 2000 Hz octave band (Figure 4.i) exhibits
 253 thinner spurious lobes than the CBF acoustic image (Figure 4.d). This first
 254 investigation allows for demonstrating that CBF and GCC similarly perform in
 255 terms of acoustic imaging in the case of a single source in front a microphone
 256 array.

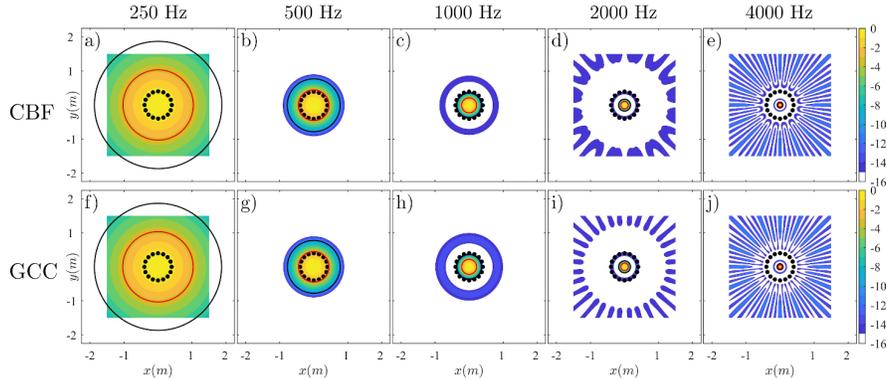


Figure 4: Acoustic images obtained with CBF (top) and GCC (bottom). The source signal is a white noise and the microphone signals are filtered in the 250 Hz (a-f), 500 Hz (b-g), 1000 Hz (c-h), 2000 Hz (d-i) and 4000 Hz (e-j) octave bands. A 16-microphones circular array is used. The red and black lines are the EAR criterion at -3 dB and -9 dB. The black dots denote the microphone positions. (color online)

257 3.2. Two sources with different amplitudes

258 In this section, the performance of both techniques is investigated in the case
 259 of two sources with different amplitudes. A 16-microphones circular array with
 260 0.4 m radius is used. The source array distance is 1.2 m. The source amplitude
 261 difference is 6 dB and the source positions are $x = -0.3$ m and $x = 0.3$ m (with
 262 $y = 0$). The signal duration is 1 s and the octave bands 1000 Hz and 2000 Hz
 263 are considered. The acoustic images obtained and the slices along x -axis are
 264 presented in Figure 5. The EAR criterion is not considered here because the
 265 main interest is the amplitude difference. Both techniques correctly identify

266 the two sources no matter the frequency. The side lobes amplitude below -
 267 12 dB are more important with the GCC but do not prevent an efficient source
 268 localization. The slices along the x -axis, Figure 5.c-f show that both techniques
 269 estimate similar source amplitude difference in both cases (octave bands 1000 Hz
 270 and 2000 Hz). In the octave band 1000 Hz, the source amplitude difference is
 271 slightly overestimated by both techniques, probably due to the presence of the
 272 side lobes. In the octave band 2000 Hz, the source amplitude difference is of
 273 6.2 dB for both techniques which is close to the initial value. Therefore, both
 274 techniques provide similar source amplitude difference estimation.

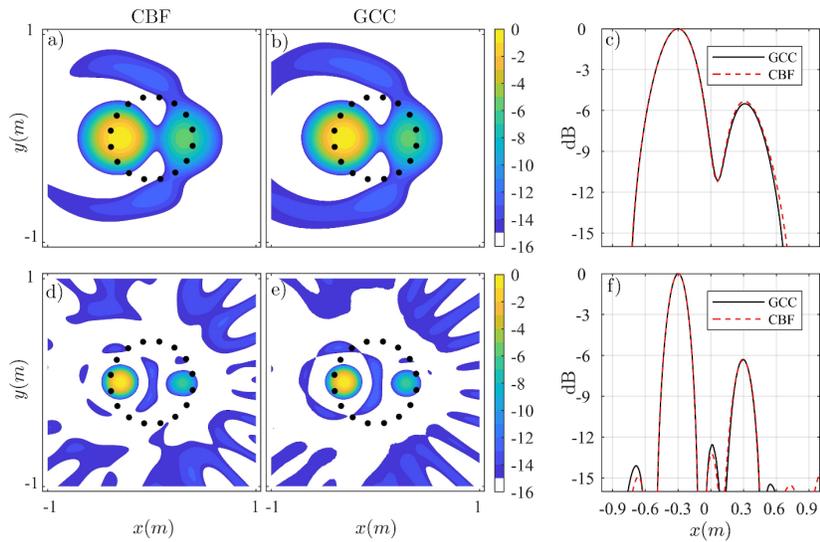


Figure 5: Acoustic images obtained with CBF (left) and GCC (center) for microphone signals filtered in the 1000 Hz (a-b) and 2000 Hz (d-e) octave bands. c-f) Slices along the line $x = 0$. A 16-microphones circular array is used. The black dots denote the microphone positions. (color online)

275 *3.3. Influence of the signal-to-noise ratio*

276 In this section, the performance of both techniques is investigated in the
277 case of different Signal-to-Noise Ratio (SNR). A 16-microphones circular array
278 with 0.4 m radius is used. The source array distance is 1.2 m. The signal
279 duration is 1 s and the octave band 2000 Hz is considered. The SNR ranges from
280 5 dB to -15 dB. The acoustic images obtained are presented in Figure 6. With
281 SNR=5 dB, the GCC acoustic image does not seem to be affected by the additive
282 noise while some spurious lobes appear around the main lobe with the CBF.
283 When the SNR decreases to -5 dB, the CBF acoustic image is contaminated by
284 the presence of random spurious lobes. On the other hand, the GCC acoustic
285 image is less affected. Decreasing the SNR to -15 dB, increases the spurious
286 lobe amplitude for both techniques. However, the spurious lobes amplitude is
287 larger with the CBF with values reaching -6 dB (as shown by the EAR criterion
288 at -9 dB).

