# Considering human error in optimizing production and corrective and preventive maintenance policies for manufacturing systems

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**Abstract**

This article extends previous work on co-optimization of production and corrective and preventive maintenance including lockout/tagout. We study the impact of human error on repairable manufacturing systems subject to random failure over an infinite planning horizon and its implications for system capacity and inventory policies, and we derive an optimal policy for minimizing production cost based on machinery maintenance and inventory management while meeting market demand over an infinite horizon. A numerical example is provided to demonstrate the usefulness of the proposed approach and a sensitivity analysis is presented to confirm the efficiency of the control policy.

**Keywords:** human error, lockout/tagout, corrective maintenance, preventive maintenance, production control

## 1. Introduction

Production planning has become an important area of operations management over the past three decades[1]. This is due in large part to its ever-increasing importance in today's highly competitive business environments. Production planning and control involves solving challenging optimization problems that combine strategic and operational decisions regarding production, corrective and preventive maintenance, and stock levels[2]. Such optimization is particularly relevant in manufacturing environments made uncertain by a variety of stochastic factors such as operational activities, maintenance, injury, raw materials and demand. Implicit in optimal planning is protection of employees against the risk of injury associated with activities such as operations and maintenance. One of the procedures developed to make servicing and repair of machinery safer is lockout/tagout or LOTO. For a variety of reasons, compliance with this procedure remains poor in some manufacturing businesses, where it may be regarded as non-productive. Despite the extensive literature on flexible manufacturing systems(FMS), only a few studies have addressed the integration of LOTO into production control in stochastic environments. Although considering this integration into a manufacturing system makes the study more realistic, it complicates the optimal control problem. Incorporating accident prevention techniques such as LOTO into equipment servicing and repair has been proven to increase occupational safety significantly [3].

The economic importance of responding quickly to customer requirements has fuelled interest in production optimization. In uncertain environments, optimization is effective only if it reduces the total cost of production while increasing occupational safety[3]. Some authors attach more importance to techno-economic aspects, that is, controlling parameters that determine production and the frequency of maintenance to meet demand more reliably [4,5,6,7], while others focus on occupational health and safety, that is, preventing accidents and work-related illness by eliminating or controlling hazards [8]. To estimate the impact of maintenance on occupational risks in Quebec, the Institut de Recherche Robert Sauvé en Santé et Sécurité du Travail (IRSST) has studied fatal accidents in the province between 1999 and 2003 [9] using data available at the Commission de la Santé et de la Sécurité du Travail (CSST). According to this report, 1275 deaths resulted from workplace accidents during this period, of which 163 (13%)

occurred during maintenance activities such as machine upgrading, testing, troubleshooting, inspection, modification, repair and monitoring. This study made it clear that a significant proportion of workplace fatalities in manufacturing settings was associated with maintenance activities, which highlights the importance of applying LOTO and hence of considering it in planning models.

Several methods have been used to determine optimal control policies for FMS control problems. These include the Kushner algorithm [10], artificial intelligence based on genetic algorithms [11], heuristics [12] and simulation [13]. Rishel [14] has identified the sufficient and necessary conditions for optimizing a finite-state Markov process using dynamic programming. Older and Suri [15] have extended Rishel's formalism by presenting FMS production planning optimality as a problem of stochastic control subject to random failure. They obtained dynamic programming equations of optimal control, while the differential equations remained unsolved due to their complexity. Kimemia and Gershwin[16] modelled stochastic manufacturing systems using homogeneous Markov processes with a constant transition rate. They determined production policies in which inventory and order backlog costs are minimized based on the rate of production by the machines. Akella and Kumar [17] used homogeneous Markov chains to model a manufacturing system consisting of one machine producing one part type, and Hamilton-Jacobi-Bellman (HJB) equations to determine an optimal control policy, namely the Hedging Point Policy (HPP). In the case of complex manufacturing systems, no analytical solution to HJB equations is currently possible. To find the optimal solution to the stochastic control optimization problem, Yan and Zhang [18] used a numerical method based on the Kushner approach [10] to manufacturing systems producing several parts. Addressing realistic manufacturing systems, Kenné and Nkeungoue [19] modeled machine failure and repair using non-homogenous Markov processes, showing that machine failure probability increases with machinery age. While several studies have provided adequate descriptions of the theoretical basis of control system optimization, the impact of human error (e.g. carelessness, forgetfulness, inattentive or reckless behaviour) on production costs in manufacturing settings needs closer examination. Human error and its frequency during maintenance activities depend significantly on the machinery under repair and, in principal, on the type of industrial sector. Occupational safety researchers have confirmed in several studies the importance of controlling accident and incident factors during maintenance activities [3,13]. This raises two important research questions: *1) How can human safety policies in manufacturing environments be improved?2) What is the impact of human error during maintenance activities on production costs?*

One possible answer to the first question is rigorous application of LOTO procedures, which consist of locking a machine with a padlock after discharging all residual energy sources (electrical, hydraulic, etc.). LOTO prevents premature starting of equipment during a maintenance intervention. It is unfortunate that some companies allow their employees to bypass LOTO procedures during equipment maintenance. This alone explains much of the greater number of accidents that these companies experience[20]. In answer to the second question, Emami-Mehrgani et al. [13,21] have studied two analytical models combining production, LOTO and corrective maintenance policies in a passive redundancy system. They examined a manufacturing system consisting of two non-identical machines with passive redundancy and a system consisting of three non-identical machines (a third in series with the previous two), producing one part type. They illustrate that passive redundancy optimizes production and maintenance costs while increasing occupational safety. In many manufacturing scenarios, human participation is a critical element affecting system performance and reliability. Ferguson et al. [22] and Mason and Rushworth [23] have shown that human error decreases safety and can cause damage to machinery or reduce machine reliability. Njike et al. [24] have demonstrated that the frequency of machine failure is related not only to machine age but also to machine operator performance. The second question nevertheless remains a production control and operations management issue. In this article, we address this issue in a way that is more relevant to a repairable manufacturing system. We consider a system consisting of one machine producing one part type with random failure. We develop a non-homogeneous Markov process in which human error during maintenance activities including LOTO is integrated over an infinite horizon. The control optimization problem consists of determining policies that justify maintenance activities including LOTO and optimize production rate given the changing states of a machine over an infinite horizon. We formulate our system as a stochastic dynamic programming problem. Obtaining the optimal solution analytically remains a challenge for this type of problem. We illustrate the corresponding optimality conditions using a set of HJB equations, which are difficult-to-solve non-linear partial differential equations. We further provide some numerical examples to obtain a solution to the problem.

This article is organized as follows: Section 2 presents the notations and assumptions. Section 3 describes the research problem. Section 4 provides a numerical example and sensitivity analysis. In section 5,a discussion of the results obtained using our approach is provided. Our conclusions are presented in Section 6.

## 2. Assumptions and notations

### *2.1. Assumptions*

The following assumptions apply throughout this article:

1. ALL maintenance, corrective or preventive, is carried out with LOTO.
2. Corrective and preventive maintenance may be carried out with or without human error.
3. Mean corrective and preventive maintenance time without human error is shorter than mean corrective and preventive maintenance time with human error.
4. Mean preventive maintenance time is shorter than mean corrective maintenance time.
5. After each corrective or preventive maintenance activity, machine performance is restored to the level of the brand new machine.
6. Frequency of machine failure is a continuous function of machine age. It remains undisturbed by corrective or preventive maintenance.

### *2.2. Notations*

Throughout this article, the following notations are used:

x : inventory level

$d$ : demand

$\alpha$ : machine state or mode

$a$ : machine age

$\rho$ : discount

$J(.)$: total cost

$\Gamma(\alpha)$: set of admissible decisions applicable to mode $\alpha$

$\Theta(\cdot)$: system mode $\alpha \in A$

$A$: set of machine states or modes

$c^{+}$: manufactured unit holding cost per unit of time

$c^{-}$: order backlog unit cost per unit of time

$c_r$: corrective maintenance cost

$c_{mp}$: preventive maintenance cost

$c_{tagout}$: LOTO cost

$c^{\alpha}$: maintenance cost

$u^{\max}$: maximal production rate

$Q(.)$: transition matrix

$q_{\alpha\beta}$ : rate of mode transition from $\alpha$ to $\beta$

$k_{21}$: frequency of corrective maintenance without human error

$k_{21}^{\min}$ : minimal frequency of corrective maintenance without human error

$k_{21}^{\max}$ : maximal frequency of corrective maintenance without human error

$k_{41}$: frequency of corrective maintenance with human error

$k_{41}^{\min}$ : minimal frequency of corrective maintenance with human error

$k_{41}^{\max}$ : maximal frequency of corrective maintenance with human error

$k_{13}$: frequency of preventive maintenance without human error

$k_{13}^{\min}$ : minimal frequency of preventive maintenance without human error

$k_{13}^{\max}$ : maximal frequency of preventive maintenance without human error

$k_{35}$ : frequency of preventive maintenance with human error

$k_{35}^{\min}$ : minimal frequency of preventive maintenance with human error

$k_{35}^{\max}$ : maximal frequency of preventive maintenance with human error

$h_x$ : increment of the finite difference interval of variable $x$

$h_a$ : increment of the finite difference interval of variable $a$

$v_x(\cdot)$ : value function given a small increment in the direction of $x$

$v_a(\cdot)$ : value function given a small increment in the direction of $a$

$v^h(\cdot)$ : approximation of the value function given a small increment in the direction of $x$ and $a$

**3. Problem statement**

The system under study consists of one machine producing one part type. The machine is subject to random breakdown and repair, is either operational or shutdown, and shall be considered as a production unit installed in a facility that manufactures and stocks the part. LOTO is applied during all maintenance activities, meaning that the machine is always locked for corrective and preventive maintenance and unlocked thereafter. Three situations thus arise: the machine is operational; the machine is under repair (corrective maintenance); the machine is being serviced (preventive maintenance). The effect of human error during maintenance activities is modelled (corrective and preventive maintenance with human error), meaning that an improperly conducted maintenance activity leading to repeating the activity before resuming production is counted. We describe the machine capacity using a finite-state Markov chain. Based on the above description of the system, the machine has five states: $\xi(t)=1$ if the machine is operational, $\xi(t)=2$ if the machine is in corrective maintenance without human error, $\xi(t)=3$ if the machine is in preventive maintenance without human error, $\xi(t)=4$ if the machine is in corrective maintenance with human error, and $\xi(t) = 5$ if the machine is in preventive maintenance with human error.