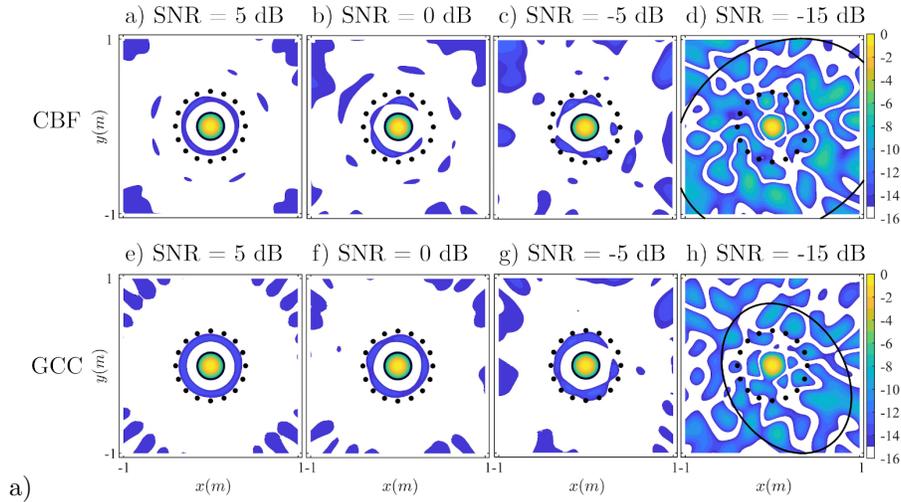


Figure 6: Acoustic images obtained with CBF (top) and GCC (bottom). The SNR are 5 dB (a-e), 0 dB (b-f), -5 dB (c-g) and -15 dB(d-h). The source signal is a white noise and the microphone signals are filtered in the 2000 Hz octave band. A 16-microphones circular array is used. The black lines are the EAR criterion at -9 dB. The black dots denote the microphone positions. (color online)

289 3.4. Computation time

290 In this section, the computation time of both techniques is compared. The
 291 algorithms are coded with Matlab using classical functions, no optimization
 292 toolbox such as parallel computing are used. The codes, partially described
 293 in Figure 1, were run onto a personal laptop (Intel i7-7600 at 2.8 GHz, 16 Go
 294 Ram).

295 First the influence of the number of scan points on the computation time is
 296 investigated. The array is circular with 16 microphones and only one frequency
 297 bin is used to compute the CBF (in order to avoid frequency averaging). The
 298 number of scan points ranges from 10 to 10^6 and the computation time is dis-
 299 played in Figure 7.a. In between 10 and 10^4 , the GCC is four times faster than
 300 the CBF. When the number of scan point is equal to $4 \cdot 10^5$, the computation

301 time is similar for both techniques. Above 4.10^5 points, the CBF becomes faster
302 than GCC; however the time difference is only 1 sec at 10^6 points.

303 Now, the number of scan points is set to 4.10^4 (200×200 grid size) and
304 the number of microphones ranges from 4 to 128. The computation time is
305 shown in Figure 7.b. The computation times are similar when the number
306 of microphones is equal to 10 but become largely different when the number
307 of microphones increases. For instance, with 32 microphones the CBF is four
308 times longer than the GCC. This fact can be easily explained by the computation
309 process of both techniques. Indeed, the GCC requires only the microphone pairs
310 ($M_p = (M \times (M - 1))/2$), while the CBF requires the square of the number of
311 microphones ($M^2 > M_p$).

312 Finally, the computation time of both techniques is compared when octave
313 bands are considered (from 250 Hz to 8000 Hz). In this case, the number of
314 microphones and scan points are 16 and 4.10^4 , respectively. The frequency
315 resolution (of the FFT) is an important parameter in this case because it de-
316 termines the number of averages required in the CBF calculation. An arbitrary
317 frequency resolution of 10 Hz is chosen and the result is displayed in Figure 7.c.
318 As expected, the CBF computation time increases with the octave band be-
319 cause the number of frequency bins, required for the average, increases. The
320 GCC computation time is the same for each octave band. Therefore, the GCC
321 is much more efficient than the CBF when acoustic images have to be displayed
322 for octave bands.

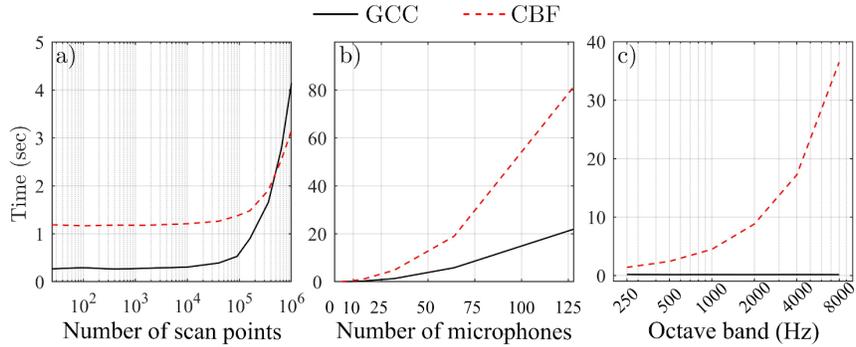


Figure 7: Computation times of the CBF (dashed red line) and the GCC (black line) versus a) the number of scan points, b) the number of microphones and c) the octave bands considered.