The manufacturing system dynamics are hybrid and consist of a discrete element $\xi(t)$ that describes machine state and a continuous element $x(t)$ that represents inventory level. The difference between cumulative production and demand, $x(\cdot)$,can be positive if inventory costs $c^{+}$ have been considered, or negative if backlog costs $c^{-}$ have been considered. The dynamics are described as follows:

$$\begin{aligned} &\frac{dx}{dt} = u(\cdot) - d, \quad x(0) = x, \\ &\frac{da}{dt} = f(u(\cdot)), \quad a(0) = a, \\ &a(T) = 0 \end{aligned} \tag{1}$$

where $u(\cdot)$, $d$, $x$, $a$, and $\mathrm{T}$ are respectively production rate, demand, initial surplus, initial machine age, and most recent machine restart time, and $f(u(\cdot))$ is machine aging expressed as an increasing function of production rate.

The discrete part of the hybrid state $\xi(t)$ of the manufacturing system is a time-continuous Markov process taking a value in $A = \{1, 2, 3, 4, 5\}$ where $A$ presents the set of machine states or modes.

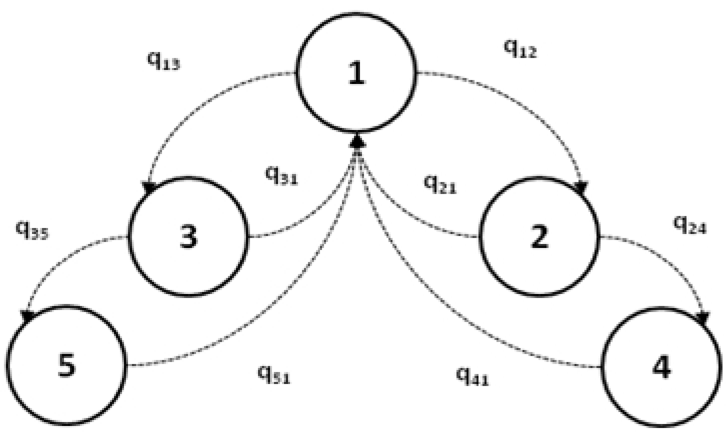



Figure 1. State transition diagram

Figure 1 depicts the Markov states of this manufacturing system. The machine is either operational or in maintenance, which may be corrective (repair) or preventive (servicing).

Human error is defined as an improper procedure that increases the mean maintenance time [25]. This corresponds to jumps from states 2 to 4or 3 to 5, meaning that if the procedure is improper at state 2 or 3, it is repeated to complete the maintenance, thus lengthening the maintenance time.

The transition rate matrix $Q$ of the stochastic processes $\xi(t)$ is defined such that it satisfies the following conditions:

$$Q(\cdot) = \{q_{\alpha\beta}(\cdot)\}, \text{ with } q_{\alpha\beta} \geq 0 \text{ if } \alpha \neq \beta \tag{2}$$

$$q_{\alpha\alpha}(\cdot) = -\sum_{\beta\neq\alpha} q_{\alpha\beta}(\cdot), \text{ where } \alpha, \beta \in \mathrm{A}. \tag{3}$$

The transition probabilities associated with the transition rate $q_{\alpha\beta}$ are given as:

$$p\left[\xi(t+\delta t)=\beta \middle| \xi(t)=\alpha\right] = \begin{cases} q_{\alpha\beta}(\cdot)\delta t+o(\delta t) & if \ \alpha\neq\beta, \\ 1+q_{\alpha\beta}(\cdot)\delta t+o(\delta t) & if \ \alpha=\beta. \end{cases} \tag{4}$$

With $\lim_{\delta t\to 0}(o(\delta t)/\delta t) = 0 \ \forall \alpha, \beta \in A$.It should be noted that $o(\delta t)$ describes the noise or perturbation that may occur at each transition. In the Markov process, we consider $\lim_{\delta t\to 0}(o(\delta t)/\delta t) = 0$ in order to eliminate this perturbation.

The transition rate matrix $Q$ is given as follows:

$$Q(k_{13}, k_{21}, k_{35}, k_{41}) = \begin{bmatrix} q_{11} & q_{12} & k_{13} & 0 & 0 \\ k_{21} & q_{22} & 0 & q_{24} & 0 \\ q_{31} & 0 & q_{33} & 0 & k_{35} \\ k_{41} & 0 & 0 & q_{44} & 0 \\ q_{51} & 0 & 0 & 0 & q_{55} \end{bmatrix}, \tag{5}$$

We now define the set of admissible decisions and control policies (control variables) at mode $\alpha(t)$ as in equation (6).

$$\Gamma(\alpha) = \left\{ \begin{array}{l} (u(\cdot), k_{13}(\cdot), k_{21}(\cdot), k_{35}(\cdot), k_{41}(\cdot)) \in R^5, \ 0 \leq \| u(\cdot) \|_\infty \leq u^{\max}, \ k_{13}^{\min}(\cdot) \leq k_{13}(\cdot) \leq k_{13}^{\max}(\cdot), \\ k_{21}^{\min}(\cdot) \leq k_{21}(\cdot) \leq k_{21}^{\max}(\cdot), \ k_{35}^{\min}(\cdot) \leq k_{35}(\cdot) \leq k_{35}^{\max}(\cdot), \ k_{41}^{\min}(\cdot) \leq k_{41}(\cdot) \leq k_{41}^{\max}(\cdot) \end{array} \right\}, \tag{6}$$

The cost function $J(\cdot)$ given by equation (7) is minimized through the control problem, of which the solution is an admissible control law $\mathrm{B}(\cdot) = (u, k_{13}, k_{21}, k_{35}, k_{41})$.

$$J\left(a,x,\alpha,u,k_{13},k_{21},k_{35},k_{41}\right)=E\left\{\int_0^\infty e^{-\rho t} g\left(a,x,\alpha,u,k_{13},k_{21},k_{35},k_{41}\right)dt \,\middle|\, x(0)=x,\ \xi(0)=\alpha,\ a(0)=a\right\}, \qquad (7)$$

Where $E\{\square\}$ is the expected value function for determining the expected production cost based on the instantaneous cost over an infinite planning horizon. It should be noted that the instantaneous cost varies at each time lapse. In equation (7), $\rho$ is discount rate and $\mathrm{g}(a,x,\alpha,\cdot)=h(\cdot)+w(\cdot)=c^{+}x^{+}+c^{-}x^{-}+c^{\alpha}$ is the instantaneous cost. Hence, we have $h(\cdot)=c^{+}x^{+}+c^{-}x^{-}$ and $w(\cdot)=c^{\alpha}$ .In the instantaneous cost equation, $c^{+}$ , $c^{-}$ and $c^{\alpha}$ are respectively the inventory and order backlog costs per unit, and maintenance cost.

$x^{+}=\max(0,x)$, $x^{-}=\max(-x,0)$
and

$$\begin{aligned}c^{\alpha}&=(c_r+c_{tagout})\mathrm{Ind}\{\alpha=2\}+(c_{pm}+c_{tagout})\mathrm{Ind}\{\alpha=3\}\\&+\beta(c_r+c_{tagout})\mathrm{Ind}\{\alpha=4\}+\beta(c_{pm}+c_{tagout})\mathrm{Ind}\{\alpha=5\}\end{aligned}$$

with:

$$\mathrm{Ind}\{\Theta(\cdot)\}=\begin{cases}1 & \text{if} \quad \Theta(\cdot)\,\text{is true}\\ 0 & \text{otherwise}\end{cases}$$

Where $\beta>1$is the cost index and $\Theta(\cdot)$ represents the system mode $\alpha\in A$.For instance, if the system is in state 2 ( $\alpha=2$ ), then maintenance cost $c^{\alpha}=c_r+c_{tagout}$ . That is, $\mathrm{Ind}\{\alpha=2\}=1,\mathrm{Ind}\{\alpha=3\}=0,\mathrm{Ind}\{\alpha=4\}=0$ and $\mathrm{Ind}\{\alpha=5\}=0$. It should be noted that a state 2 or 3 improper maintenance procedure will lead to a machine malfunction requiring diagnosis followed by a suitable repair procedure, increasing process time and hence production cost. To model this cost we introduce a parameter $\beta>1$.

The manufacturing system should be able to meet demand, meaning that the demand should not be higher than the average production capacity, as described by Akella and Kumar [17].

Let $v(a,x,\alpha)$ denote the value function (minimal discounted cost) for equation 7 as given by equation 8.

$$v(a,x,\alpha)=\min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(\alpha)} J(a,x,\alpha,u,k_{13},k_{21},k_{35},k_{41}),\forall\alpha\in A \tag{8}$$

If the appropriate assumptions are respected, the value function $v(a,x,\alpha)$ satisfies a set of Hamilton-Jacobi-Bellman (HJB) partial differential equations (Appendix A).

## 4. Numerical example and sensitivity analysis

As mentioned, the manufacturing system consists of one machine producing one part type. A five-state Markov process $\xi(t)\in$ A defines the system capacity. The generator matrix $Q(\cdot)$ defined above (5)is described more explicitly as follows:

$$Q(k_{13},k_{21},k_{35},k_{41})=\begin{bmatrix} -(q_{12}(a)+k_{13}) & q_{12}(\alpha) & k_{13} & 0 & 0 \\ k_{21} & -(k_{21}+q_{24}) & 0 & q_{24} & 0 \\ q_{31} & 0 & -(q_{31}+k_{35}) & 0 & k_{35} \\ k_{41} & 0 & 0 & -k_{41} & 0 \\ q_{51} & 0 & 0 & 0 & -q_{51} \end{bmatrix}$$

and

$$q_{12}(a(t))=B_{12}^{\infty}\left[1-e^{-(B_t\times a(t)^2)}\right], \tag{9}$$

Equation 9 describes the impact of machine age on its probability of failure $q_{12}(a(t))$ as illustrated by Kenné and Nkeungoue [19]. Constants $B_t$ and $B_{12}^{\infty}$ represent the frequency of failure respectively at age $a(t)$ and infinity. The mean time between failures (MTBF) is subsequently machine-age-dependent and is given by $\text{MTBF}(a)=\dfrac{1}{q_{12}(a)}$ and $\text{MTBF}(\infty)=\dfrac{1}{B_{12}^{\infty}}$ at age $a$ and $\infty$. We refer our readers to Rausand and Hoyland [26] for more details on the definition of MTBF.

Let $\text{B}(\cdot)=(u,k_{13},k_{21},k_{35},k_{41})$ .The following five equations are obtained from the discrete dynamic programming equation A.2.16 (see appendix A for more details):

$$v^h(\alpha,\text{x},1)=\min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(1)}\left(\rho+\frac{|u-d|}{h_x}+\frac{Ku}{h_a}+q_{12}+k_{13}\right)^{-1}\left\{\begin{array}{l}\dfrac{|u-d|}{h_x}(v^h(\text{x}+\text{h}_x,\alpha,1)k^{+}+v^h(\text{x-h}_x,\alpha,1)k^{-}) \\ +\dfrac{Ku}{h_a}(v^h(\text{x}+\text{h}_\alpha,\text{x},1)+g(\text{x},\alpha,1) \\ +q_{12}v^h(\text{x},\alpha,2)+k_{13}v^h(\text{x},\alpha,3) \end{array}\right\}, \tag{10}$$

$$v^h(\alpha,\mathrm{x},2) = \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(2)} \left(\rho + \frac{d}{h_x} + k_{21} + q_{24}\right)^{-1} \begin{Bmatrix} \frac{d}{h_x} v^h(\mathrm{x\text{-}h}_x,\alpha,2) + g(\mathrm{x},\alpha,2) + k_{21} v^h(\mathrm{x},\alpha,1) \\ + q_{24} v^h(\mathrm{x},\alpha,4) \end{Bmatrix}, \tag{11}$$

$$v^h(\alpha,\mathrm{x},3) = \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(3)} \left(\rho + \frac{d}{h_x} + q_{31} + k_{35}\right)^{-1} \begin{Bmatrix} \frac{d}{h_x} v^h(\mathrm{x\text{-}h}_x,\alpha,3) + g(\mathrm{x},\alpha,3) + q_{31} v^h(\mathrm{x},\alpha,1) \\ + k_{35} v^h(\mathrm{x},\alpha,5) \end{Bmatrix}, \tag{12}$$

$$v^h(\alpha,\mathrm{x},4) = \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(4)} \left(\rho + \frac{d}{h_x} + k_{41}\right)^{-1} \left\{ \frac{d}{h_x} v^h(\mathrm{x\text{-}h}_x,\alpha,4) + g(\mathrm{x},\alpha,4) + k_{41} v^h(\mathrm{x},\alpha,1) \right\}, \tag{13}$$

$$v^h(\alpha,\mathrm{x},5) = \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(5)} \left(\rho + \frac{d}{h_x} + q_{51}\right)^{-1} \left\{ \frac{d}{h_x} v^h(\mathrm{x\text{-}h}_x,\alpha,5) + g(\mathrm{x},\alpha,5) + q_{51} v^h(\mathrm{x},\alpha,1) \right\}, \tag{14}$$

Where $v^h(\alpha,\mathrm{x},\cdot)$ is an approximation of the value function for small increments in the direction of $x$ and $a$. In this study, we consider the computational domain defined as follows:

$G_{ax}^h = \{(x,a): -10 \le x \le 30;\ 0 \le a \le 150\}$

Where age unit $a$ is arbitrary. Note that we opted not to consider a specific time unit, since it can be defined on a weekly, monthly or yearly basis depending on the particular application. Stock unit $x$ corresponds to the number of parts. We assume $h_x = 2$ and $h_a = 2$.

It should be noted that machine characteristics must be defined in terms of system feasibility, meaning that the manufacturing system average capacity is not less than the demand. A manufacturing system is feasible, as described in Akella and Kumar [17], when:

$$\pi_1 u^{\max} \geq d \tag{15}$$

It should be noted that the limitation probabilities are obtained for a system that follows a Markov process as follows:

$$\begin{cases} \pi(\cdot)Q(\cdot) = 0 \\ \sum_{i=1}^{n} \pi_i = 1 \quad \text{where} \quad n = 5 \end{cases} \tag{16}$$

with:

$\pi(\cdot)$ : Limiting probabilities

$Q(\cdot)$ : Transition matrix rates

The vector of limiting probabilities $(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$ indicates the average time for which the system stays in mode $i$ = 1 to 5.

From equation (16) based on the theory illustrated by Ross [27], we have for the manufacturing system under consideration:

$$\begin{cases} \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 & \pi_5 \end{bmatrix} \begin{bmatrix} -(q_{12}(a)+k_{13}) & q_{12}(\alpha) & k_{13} & 0 & 0 \\ k_{21} & -(k_{21}+q_{24}) & 0 & q_{24} & 0 \\ q_{31} & 0 & -(q_{31}+k_{35}) & 0 & k_{35} \\ k_{41} & 0 & 0 & -k_{41} & 0 \\ q_{51} & 0 & 0 & 0 & -q_{51} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1 \end{cases}$$

Solving equation system (16) gives:

$$\pi_1 = \frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \tag{17}$$

$$\pi_2 = \frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \times \frac{q_{12}(a)}{k_{21}+q_{24}} \tag{18}$$

$$\pi_3 = \frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \times \frac{k_{13}}{q_{31}+k_{35}} \tag{19}$$

$$\pi_4 = \frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \times \frac{q_{12}(a)}{k_{21}+q_{24}} \times \frac{q_{24}}{k_{41}} \tag{20}$$

$$\pi_5 = \frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \times \frac{k_{13}}{q_{31}+k_{35}} \times \frac{k_{35}}{q_{51}} \tag{21}$$

Given that the manufacturing system under consideration is operational only in mode 1 and by definition feasible only under the condition stated in equation 15,that is, $\delta \geq 0$, where $\delta$ is difference between the capacity and the demand( $\pi_1 u^{\max} - d$ ) the feasibility of the considered manufacturing system depends on $\pi_1$; hence:

$$\frac{1}{1+\frac{q_{12}(a)}{k_{21}+q_{24}}+\frac{k_{13}}{q_{31}+k_{35}}+\frac{q_{12}(a)q_{24}}{k_{41}k_{21}+k_{41}q_{24}}+\frac{k_{13}k_{35}}{q_{51}q_{31}+q_{51}k_{35}}} \times u^{\max} \geq d \tag{22}$$

The machine failure rate for each value of its age is plotted in Figure 2, where $B_{12}^{\infty} = 0.1$and $B_t = 1 \times 10^{-5}$ are selected to obtain a failure probability trajectory according to the machine age as described by Kenné and Nkeungoue [19].

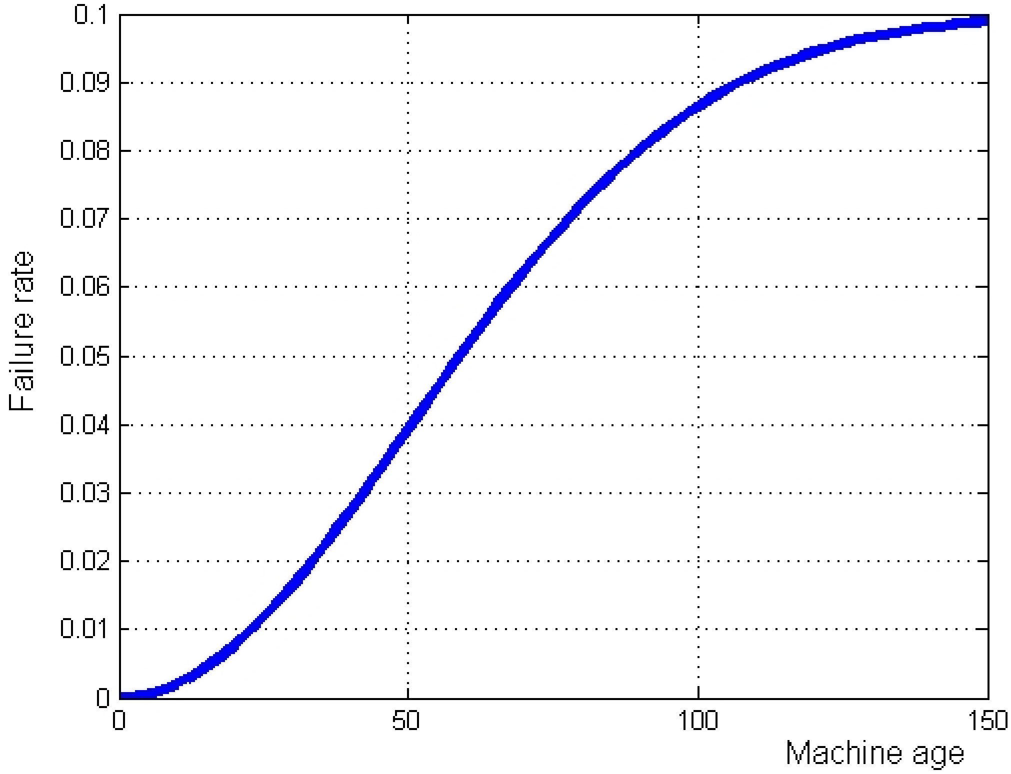



Figure 2. Age-dependent frequency of machine failure

After tuning to reflect the behaviour of the developing policy more accurately, the parameters for the described manufacturing system were set to specific values shown in Table 1,without loss of generality.