323 4. Experimental tests

324 The acoustic images provided by both techniques are now compared with
 325 experimental data obtained in a hemi-anechoic chamber. Various microphone
 326 array geometries and sources are considered. The aim is not to characterize
 327 the source behavior but to assess the performance of each technique for acoustic
 328 imaging. The colorbar is set to -10 dB to focus on acoustic imaging performance.

329 4.1. Two loudspeakers in front of a circular microphone array

330 During the first experiment, two loudspeakers (Eris E5 PreSonus), spaced
 331 by 50 cm were set in front of a 16-microphones circular array located at 1.2 m
 332 away. The loudspeakers signals were uncorrelated white noises with the same
 333 level and were generated with a PXI-4461 Sound and Vibration Module. The
 334 loudspeaker crossover frequency, which separates the signal of the tweeter (top)
 335 and the woofer (bottom), was of 3000 Hz. The acoustic signals were recorded
 336 with Brüel&Kjaer microphones type 4935 and 12-Ch input module type 3038B
 337 and sampled at 65,536 Hz. The microphone array aperture was 86 cm in both
 338 directions (see black dots in Figure 8.a). The scan zone, where the source

339 positions were sought, is a square with side equal to 1 m and containing 160,801
 340 points (401×401). The octave band 1000 Hz and 4000 Hz were considered.

341 The acoustic images obtained with the CBF and GCC are shown in Figure 8
 342 for both octave bands investigated. Overall, the acoustic images are similar and
 343 detect the woofer when the 1000 Hz octave band is considered or the tweeter
 344 when the 4000 Hz octave band is considered. The main lobe width and the
 345 side lobes level are also equivalent. In this case, both techniques yield similar
 346 results.

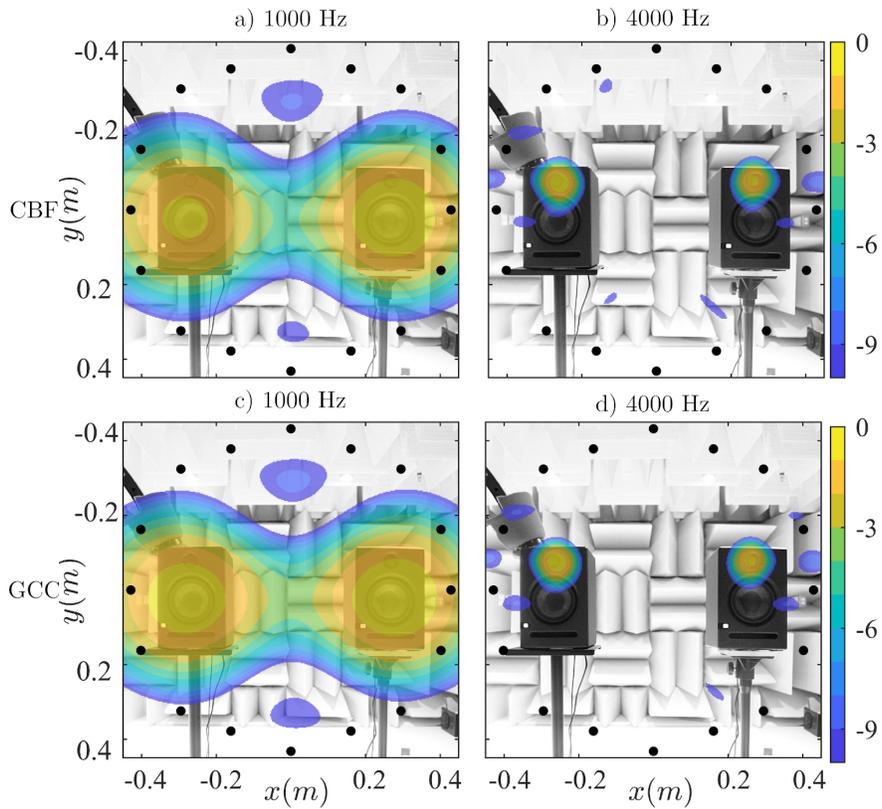


Figure 8: Acoustic images obtained with the CBF (top) and GCC (bottom). The source signals were white noises and the microphone signals were filtered in the 1000 Hz (a-c) and 4000 Hz (b-d) octave bands. A 16-microphones circular array in front of two loudspeaker with the same level was used. The black dots denote the microphone positions. (color online)

347 *4.2. Leaf blower in front of a spiral-arm microphone array*

348 The source considered now is a leaf blower which generates a stationary noise
349 with a more complex frequency content. This hand-tool generates a tonal peak
350 in mid frequencies (at 2631 Hz) and a broadband bump at higher frequencies
351 (between 3400 and 4000 Hz). The array geometry is multi-arms logarithmic spi-
352 ral shape with 41 microphones. First, the position of the source which generates
353 the tonal peak at 2631 Hz is sought (9.a-b). Both techniques indicate that the
354 position of this source is the air exhaust, the acoustic images are similar.

355 Then, the position of the source which generates the broadband bump is
356 sought (9.c-d). Again, the acoustic images are similar and point to the engine
357 as the main source with a secondary source at the air exhaust. Therefore in the
358 case of a real source, both techniques provide similar results even if the noise
359 radiated is tonal or broadband.