Table 1

| $c^-$ | $c^+$ | $c_r$ | $c_{mp}$ | $c_{tagout}$ | $u^{\max}$ | $d$ | $k_{21}^{\min}$ | $k_{21}^{\max}$ | $k_{13}^{\min}$ |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 30 | 8000 | 500 | 100 | 0.32 | 0.25 | 0.083 | 0.25 | $10^{-6}$ |
| $k_{13}^{\max}$ | $k_{41}^{\min}$ | $k_{41}^{\max}$ | $k_{35}^{\min}$ | $k_{35}^{\max}$ | $q_{24}$ | $q_{31}$ | $q_{51}$ | $\rho$ | |
| 0.05 | 0.1 | 0.3 | $2\times10^{-7}$ | 0.01 | 0.1 | 0.5 | 0.417 | 0.01 | |

We note that there are no benchmarks in the literature and we therefore generated a dataset that better reflects the behaviour of the system, while maintaining feasibility conditions. There is thus a trade-off between production rate and demand since the order backlog cost is considered greater than the inventory holding cost. It should be noted that there is also a trade-off between the frequency of corrective maintenance and the production rate, since more frequent corrective maintenance means more shutdowns. The frequencies of corrective and preventive maintenance are chosen accordingly to reflect system behaviour/performance more accurately. Human error, described as improper procedure, is assumed to increase the mean time spent on corrective and preventive maintenance (including LOTO) by 20% (see [28] for details).

In this article, the policy improvement technique is used to solve the optimization approximation problem, namely equations 10 through 14.The numerical solution that meets the optimality conditions is used to obtain the control policies. We describe the policy improvement technique as follows:

**Step 1.** (Initialization) Choose $\delta \in \mathbb{R}^+, set\, n := 1, (v^h(x,k))^n := 0, \forall \alpha \in A, (a,x) \in G_{ax}^h$

**Step 2.**Compute $(v^h(a,x,\alpha))^{n-1} := (v^h(a,x,\alpha))^n,\ \forall \alpha \in A,\ (a,x) \in G_{ax}^h.$

**Step 3.** Compute the corresponding value function to obtain the control policy $(u, k_{13}, k_{21}, k_{35}, k_{41})$

**Step 4.** Test the convergence

$$\begin{aligned}
&\overline{c} = \min\left\{(v_n^h(a,x,\alpha) - (v_{n-1}^h(a,x,\alpha)\right\},\\
&c_{\min} = \left[\rho/(1-\rho))\right]\overline{c},\\
&\underline{c} = \max\left\{(v_n^h(a,x,\alpha) - (v_{n-1}^h(a,x,\alpha)\right\},\\
&c_{\max} = \left[\rho/(1-\rho))\right]\underline{c},\\
&\text{If } \left|c_{\max} - c_{\min}\right| \le \delta, \text{ then stop, else } n = n+1 \text{ and go to step 2.}
\end{aligned}$$

Where $\delta \in \mathbb{R}^+$ is a threshold value corresponding to the convergence of the algorithm. The production policy structure $u(\cdot)$ recommends that if the stock level is lower than a threshold level, the facility should produce at maximal capacity; if the stock level is equal to the associated critical threshold, the facility should produce at the rate that meets demand; otherwise production should be halted. Such a structure is commonly called the hedging point policy (HPP) in the literature on production control [17].

For illustrative purposes, results obtained for the control variables $(u, k_{13}, k_{21}, k_{35}, k_{41})$ of a machine producing a single type of part are shown in Figures3 through 5(b).

For the parameters mentioned in Table 1, the machine-age-dependent threshold value is defined using the trend plotted in Figures 3 through 5(b). The production policy is therefore stated as follows:

$$u(1, x, a) = \begin{cases} u^{\max} & \text{if } x(t) < X^*(a), \\ d & \text{if } x(t) = X^*(a), \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Where $X^*(a)$ is optimal stock level at age $a(t)$. Equation (23) determines the production policy for the manufacturing system under consideration. Recall that the machine is operational only in mode $\alpha = 1$.

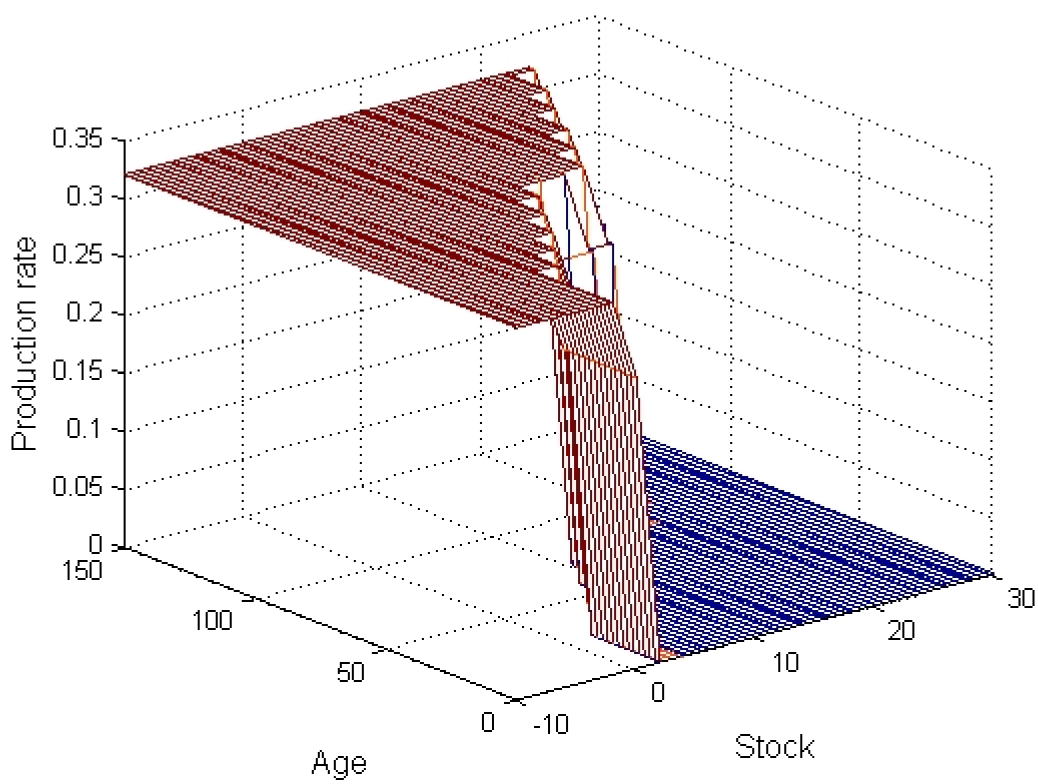



Figure 3. Machine production rate

Figure 3 proposes different zones for the production rate. Each of these corresponds to an optimal production policy based on inventory level and machine age. The optimal policy appears to be to maximize the production rate when the current stock level is below an age-dependent threshold value. In this zone, machine failure is frequent because of aging. This policy also suggests that when the current stock level is equal to an age-dependent threshold value, production should be set at a rate corresponding to demand. Under any other condition, production should be halted.

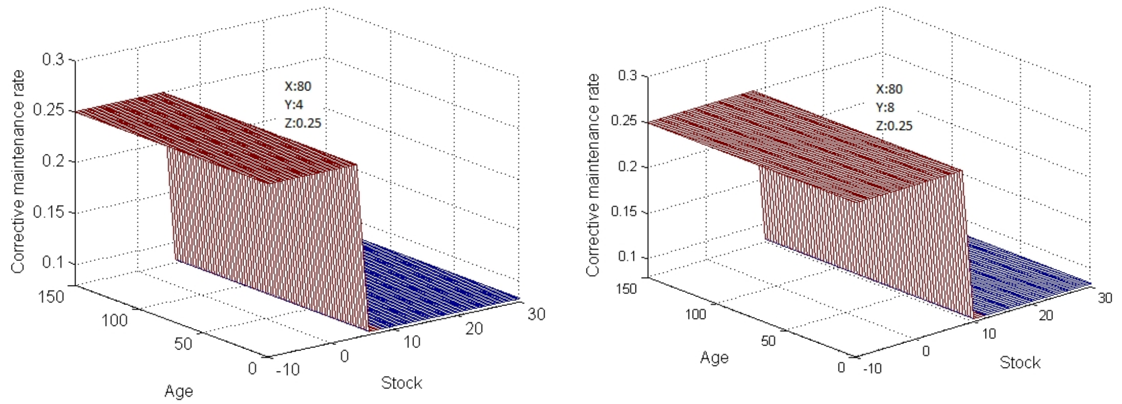
The corrective maintenance policies with and without human error are plotted in Figures 4(a) and 4(b). Figure 4(a) shows that the computational domain $(x,a)$ is divided into two regions, one in which the frequency of corrective maintenance without human error is set at its maximal value, resulting in order backlog situations, and one in which it is set at its minimal value, resulting in comfortable stock levels. Figure 4(b) has the same structure. At a given machine age, higher levels of inventories are recommended when corrective maintenance (including LOTO) might be conducted improperly. For instance, the penalty on inventory level for age *80* and a corrective maintenance frequency of *0.25* is an increase of *29%*. The corresponding optimal policies have a bang-bang structure and are given by equations 24 and 25.

$$k_{21}(2,x,a) = \begin{cases} k_{21}^{\max} & \text{if } x(\cdot) < C^*(a), \\ k_{21}^{\min} & \text{otherwise.} \end{cases} \tag{24}$$

Where $C^*(a)$ represents the optimal stock level at which the frequency of corrective maintenance without human error should be changed from $k_{21}^{\min}$ to $k_{21}^{\max}$.

$$k_{41}(4,x,a) = \begin{cases} k_{41}^{\max} & \text{if } x(\cdot) < D^*(a), \\ k_{41}^{\min} & \text{otherwise.} \end{cases} \tag{25}$$

Where $D^*(a)$ represents the optimal stock level at which the corrective maintenance with human error should be changed from $k_{41}^{\min}$ to $k_{41}^{\max}$.




Figures 4(a) and 4(b). Frequency of machine corrective maintenance including LOTO (a) without human error and (b) with human error

The preventive maintenance rates without and with human error are plotted in Figures 5(a) and 5(b). Figure 5(a) shows that the computational domain $(x,a)$ is divided into two regions. The frequency of preventive maintenance without human error is set at its maximum for backlog situations and at zero for large stock levels. The zone in the domain $(x,a)$ where this frequency is maximal increases with machine age for significant stock levels. The policy recommends increasing inventory levels as machine age increases to meet demand when preventive maintenance might be conducted improperly. For example, at age *80* and a preventive maintenance frequency of *0.05*, the inventory level penalty is an increase of *25%*.

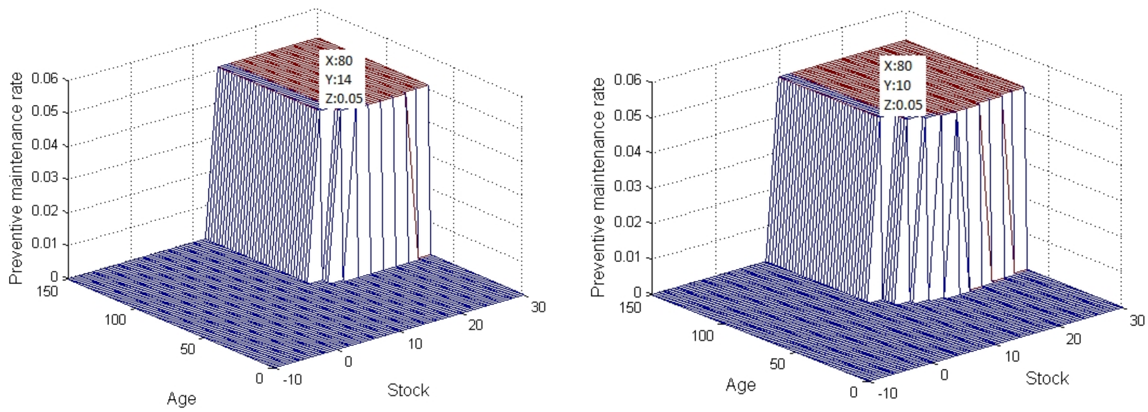
The corresponding optimal policies have a bang-bang structure, as was the case for the corrective maintenance policies with or without human error. These control policies are given by equations 26 and 27.

$$k_{13}(1,x,a)=\begin{cases} k_{13}^{\max} & \text{if } \; x(\cdot)<E^{*}(a), \\ k_{13}^{\min} & \text{otherwise.} \end{cases} \tag{26}$$

Where $E^{*}(a)$ is a machine-age function that gives the optimal stock level at which it is necessary to change the frequency of preventive maintenance without human error from $k_{13}^{\min}$ to $k_{13}^{\max}$.

$$k_{35}(3,x,a)=\begin{cases} k_{35}^{\max} & \text{if } \; x(\cdot)<F^{*}(a), \\ k_{35}^{\min} & \text{otherwise.} \end{cases} \tag{27}$$

Where $F^*(a)$ is a machine-age function that gives the optimal stock level at which it is necessary to change the frequency of preventive maintenance with human error from $k_{35}^{\min}$ to $k_{35}^{\max}$.




Figures 5(a) and 5(b). Frequency of machine preventive maintenance including LOTO (a) without human error and (b) with human error

The variability was analyzed to evaluate the sensitivity of the results to the policies. The influences of the backlog cost on the production threshold, the corrective maintenance cost and the preventive maintenance cost as a function of machine age are plotted in Figures 6 through 8.

Figure 6 shows that it is not necessary to keep large inventories if the manufacturing system is in its youthful phase, that is, while the production machinery is highly reliable and can meet demand. However, as the machine ages, storing more finished inventory is recommended to meet demand as it peaks. In other words, the manufacturing system must prepare for shutdowns at increasing frequency. Furthermore, if the order backlog cost increases at any machine age, the consequences of stock shortfall become greater and stock levels should therefore be increased.

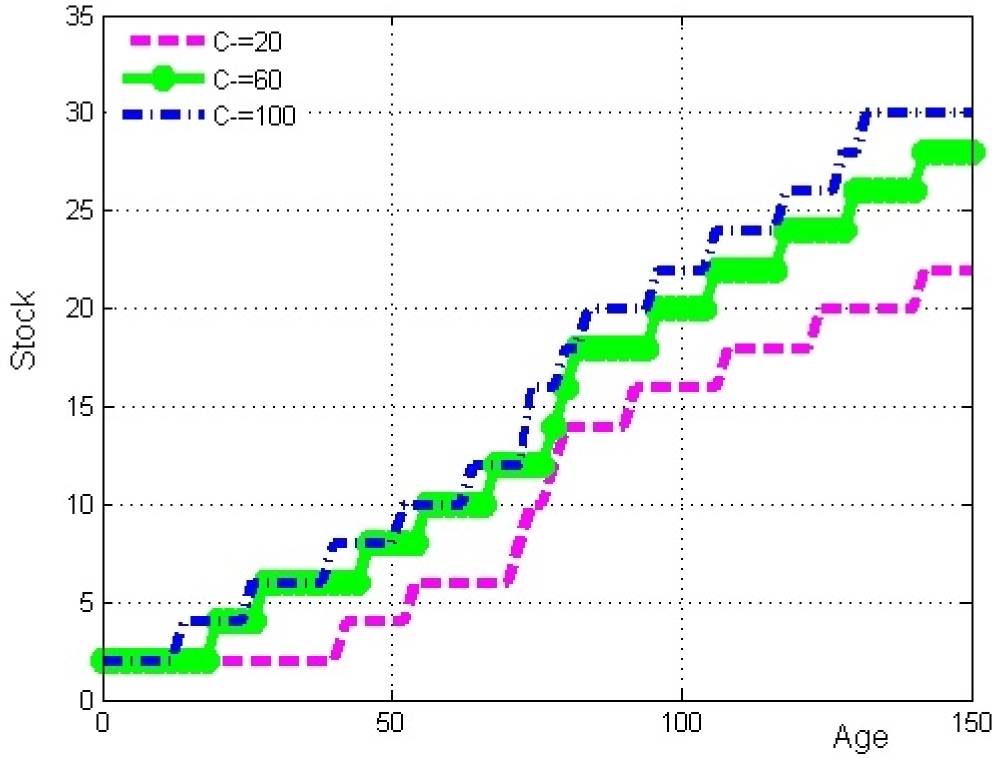



Figure 6. Threshold value versus machine age

C- Backlog cost

Figure 7 displays the relationship between corrective maintenance, inventory level and machine age. This relationship indicates that if the corrective maintenance cost increases, the stock level at any machine age should be increased to avoid possible shortages, all the more if that maintenance activity is subject to improper procedure by the technician. In the latter case, the probability of incident or accident is also greater. To reduce this probability, the production speed must not be increased quickly and worker training should be improved [29].

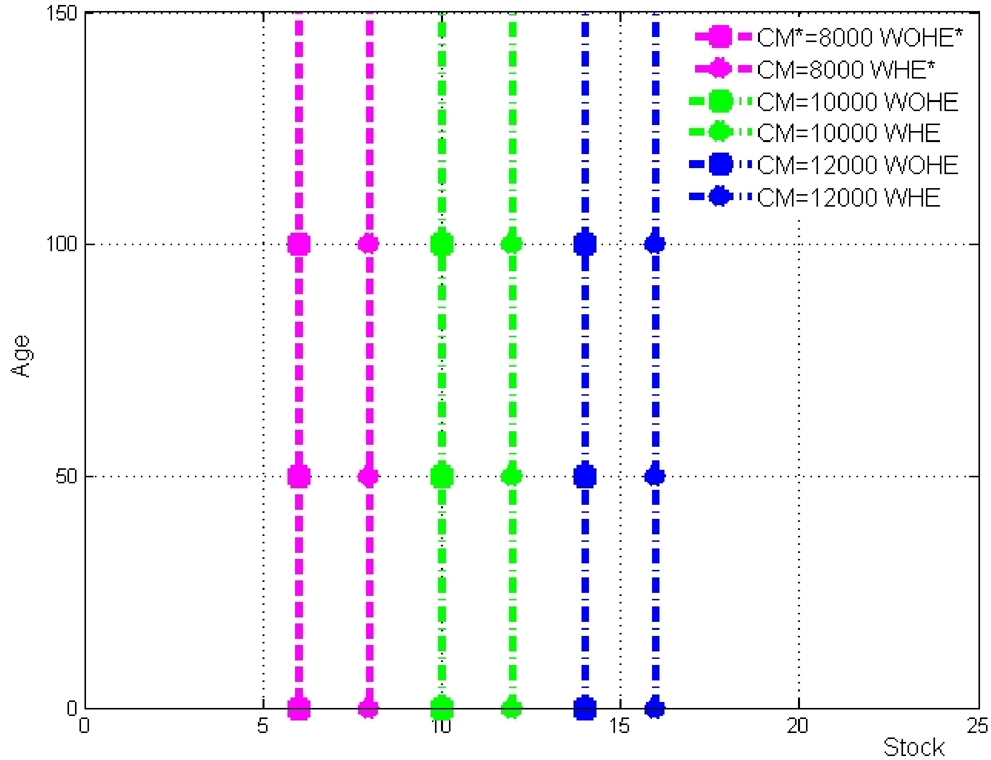



Figure 7. Frequency of corrective maintenance without and with human error, versus machine age

The abbreviations in Figure 7 refer to the following:

$\mathrm{CM}^*$ : corrective maintenance cost

$\mathrm{WOHE}^*$: without human error

$\mathrm{WHE}^*$ : with human error

Figure 8 describes the recommended inventory level at a given the machine age and preventive maintenance cost. The dotted line represents the recommended inventory when the effect of human error during preventive maintenance is integrated. Each line divides the $(x,a)$ domain into two zones. In zone I, the production policy recommends not to impose preventive maintenance, since the probability of machine failure is almost zero (e.g. the machine is brand new). The machine will certainly meet demand. Maintenance is appropriate in zone II, since the manufacturing system is aging and breakdown is more likely. To increase the life and reliability of the production unit, preventive maintenance becomes indispensable, and inventory must be increased to meet demand. Furthermore, the frequency of preventive maintenance must be decreased if its cost increases. These results illustrate that inventory must be increased for any preventive maintenance cost when it is likely that maintenance procedures will be improper and the probability of accident will increase. This is consistent with previous study [29] of the impact of increased production speed and reduced training on the probability of incidents and accidents.