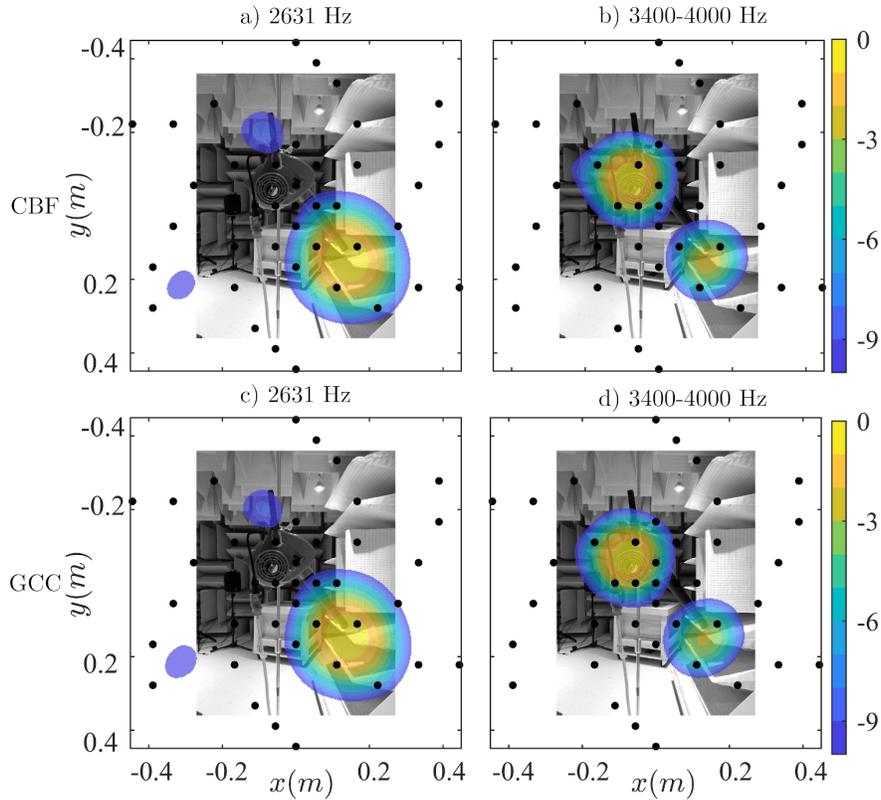


Figure 9: Acoustic images obtained with the CBF (top) and GCC (bottom). The microphone signals were filtered at 2631 Hz (a-c) and between 3400 Hz and 4000 Hz (b-d). A 41-microphones multi-arms logarithmic spiral array in front of a leaf blower was used. The black dots denote the microphone positions. (color online)

360 4.3. Nail gun in front of a spiral-arm microphone array

361 The source is now a pneumatic nail gun which typically generates impulsive
 362 noise. When the trigger nail gun is pulled up, the pressurized air is rushing
 363 in the chamber which moves down the piston and impacts the nail. Then, the
 364 piston goes up, the air is released by the exhaust at the top of the nail gun,
 365 and hits the high position. The noise generated by the exhaust is one of the
 366 main noise source of this nail gun. The noise frequency content being contained

367 in the high frequency, the frequency band considered ranges from 2000 Hz to
 368 5000 Hz [51]. The abilities of both techniques to localize the noise coming from
 369 the exhaust are compared in Figure 10. Both techniques provide similar results
 370 with a main lobe at the exhaust position.

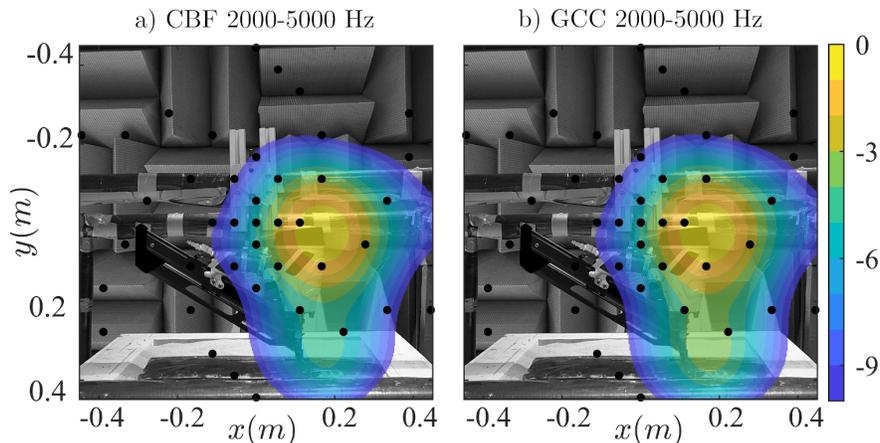


Figure 10: acoustic images obtained with the CBF (left) and GCC (right). The microphone signals were filtered between 2000 Hz and 5000 Hz. A 41-microphones spiral-arm array in front of a nail gun was considered. The black dots denote the microphone positions. (color online)

371 4.4. CBF and GCC with advanced techniques

372 It has been shown previously that CBF and GCC have a large main lobe
 373 for low frequency content or strong side lobes for higher frequencies (Figure 8).
 374 Many techniques have been developed for improving the acoustic image.

375 With the CBF, deconvolution methods have been proposed [24, 25, 26].
 376 Although these methods improve the acoustic image by narrowing the main
 377 lobe width or removing side lobe, the computation time and complexity are
 378 usually greatly increased. A full comparison of all the deconvolution techniques
 379 with the GCC is not possible here because the number of proposed methods is
 380 too large. The deconvolution method selected in this study is CLEAN-SC [35]

381 which is well known for its efficiency and short computation time.

382 This deconvolution technique is compared with the GCC based on the ge-
383 ometric mean with the weighting function ρ -PHAT-C. Although, other GCC
384 improvements exist [52, 53], the latter has been chosen due to its performance
385 with acoustic imaging [41]. Both improvements of the GCC are almost straight-
386 forward. Indeed, the geometric mean only modifies the average process of Eq. 10
387 by a product and the weighting function only divides the cross spectrum of the
388 microphone signal by its absolute value at the power ρ and coherence of the mi-
389 crophone signals. These improvements do not greatly increase the computation
390 time and complexity and are not based on iterative process. This technique is
391 called GEO- ρ -PHAT-C.

392 Both techniques are compared in the case of two loudspeakers in front of a
393 circular microphone array (Figure 11). First, the frequency band considered is
394 the 500 Hz octave band. In this case, the loudspeakers are too close and the
395 CBF and GCC are not able to separate them (Figure 11.a-c). The result is a
396 main lobe in between the loudspeaker created by the merging of the main lobes.
397 Although the frequency content is low, the GEO- $\rho = 1, 3$ -PHAT-C is able to
398 separate both loudspeakers and to identify the woofers as the main sources.
399 The CLEAN-SC technique being based on the peak value of the CBF exhibits
400 a source in between the loudspeakers. This technique is not able to separate
401 both sources.

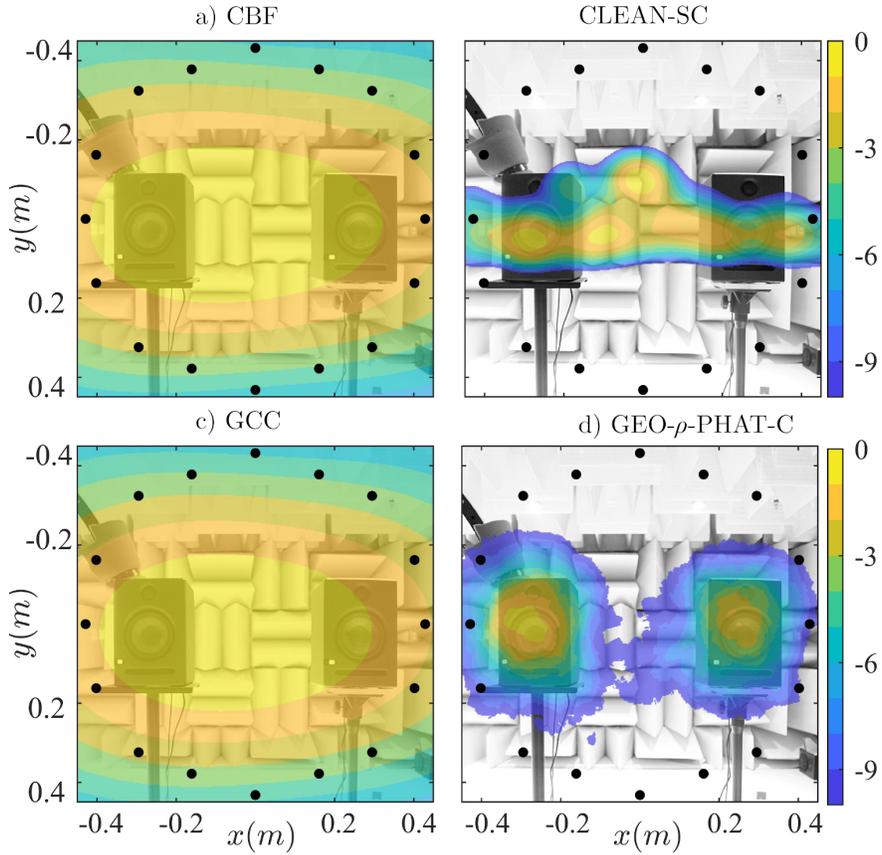


Figure 11: Acoustic images obtained a) CBF, b) CLEAN-SC, c) GCC and d) GEO- ρ -PHAT-C. A 16-microphones circular array in front of two loudspeaker was used. The microphone signals were filtered in the 500 Hz octave band. The black dots denote the microphone positions. (color online)

402 Then, the third octave band 3150 Hz is investigated. This frequency band
 403 is slightly larger than the cross over frequency between the woofer and tweeter.
 404 The CBF and GCC provide similar acoustic images with main lobes a little
 405 bit below the tweeter positions and strong side lobes (Figure 12). The GEO-
 406 $\rho = 1,3$ -PHAT-C perfectly detects both tweeters and removes all side lobes.
 407 The CLEAN-SC also removes the side lobes but identify the main sources such

408 as the CBF i.e. a little bit below the tweeter positions.

409 To conclude, the GEO- ρ -PHAT-C provides similar results than CLEAN-SC
410 but without using iterative process and without increasing the computation time
411 and complexity.