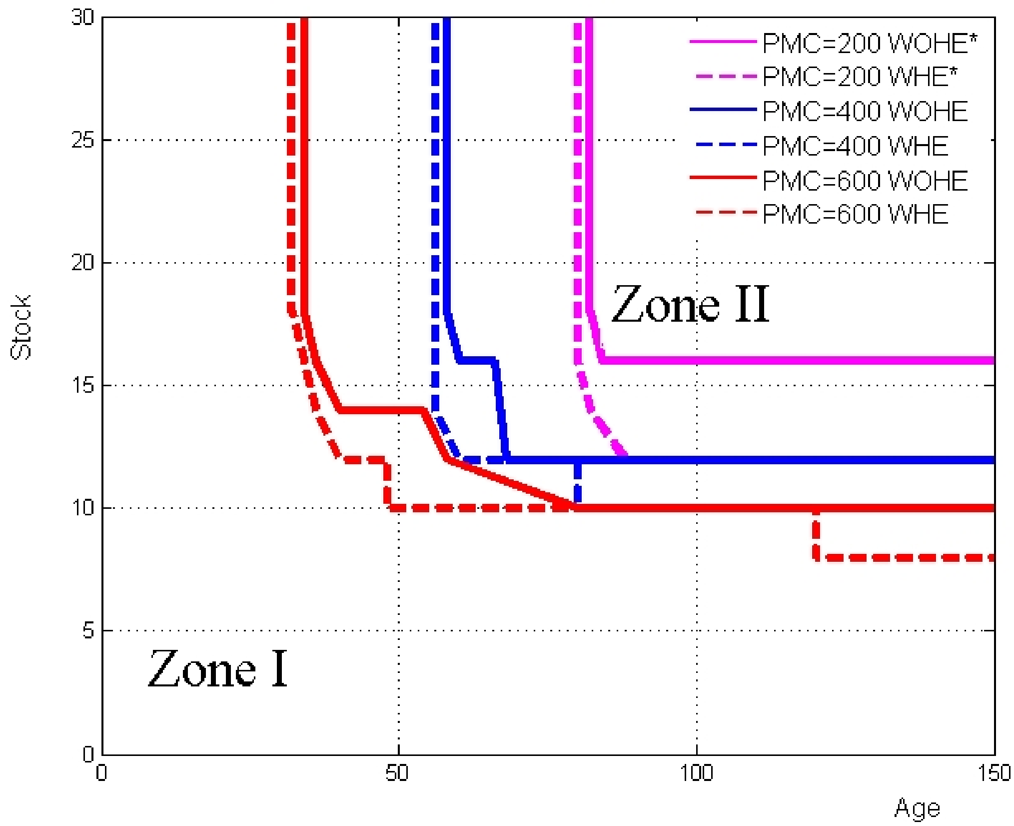



Figure 8 here. Frequency of preventive maintenance without and with human error versus machine age

The abbreviations in figure 8refer to the following:

PMC* : preventive maintenance cost

WOHE*: without human error

WHE* : with human error

## 5. Discussion

The motivation for this study is the concept of corporate social responsibility as presented elsewhere [30, 31]and the fundamental message of the Brundtland Commission on social, economic and environmental integration and the contribution of workers to sustainable development [32]. These studies highlight the importance of seizing opportunities to achieve better integration of occupational safety and day-to-day management of manufacturing processes. The results presented in this article could help corporate managers to realise the impact of human error on production costs as well as accidents and occupational health hazards in uncertain manufacturing environments. This paper contributes to a stream of research that emphasises integrating human aspects into operations models as called for by [29] and as exemplified by the integration of human aspects like biomechanical loading and fatigue into discrete event simulation [33,34], connecting learning and forgetting into mathematical models of Dual Resource Constrained (DRC) systems [35,36,37], modelling production costs in ways that include employee health hazards [38], and incorporating human aspects into industrial engineering design tools to support the production system design process [39]. This current study contributes to this agenda by providing further examples of novel approaches to integrating human aspects into engineering design and decision making tools that can support better design choices for both improved system safety and long term system performance.

Already in the 1960s, Rook [40] illustrated that 82% of 23,000 production defects originating on assembly lines were due to human error. At the beginning of the 2000s, Shibata [41] reported that on average, 42% of the assembly defects in products such as compact disc/mini disc dual-deck players were due to human error. This work also revealed that assembly process complexity increases the human error rate and hence the occurrence of defects. To discuss the impact of human error on assembly process production quality and defective parts, Bubb [42] used field data from the electronic assembly line sector and recommended using methods such as THERP (Technique for Human Error Rate Prediction) to reduce the probability of human error. Liu et al [43] later showed the direct relationship between human error and economic loss in production and developed a loss estimation model in which the impact of human error is categorized as minor, medium, or severe. Considering 33 types of cost factor, the effect of complexity on quality in consumer audio equipment assembly has been analyzed at various sites around the world using a vast field dataset [41]. In the recent literature, researchers now recommend integrating production and human factors [44] as well as maintenance [45] and human error probability (HEP) to improve production system reliability [46]. The group headed by B.S. Dhillon has contributed substantially to our current comprehension of maintenance and maintenance errors. The consensus in the recent literature is that "most human errors occur in the inspection and maintenance phases" [46]. The main sources of human error in maintenance have been identified as "lack of communication, distraction, lack of resources, stress, complacency, lack of teamwork, pressure, lack of awareness, lack of knowledge, fatigue, lack of assertiveness and social norms" [45]. For example, in a study of chemical processing, 21 % of maintenance-related major accidents were found to be the result of bypassing safety measures (active failure), while 69% were caused by "deficient planning/scheduling/fault diagnosis", a form of latent failure [47]. Quality inspection errors have been modeled recently [48]. HEP has been assessed in LOTO pump maintenance activities using the success likelihood index method or SLIM [46]. Nevertheless, the difficulties of quantifying error occurrence in maintenance tasks are immense, due to the diversity of the activities [49].The literature on LOTO focuses on the sole utilization of LOTO in the procedural form focusing on how LOTO should be implemented [50,51]. While appropriate LOTO procedural routines are indeed needed they are insufficient. The problem that persists is the lack of inclusion of the LOTO procedures into production planning and control. To cope with this shortcoming Charlot et al. [3] integrated LOTO control into production planning. They showed that this integration into production planning makes the implementation of safety measures to reduce risk of accidents easier. Moreover, Emami-Mehrgani et al. [13,21] showed that by integrating the LOTO into production capacity control planning for a passive redundancy system, the system becomes less vulnerable to changes in shortage and inventory costs by meeting the demand permanently. They also disclosed that the integration of LOTO in passive redundancy system allows managers to respect the essential space and time requirements during machine repair actions. With the appropriate consideration of space and time issues, the LOTO procedure can be put in place effectively. In this article, we address corrective and preventive maintenance with LOTO while considering human error – a novel approach. By relying on the literature and the fact that “most human errors occur in the inspection and maintenance phases” [46] the importance of this research work, to improve system performance by attending to critical human aspects, is highlighted.

In this study, we built our policy on the assumption that thanks to safety measures such as LOTO as well as appropriate maintenance and production planning, the risk of accident during maintenance activities is negligible, although human error may occur in the procedures. We are interested in the impact of such error on inventories, production capacity and the cost associated with maintenance and safety. LOTO contributes to the cost associated with the frequency of preventive and corrective maintenance. However, this cost should be acceptable compared to the cost of accidents or managing preventable hazards [52]. The impact of human error during maintenance activity on production costs appears to be twofold: the additional downtime required to rectify inadequate repairs, as well as the additional LOTO cost.

The analytical model developed here was preferred because of the smaller number of parameters and assumptions required. Such a model may be very useful in support of results obtained by numerical methods, due to the flexibility it offers when the parameter values are changed. However, the policies obtained have to be robust. That is, they must propose sound responses to uncertainties (e.g. breakdown and human error) and to the rapidly changing conditions under which manufacturing systems operate.

The sensitivity analysis, results and observations presented in this article validate the effectiveness and usefulness of the proposed model, meaning that the control policy is well developed. As claimed above, we have defined the optimal policies for production and corrective and preventive maintenance including LOTO, for a manufacturing system producing one part type, said system subject to random failure and to human error during maintenance activities. The policies thus developed could be extended to address real-scale manufacturing systems involving more than one machine and multiple part types. However, this makes the problem more complex. A certain number of conditions have to be met to make efficient use of this model. Close monitoring and an efficient time management system are required. In addition, a breakdown history recording system has to be in place to estimate probabilities of occurrence and the costs of intervention of one kind or another.

## 6. Conclusion

In this study of the impact of human error on corrective and preventive maintenance activities including LOTO and the associated production cost, we developed a control policy based on an extension of the hedging point structure. A parameterized near-optimal control policy was derived from the numerical solution obtained. By running a numerical example and sensitivity analysis, we showed how human error throughout the maintenance activities increases the total production cost. We note that increasing production speed and reducing worker training increase human error and subsequently the total production cost while decreasing worker safety. The total production cost is also subject to increases due to the larger inventory needed to meet demand while machinery is subject to breakdown at increased frequency.

## Acknowledgments

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**Appendix A. Optimal conditions and numerical approach**

In this section, the assumptions and lemmas underlying the analysis are described.

Assumptions A.1.

A.1.1) $h(\cdot)$ is a non-negative convex function with $h(0)=0$. It has positive constants $\mathrm{C_g}$ and $\mathrm{K_g}$ such that:

$$h(x) \le \mathrm{C}_g(1+|x|^{K_g})$$

and

$$|h(x_1)-h(x_2)| \le \mathrm{C}_g(1+|x_1|^{K_g}+|x_2|^{K_g})|x_1-x_2|$$

A.1.2) $w(\cdot)$ is a non-negative function with $w(0)=0$ and is twice differentiable and either strictly convex or linear.