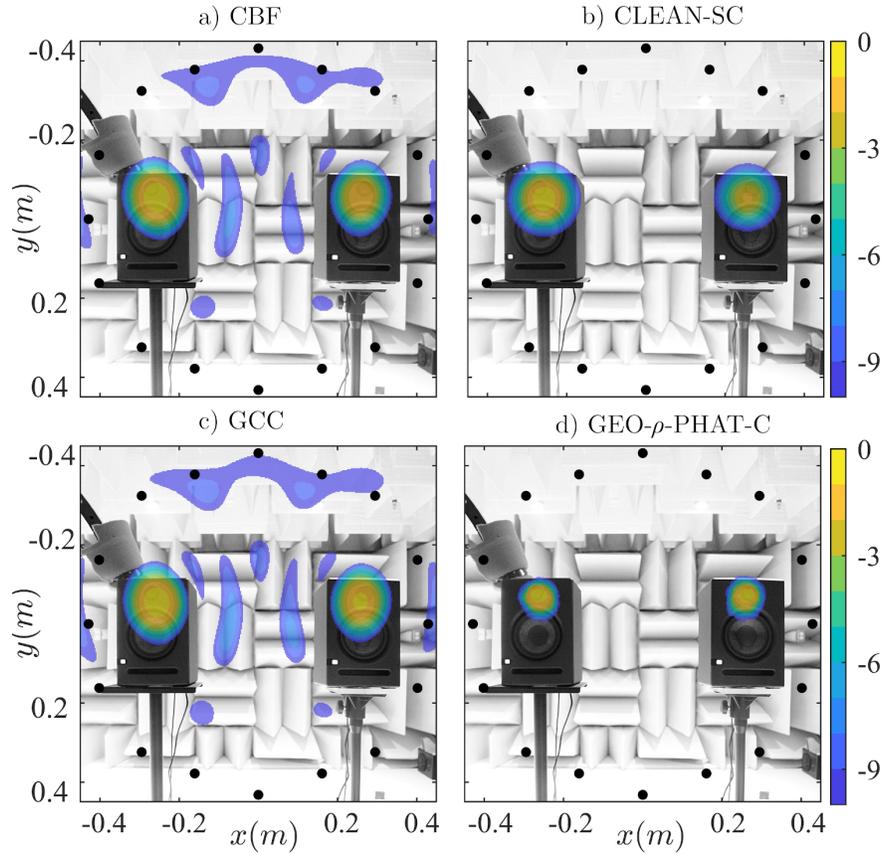


Figure 12: Acoustic images obtained a) CBF, b) CLEAN-SC, c) GCC and d) GEO- ρ -PHAT-C. A 16-microphones circular array in front of two loudspeaker was used. The microphone signals were filtered in the 3150 Hz third octave band. The black dots are the microphone positions. (color online)

412 5. Conclusion

413 Microphone arrays have become a standard tool to perform source localiza-
414 tion or acoustic imaging. The post-processing of the microphone signals can
415 be done in the time or frequency domain. In acoustic imaging, the standard
416 technique is the conventional beamforming (CBF) based on the cross spectral
417 matrix (frequency domain). However, acoustic images can be also obtained with
418 the time domain technique such as the generalized cross-correlation (GCC). The
419 objective of this work was to compare both techniques.

420 First, the computation of the CBF and GCC has been presented in details
421 in the first section. Then, a criterion based on a covariance ellipse, denoted
422 EAR, has been introduced to compare the acoustic images obtained numeri-
423 cally with each technique. This criterion allows for characterizing either the
424 main lobe surface or side lobe influence. The CBF and GCC provided similar
425 EAR criterion in the case of a single source. The abilities of both techniques
426 for estimating a source amplitude difference or for localizing a single source in
427 presence of an additive noise have also shown similar results. The main differ-
428 ence was the computation time which was faster with the GCC than the CBF
429 (at the exception of very large grids).

430 Then, experimental data were used in order to compare both techniques, the
431 sources were two loudspeakers, a leaf blower (stationary tonal and broadband
432 noise) or a nail gun (impulsive noise). Again, the acoustic images provided by
433 both techniques were similar. The main difference was when improved tech-
434 niques were used. The deconvolution technique CLEAN-SC (usually used with
435 the CBF) was compared with the GCC with the geometric mean associated to
436 an improved weighting function. Both techniques were selected for their fast
437 computation time. In the case of low frequency sources, the improved GCC
438 outperformed the CBF, GCC and CLEAN-SC and was able to separate them.

439 For higher frequency source, the improved GCC enhanced the noise source map
440 (as compared to GCC and CBF) in similar way than CLEAN-SC.

441 To conclude, the CBF and GCC provide similar performance, the main
442 differences lies in the computation time and the use of improved techniques.
443 A next step of this work would be a larger comparison of time and frequency
444 domain techniques based on sparse representation.

445 **References**

446 **References**

- 447 [1] U. Michel and B. Barsikow, Localisation of sound sources on moving vehi-
448 cles with microphone arrays. Proceedings of Euro-Noise, May 19–21 2003,
449 Naples, Italy, 2003
- 450 [2] U. Michel, B. Barsikow, P. Böhning and M. Hellmig, Localisation of moving
451 sound sources with phased microphone arrays. Inter-Noise, 22–25 August
452 2004, Prague, Czech Republic, 2004
- 453 [3] S.N.Y Gerges, W.D. Fonseca, W.D. and R.P. Dougherty, State of the art
454 beamforming software and hardware for application. 16th International
455 Congress on Sound and Vibration, July 5–9, Kraków, Poland (2009)
- 456 [4] P. Sijtsma and R. Stoker, R. Determination of absolute contributions
457 of aircraft noise components using fly-over array measurements. 10th
458 AIAA/CEAS Aeroacoustics Conference, May 10–12, Manchester, United
459 Kingdom, AIAA Paper 2004-2958, 2004
- 460 [5] R. Merino-Martinez, M. Snellen and D.G Simons Functional beamforming
461 applied to imaging of flyover noise on landing aircraft. *J. Aircr.* 53(6),
462 1830–1843, 2016