Lemmas A.1:

A.1.1) if $g(\alpha,x,a,\cdot)$ is convex, then $v(\alpha,x,a)$ is convex in $(x,a)$ for each $\alpha \in A$.

A.1.2) if $g(\alpha,x,a,\cdot)$ is locally Lipschitz, meaning that positive constants $C_g$ and $k_g$ exist such that:

$g(\alpha,x,a,\cdot) \le \mathrm{C}_g(1+|x|^{K_g}+|a|^{K_g})$, then for all $x_1,a_1,u,k_{13},k_{21},k_{35},k_{41},x_2$ and $a_2$ we have:

$$\begin{aligned}&|g(\alpha,x_1,a_1,\cdot)-g(\alpha,x_2,a_2,\cdot)|\\ &\le \mathrm{C}_g(1+|x_1|^{K_g}+|x_2|^{K_g}+|a_1|^{K_g}+|a_2|^{K_g})(|x_1-x_2|+|a_1-a_2|)\end{aligned}$$

Hence, $v(\alpha,x,a)$ is locally Lipschitz which means:

$$|v(\alpha,x_1,a_1)-v(\alpha,x_2,a_2)| \le \mathrm{C}_g(1+|x_1|^{K_g}+|x_2|^{K_g}+|a_1|^{K_g}+|a_2|^{K_g})(|x_1-x_2|+|a_1-a_2|)$$

Proof:

For A.1.1, we now show that $J(\alpha,x,a,\cdot)$ is jointly convex.

Let $a_1,x_1,a_2$ and $x_2$ be arbitrary initial values, $B^1(\cdot)$ and $B^2(\cdot)$ to be some admissible controls. Let $a_{11}(t), x_{11}(t), a_{22}(t)$ and $x_{22}(t)$ with $t \ge 0$ denote the trajectories related to $(x_1,a_1,B^1(\cdot))$ and $(x_2,a_2,B^2(\cdot))$.

For any $\mu \in [0,1]$,

$$\mu J\left(\alpha, x_1, a_1, B^1(\cdot)\right) + (1-\mu) J\left(\alpha, x_2, a_2, B^2(\cdot)\right)$$
$$= E\left[\int_0^\infty e^{-\rho t}(\mu g(\alpha, x_1, a_1, B^1(\cdot)) + (1-\mu) g(\alpha, x_2, a_2, B^2(\cdot)))dt\right]$$
$$= E\left[\int_0^\infty e^{-\rho t}(\mu h(x_1) + \mu w(a_1) + (1-\mu)h(x_2) + (1-\mu)w(a_2))dt\right]$$
$$= E\left[\int_0^\infty e^{-\rho t}(\mu h(x_1) + (1-\mu)h(x_2))dt\right] + E\left[\int_0^\infty e^{-\rho t}(\mu w(a_1) + (1-\mu)w(a_2))dt\right]$$

With assumptions A.1.1 and A.1.2 we have:

$$\mu J\left(\alpha, x_1, a_1, B^1(\cdot)\right) + (1-\mu) J\left(\alpha, x_2, a_2, B^2(\cdot)\right) \geq E\left[\int_0^\infty e^{-\rho t} g(\alpha, x(t), a(t), B(t))dt\right]$$

where $B(t) = \mu B^1(t) + (1-\mu)B^2(t)$ , $x(t)$ and $a(t)$ denote the trajectory with initial values

$x = \mu x_1(t) + (1-\mu)x_2(t) = \mu x_1 + (1-\mu)x_2$,
$a = \mu a_1(t) + (1-\mu)a_2(t) = (\mu a_1 + (1-\mu)a_2$
and admissible controls $B(\cdot)$.

Therefore, we obtain:

$$\mu J(\alpha, x_1, a_1, B^1(\cdot)) + (1-\mu)J(\alpha, x_2, a_2, B^2(\cdot)) \geq J(a, \mu x_1 + (1-\mu)x_2, \mu a_1 + (1-\mu)a_2, \mu B^1(\cdot) + (1-\mu)B^2(\cdot))$$

meaning that $J\left(a, x, \alpha, \cdot\right)$ is jointly convex and thus $v\left(a, x, \alpha\right)$ is convex.

For A.1.2, we take into consideration an admissible control $B(\cdot)$ where $a_{11}(\cdot), x_{11}(\cdot), a_{22}(\cdot)$ and $x_{22}(\cdot)$ denote the state trajectories under $B(\cdot)$ with initial values $a_1, x_1, a_2$ and $x_2$ . Then, we have:

$$|x_{11}(t) - x_{22}(t)| + |a_{11}(t) - a_{22}(t)| \leq (|x_1 - x_2| + |a_1 - a_2|),$$
$$|x_1(t)| + |a_1(t)| \leq C_1(1 + |x_{11}| + |a_{11}|) \text{ and } |x_2(t)| + |a_2(t)| \leq C(1 + |x_{22}| + |a_{22}|)$$

Assuming a locally Lipschitz condition, a constant $C_1$ exists independently of $B(\cdot)$ , $x_1, a_1, x_2$ and $a_2$ such that :

$$\left|J\left(\alpha, x_1, a_1, B(\cdot)\right) - J\left(\alpha, x_2, a_2, B(\cdot)\right)\right| \leq C_1 (1 + |x_1|^{K_g} + |x_2|^{K_g} + |a_1|^{K_g} + |a_2|^{K_g})(|x_1 - x_2| + |a_1 - a_2|)$$

From this, we obtain:

$$\left|v\left(\alpha,x_1,a_1\right)-v\left(\alpha,x_2,a_2\right)\right| \le \sup_{B(\cdot)\in\Gamma(\alpha)}\left|J\left(\alpha,x_1,a_1,B(\cdot)\right)-J\left(\alpha,x_2,a_2,B(\cdot)\right)\right|$$

Hence:

$$\left|v\left(\alpha,x_1,a_1\right)-v\left(\alpha,x_2,a_2\right)\right| \le C_1\left(1+\left|x_1\right|^{K_g}+\left|x_2\right|^{K_g}+\left|a_1\right|^{K_g}+\left|a_2\right|^{K_g}\right)\left(\left|x_1-x_2\right|+\left|a_1-a_2\right|\right)$$

Lemma A.2:

The value function $v(\alpha,x,a)$ is the unique viscosity solution to the HJB equations:

$$\rho v\left(\alpha,x,a\right)= \min_{B(\cdot)\in\Gamma(\alpha)}\left\{\begin{array}{l}\left(u-d\right)\dfrac{\partial}{\partial \mathrm{x}}v\left(\alpha,x,a\right)+f(u)\dfrac{\partial}{\partial a}v\left(\alpha,x,a\right)+g\left(\alpha,\cdot\right)\\ +\sum\limits_{\alpha\neq\beta}\left[q_{\alpha\beta}(\cdot)v\left(\cdot,\beta\right)-v\left(\cdot,\alpha\right)\right]\end{array}\right\}, \forall\alpha,\beta\in A \qquad \text{(A.2.1)}$$

where $\dfrac{\partial}{\partial \mathrm{x}}v\left(\alpha,x,a\right)$ and $\dfrac{\partial}{\partial a}v\left(\alpha,x,a\right)$ are the partial derivatives of the value function $v\left(\alpha,x,a\right)$.

Proof:

This involves showing that $v(\cdot)$ is both a viscosity sub-solution and viscosity super-solution of equation (A.2.1) by using the procedure presented by Sethi and Zhang [53]. The reader is referred to Yong and Zhou [54] for more details.

Let us consider $C^1(R^+\times R^{++})$ to be a class of continuously differentiable function defined on $R^+\times R^{++}$. Set $\alpha, a_0$ and $x_0$. Then, let $\vartheta(\cdot)\in C^1(R^+\times R^{++})$ be such that:

$v\left(\alpha,x,a\right)-\vartheta(x,a)$ attains its maximum at $(x=x_0, a=a_0)$ in neighbourhood $\mathrm{P}(x_0,a_0)$.

Let $\tau$ denote the first jump time of $\xi(t)$. We take into consideration $B(t)=B$ for $0\le t\le\tau$, where $\mathrm{B}\in\Gamma(\alpha)$ is a constant. In addition, let $\varepsilon\in(0,\tau]$ be such that $(x,a)$ starts at $(x_0,a_0)$ and stays in $\mathrm{P}(x_0,a_0)$ for $0\le t\le\varepsilon$. We define:

$$\gamma(\xi,x,a)=\begin{cases}\vartheta(x,a)+v(\alpha,x_0,a_0)-\vartheta_0(x,a) & \text{if} \quad \xi(t)=\alpha\\ v(\alpha,x,a) & \text{if} \quad \xi(t)\neq\alpha\end{cases} \qquad \text{(A.2.2)}$$

Then, by Dykin's formula and the fact that $\xi(t)=\alpha$ and $0\le t\le\varepsilon$,

$$Ee^{-\rho\varepsilon}\gamma(\xi(\varepsilon),x(\varepsilon),a(\varepsilon))-v(a,x_0,a_0)=E\int_0^{\varepsilon}e^{-\rho t}\begin{bmatrix}-\rho\gamma(\alpha,x(t),a(t))+\vartheta_X(x,a)\big(u(t)-d\big)\\ +\vartheta_a(x,a)f(u)+\sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)\gamma(\cdot,\beta)-\gamma(\cdot,\alpha)]\end{bmatrix}dt \qquad (A.2.3)$$

$v(\alpha,x,a)-\vartheta(x,a)$ reaches its maximum

at $P(x_0,a_0)$ and $(x(t),a(t))\in P(x_0,a_0)$ for $0\le t\le\varepsilon$.