- 463 [6] C. Camier T. Padois, J. Provencher, P-A. Gauthier, A. Berry, J-F. Blais, M.
464 Patenaude-Dufour and R. Lapointe R. Fly-over source localization on civil
465 aircraft. 19th AIAA/CEAS Aeroacoustics Conference, May 27–29, Berlin,
466 Germany, AIAA Paper 2013-2261, 2013
- 467 [7] T. Padois and A. Berry Application of acoustic imaging techniques on
468 snowmobile pass-by noise. *J. Acoust. Soc. Am.* 141(2), EL134–EL139, 2017
- 469 [8] E. G. Williams, J. D. Maynard, and E. Skudrzyk. Sound source recon-
470 structions using a microphone array. *J. Acoust. Soc. Am.* 68 (1), 340–344
471 (1980).
- 472 [9] E. G. Williams and J. D. Maynard. Holographic Imaging without the wave-
473 length resolution limit. *Phys. Rev. Lett.* 45, 554–557 (1980).
- 474 [10] U. Michel, History of acoustic beamforming, 1st Berlin Beamforming Con-
475 ference, 2006.
- 476 [11] T.J. Mueller, Aeroacoustic measurements, Springer-Verlag, 2002.
- 477 [12] D.H. Johnson and D.E. Dudgeon, Array Signal Processing, Concepts and
478 Techniques. P T R Prentice Hall, Englewood Cliffs, 1993.
- 479 [13] T.F. Brooks and W.M. Humphreys, Effect of directional array size on the
480 measurement of airframe noise components, 5th AIAA Aeroacoustics Con-
481 ference, Bellevue, Washington, 1999.
- 482 [14] E. Sarradj, Three-dimensional acoustic source mapping with different
483 beamforming steering vector formulations, *Advances in Acoustics and Vi-*
484 *bration*, 1-12, 2012.
- 485 [15] J. Fischer and C. Doolan, Beamforming in a reverberant environment us-
486 ing numerical and experimental steering vector formulations, *Mech. Syst.*
487 *Signal Process.* 91,10-22, 2017

- 488 [16] J.J. Christensen and J. Hald, Technical Review Beamforming, Brüel&Kjær
489 Technical Review, 2004.
- 490 [17] A. Cigada, F. Ripamonti, M. Vanali, The delay & sum algorithm applied
491 to microphone array measurement: Numerical analysis and experimental
492 validation, *Mech. Syst. Signal Process.* 21 (6), 2645–2664, 2007.
- 493 [18] Z. Prime and C. Doolan, A comparison of popular beamforming ar-
494 rays, Proceedings of ACOUSTICS 2013—Victor Harbor, Australia, 17-20
495 November, 2013.
- 496 [19] Z. Prime, C. Doolan and B. Zajamsek Beamforming array optimisation and
497 phase averaged sound source mapping on a model wind turbine, InterNoise,
498 Melbourne Australia, 16-19 November, 2014.
- 499 [20] E. Sarradj, A generic approach to synthesize optimal array microphone
500 arrangements, 6th Berlin Beamforming Conference, 2016.
- 501 [21] W-H. Liao, Y. Mitsufuji, K. Osako and K. Ohkuri Microphone Array Ge-
502 ometry for Two Dimensional Broadband Sound Field Recording, 145th
503 convention Audio Engineering Society, New York, USA, 17-20 October,
504 2018.
- 505 [22] F. Ramos do Amaral, J. C. Serrano Rico and M. A. Faraco de Medeiros
506 Design of microphone phased arrays for acoustic beamforming, *Journal of*
507 *the Brazilian Society of Mechanical Sciences and Engineering*, July, 2018.
- 508 [23] E. Arcondoulis and Y. Liu An iterative microphone removal method for
509 acoustic beamforming array design, *J. of Sound and vib*, July, 442, 552-
510 571, 2019.
- 511 [24] Q. Leclère, A. Pereira, C. Bailly, J. Antoni, and C Picard, A unified formal-

- 512 ism for acoustic imaging based on microphone array measurements, *Int. J.*
513 *Aeroacoust.* 16(4–5), 431–456, 2017.
- 514 [25] R. Merino-Martinez, P. Sijtsma, M. Snellen, T. Ahlefeldt, J. Antoni, C. J.
515 Bahr, D. Blacodon, D. Ernst, A. Finez, S. Funke, T.F. Geyer, S. Haxter, G.
516 Herold, X. Huang, W.M. Humphreys, Q. Leclère, A. Malgoezar, U. Michel,
517 T. Padois, A. Pereira, C. Picard, E. Sarradj, H. Siller, D.G. Simons and
518 C. Spehr, A review of acoustic imaging methods using phased microphone
519 arrays, *CEAS Aeronautical Journal*, 1–34, 2019.
- 520 [26] P. Chiariotti, M. Martarelli and P. Castellini, Acoustic beamforming for
521 noise source localization - Reviews, methodology and applications, *Mech.*
522 *Syst. Signal Process.*, 120, 422–448, (2019).
- 523 [27] R.P. Dougherty and Y. Liu Spiral-shaped array for broadband imaging,
524 US5,838,284; 1998.
- 525 [28] J. Fischer, V. Valeau and L.E. Brizzi, Beamforming of aeroacoustic sources
526 in the time domain: An investigation of the intermittency of the noise
527 radiated by a forward-facing step *J. of Sound and vib*, 383, 464-485, 2016.
- 528 [29] H. Kook, . B. Moebs, P Davies and J. S Bolton, An efficient procedure for
529 visualizing the sound field radiated by vehicles during standardized passby
530 tests *J. of Sound and vib*, 233(1), 137-156, 2000.
- 531 [30] R.P. Dougherty, Advanced time-domain beamforming techniques, 10th
532 AIAA/CEAS Aeroacoustics Conference, American Institute of Aeronau-
533 tics and Astronautics, 2004.
- 534 [31] O. Jaeckel, Strengths and weaknesses of calculating beamforming in the
535 time domain, 1st Berlin Beamforming Conference, 2006.

- 536 [32] M. Bilodeau, N. Quaegebeur, O. Robin, P. O'Donoghue, P. Masson and
537 A. Berry, Time domain imaging of extended transient noise sources using
538 phase coherence J. Acoust. Soc. Am., 146(6), 4851–4859, 2019.
- 539 [33] D. Döblerand and R. Schröder, Contrast improvementand sound recon-
540 struction of quiet sound sources using a high dynamic range algorithm 4th
541 Berlin Beamforming Conference, 2012.
- 542 [34] R. Cousson, Q. Leclère, M.-A. Pallas and M. Bérengier, A time domain
543 CLEAN approach for the identification of acoustic moving sources, J. of
544 Sound and vib, July, 443, 47-62, 2019.
- 545 [35] P. Sijtsma, Clean based on spatial source coherence Int. J. Aeroacoustics,
546 6(4), 357–374, 2007.
- 547 [36] T. Padois, S. Sgard, O. Doutres, and A. Berry, Acoustic source localization
548 using a polyhedral microphone array and an improved generalized cross-
549 correlation technique, J. of Sound and vib, 386(6), 82–99, 2017.
- 550 [37] T. Padois, O. Doutres, S. Sgard, and A. Berry, On the use of geometric and
551 harmonic means with the generalized cross-correlation in the time domain
552 to improve noise source maps, J. Acoust. Soc. Am., 140(1), EL56–EL61,
553 2016.
- 554 [38] C.H. Knapp and G.C. Carter, The generalized correlation method for es-
555 timation of time delay, IEEE Trans. Acoust. Speech, Signal Process., 24,
556 320-327, 1976
- 557 [39] N. Quaegebeur, T. Padois, P-A. Gauthier and P. Masson, Enhancement of
558 time-domain acoustic imaging based on generalized cross-correlation and
559 spatial weighting, Mech. Syst. Signal Process., 75, 512-524, 2015.

- 560 [40] T. Padois, Acoustic source localization based on the generalized cross-
561 correlation and the generalized mean with few microphones, *J. Acoust.*
562 *Soc. Am.*, 15(2), EL393-EL398, 2018.
- 563 [41] T. Padois, O. Doutres and F. Sgard, On the use of modified phase trans-
564 form weighting functions for acoustic imaging with the generalized cross
565 correlation, *J. Acoust. Soc. Am.*, 145(3), 1546-1555, 2019.
- 566 [42] J. Velasco, D. Pizarro and J. Macias-Guarasa, Source localization with
567 acoustic sensor arrays using generative model based fitting with sparse
568 constraints, *Sensors*, 12, 13781-13812, 2012.
- 569 [43] C. Noël, V. Planeau and D. Habault. A new temporal method for the
570 identification of source directions in a reverberant hall, *J. of Sound and*
571 *vib*, 296(3), 518-538, 2006.
- 572 [44] T. Padois, O. Doutres, F. Sgard and A. Berry, Time domain localization
573 technique with sparsity constraint for imaging acoustic sources, *Mech. Syst.*
574 *Signal Process.*, 94, 85-93, 2017.
- 575 [45] U. Hamid, R. A. Qamar and K. Waqas Performance Comparison of Time-
576 Domain and Frequency-Domain Beamforming Techniques for Sensor Array
577 Processing Proceedings of 11th International Bhurban Conference on Ap-
578 plied Sciences & Technology (IBCAST) Islamabad, Pakistan, 14th-18th
579 January, 2014
- 580 [46] A.V. Oppenheim, R.W. Schaffer, J.R. Buck, *Discrete-time signal processing*,
581 Prentice-hall Signal Processing Series, 1999.
- 582 [47] T. Padois, O. Doutres, F. Sgard and A. Berry, Optimization of a spherical
583 microphone array geometry for localizing acoustic sources using the gener-

- 584 alized cross-correlation technique *Mech. Syst. Signal Process.*, 132, 546-559,
585 2019.
- 586 [48] P. Aarabi. The fusion of distributed microphone arrays for sound localiza-
587 tion, *EURASIP Journal on Applied Signal Processing*, 338-347, 2003.
- 588 [49] J.P. Dmochowski, J. Benesty and S. Affes. A generalized steered response
589 power method for computationally viable source localization, *IEEE Trans-*
590 *actions on Audio, Speech, and Language Processing*, 15(8), 2510-2526,
591 2007.
- 592 [50] R. A. Johnson and D. W. Wichern, *Applied multivariate statistical analysis*,
593 6th Ed., Upper Saddle River, NJ : Pearson Prentice Hall, Chap 2.3, 60-65,
594 2007.
- 595 [51] T. Padois, M-A. Gaudreau, P. Marcotte and F. Laville Identification of
596 noise sources using a time domain beamforming on pneumatic, gas and
597 electric nail guns *Noise Control Engr. J.* 67 (1), January-February 2019
- 598 [52] F. grondi and J. Glass SVD-PHAT: A fast sound source localization method
599 *IEEE International Conference on Acoustics, Speech and Signal Processing*
600 (ICASSP), May, 2019
- 601 [53] V. C. Raykar, R. Duraiswami, B. Yegnanarayana and S.R. Mahadeva
602 Prasanna Tracking A Moving Speaker using Excitation Source Informa-
603 tion *Eurospeech*, Geneva, 2003