Therefore:

$v(\alpha,x_0,a_0)-\vartheta_0(x,a)\ge v(\alpha,x,a)-\vartheta(x,a)$, that is:

$$\vartheta(x,a)\ge v(\alpha,x,a)-(v(\alpha,x_0,a_0)-\vartheta(x_0,a_0))\text{ for }0\le t\le\varepsilon. \qquad (A.2.4)$$

$v(\alpha,x_0,a_0)-\vartheta(x_0,a_0)$ is a constant, and when we replace $\gamma(\alpha,x,a)$ in (A.2.3) by $v(\alpha,x,a)-(v(\alpha,x_0,a_0)-\vartheta(x_0,a_0))$, we have:

$$Ee^{-\rho\varepsilon}v(a,(x(\varepsilon),a(\varepsilon))-v(a,x_0(\varepsilon),a_0(\varepsilon))\le E\int_0^{\varepsilon}e^{-\rho t}\begin{bmatrix}-\rho v(\alpha,x(t),a(t))+\vartheta_X(x,a)(u(t)-d)\\ \\ +\vartheta_a(x,a)f(u)+\sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v(\cdot,\beta)-v(\cdot,\alpha)]\end{bmatrix}dt \qquad (A.2.5)$$

And by using the optimality principle, we obtain:

$$v(a,x_0,a_0)\le E\left[\int_0^{\varepsilon}e^{-\rho t}g(\alpha,x(t),a(t),B(t))dt+e^{-\rho\varepsilon}v(\alpha,x(\varepsilon),a(\varepsilon))\right] \qquad (A.2.6)$$

Through a combination(A.2.5) and (A.2.6), we have:

$$0\le E\int_0^{\varepsilon}e^{-\rho t}\left[g(\alpha,x(t),a(t),B(t))+\vartheta_X(x,a)\big(u(t)-d\big)+\vartheta_a(x,a)f(u)+\sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v(\cdot,\beta)-v(\cdot,\alpha)]\right]dt$$

By letting $\varepsilon\to 0$, we conclude that:

$$\min_{B(\cdot)\in\Gamma(\alpha)}\left\{g(\alpha,x_0,a_0,B)+\vartheta_X(x_0,a_0)\big(u-d\big)+\vartheta_a(x_0,a_0)f(u)+\sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v(\cdot,\beta)-v(\cdot,\alpha)]\right\}-\rho v(\alpha,x_0,a_0)\ge 0$$

Hence, $v(\cdot)$ is a viscosity sub-solution of equation (A.2.1).

We now show that $v(\cdot)$ is a viscosity super-solution of equation (A.2.1) by assuming that it is not.

If $v(\cdot)$ is not a viscosity super-solution, this implies that $\alpha, x_0, a_0$ and $\delta_0 > 0$ exist such that for all
$B \in \Gamma(\alpha)$:

$$g(\alpha, x, a, B) + \vartheta_x(x,a)\left(u-d\right) + \vartheta_a(x,a) f(u) + \sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v\left(\cdot,\beta\right) - v\left(\cdot,\alpha\right)] - \rho v(\alpha, x, a) \geq \delta_0 \quad \text{(A.2.7)}$$

In neighbourhood $P(x_0, a_0)$ where $\vartheta(\cdot) \in C^1(R^+ \times R^{++})$ such that $v\left(\alpha, x, a\right) - \vartheta(x,a)$ reaches its maximum at $(x_0, a_0)$ in neighbourhood $P(x_0, z_0)$.

Then for all $(x(t), a(t)) \in P(x_0, a_0)$:

$$v(\alpha, x, a) \geq \vartheta(x,a) + v(\alpha, x_0, a_0) - \vartheta(x_0, a_0) \quad \text{(A.2.8)}$$

For any $B \in \Gamma(\alpha)$, let $\varepsilon_0$ denote a small number such that $(x,a)$ starts at $(x_0, a_0)$ and stays in $P(x_0, a_0)$ for $0 \leq t \leq \varepsilon_0$. Note that $\varepsilon_0$ is dependent on the admissible control $B(\cdot)$. Nevertheless, given that $u(t) - d$ is always bounded, there exists a constant $\varepsilon_1 \geq 0$ such that $\varepsilon_0 \geq \varepsilon_1 \geq 0$. Let $\tau$ denote the first jump time of the process $\xi(\cdot)$. Hence,

$$J(a, x_0, a_0, B(\cdot)) \leq E\left[\int_0^{\varepsilon} e^{-\rho t} g(\alpha, x(t), a(t), B(\cdot))dt + e^{-\rho\varepsilon} v(\alpha, x(\varepsilon), a(\varepsilon))\right]$$

$$\geq E\left\{\begin{array}{l}\int_0^{\varepsilon} e^{-\rho t}\left[\delta_0 - \vartheta_x(x,a)(u(t)-d) - \vartheta_a(x,a) f(u) + \rho v(\alpha, x(\varepsilon), a(\varepsilon)) + \sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v\left(\cdot,\beta\right) - v\left(\cdot,\alpha\right)]\right]dt \\ + e^{-\rho\varepsilon} v(\alpha, x(\varepsilon), a(\varepsilon))\end{array}\right\}$$

Using the differentiability of $\vartheta(\cdot)$ together with (A.2.4), we can show that

$$v(\alpha, x_0, a_0) \leq E\int_0^{\varepsilon} e^{-\rho t}\left[\rho v(\alpha, x(t), a(t)) - \vartheta_x(x,a)(u(t)-d) - \vartheta_a(x,a) f(u) + \sum_{\alpha\neq\beta}[q_{\alpha\beta}(\cdot)v\left(\cdot,\beta\right) - v\left(\cdot,\alpha\right)]\right]dt$$
$$+ e^{-\rho\varepsilon} v(\alpha, x(\varepsilon), a(\varepsilon))$$

Therefore:

$J(a,x_0,a_0,B(\cdot)) \geq v(\alpha,x_0,a_0)+\delta_0 E\int_0^{\varepsilon} e^{-\rho t}dt \geq v(\alpha,x_0,a_0)+\eta$ , which is a contradiction. This reveals that $v(\alpha,x,a)$ is a viscosity super-solution of equation (A.2.1).

Hence, $v(\alpha,x,a)$ is a viscosity solution to equation (A.2.1).

Theorem 1 (Uniqueness Theorem)

Given assumptions A.1.1 and A1.2 and since the function $w(\cdot)$ is non-negative and either strictly convex or linear with $w(0)=0$, we can conclude that the HJB equations (A.2.1) has a unique viscosity solution.

Proof:

For this proof, the reader is referred to Yong and Zhou [54].

The optimality conditions are described by these equations for a manufacturing system consisting of one machine producing one type of part. Regarding the optimality principle to control problem [55,56], we can rewrite the HJB equations (A.2.1) as follows:

$$\rho v(a,x,\alpha) = \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(\alpha)} \left\{ \begin{array}{l} (u-d)\frac{\partial}{\partial x}v(a,x,\alpha)+f(u)\frac{\partial}{\partial a}v(a,x,\alpha)+g(\alpha,x,\cdot) \\ +\sum_{\alpha\neq\beta} q_{\alpha\beta}(\cdot)[v(a,x,\beta)-v(a,x,\alpha)] \end{array} \right\}, \forall \alpha,\beta\in A \quad \text{(A.2.9)}$$

The control policy $(u^*,k_{13}^*,k_{21}^*,k_{35}^*,k_{41}^*)$ denotes a minimizer over $\Gamma(\alpha)$ of the right-hand side of equation A.2.9. The value function obtained in equation 8 is in line with this policy. Hence, the optimal control policy is based on solving equation A.2.9. Obtaining an analytical solution of equation A.2.9 is almost impossible. In the scientific literature, the numerical solution of the HJB equation A.2.9 is considered as an insurmountable challenge [57].

In this section, the numerical method for solving the optimality conditions using the Kushner method [10] is presented. This method uses an approximation scheme for the gradient of the value function $v(a,x,\alpha)$. Let $h_x$ and $h_a$ denote the step length of finite difference interval of the variables $x$ and $a$ respectively and $v^h(a,x,\alpha)$ denote a solution to HJB. The function $v(a,x,\alpha)$ may be approximated by $v^h(a,x,\alpha)$ where $v_x(a,x,\alpha)$ for a given step size $h_x$ is obtained from equation A.2.10 as follows:

$$v_x(a,x,\alpha)\times(u-d)=\left\{\begin{array}{ll}\frac{1}{h_x}\left(v^h(a,x+h_x,\alpha)-v^h(a,x,\alpha)\right)\times(u-d) & if\ (u-d)\geq 0\\ \frac{1}{h_x}\left(v^h(a,x,\alpha)-v^h(a,x\text{-}h_x,\alpha)\right)\times(u-d) & otherwise\end{array}\right\}, \qquad \text{(A.2.10)}$$

And using $h_a$, the value function given a small increment in the direction of $a$,

$v_a(a,x,\alpha)$, is approximated as

follows:

$$v_a(a,x,\alpha)\times f(u)=\frac{1}{h_a}\left(v^h(a+h_a,x,\alpha)-v^h(a,x,\alpha)\right)\times f(u), \qquad \text{(A.2.11)}$$

If we simplify equations A.2.10 and A.2.11, we obtain respectively equations A.2.12 and A.2.13:

$$v_x(a,x,\alpha)=\left\{\begin{array}{ll}\frac{1}{h_x}\left(v^h(a,x+h_x,\alpha)-v^h(a,x,\alpha)\right) & if\ (u-d)\geq 0\\ \frac{1}{h_x}\left(v^h(a,x,\alpha)-v^h(a,x\text{-}h_x,\alpha)\right) & otherwise\end{array}\right\}, \qquad \text{(A.2.12)}$$

$$v_a(a,x,\alpha)=\frac{1}{h_a}\left(v^h(a+h_a,x,\alpha)-v^h(a,x,\alpha)\right), \qquad \text{(A.2.13)}$$

If we replace (A.2.9) by (A.2.12) and (A.2.13) for the case with $(u-d)\geq 0$, we have:

$$\rho v^h(a,x,\alpha)=\min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(\alpha)}\left\{\begin{array}{l}(u-d)\frac{1}{h_x}\left(v^h(a,x+h_x,\alpha)-v^h(a,x,\alpha)\right)\\ +f(u)\frac{1}{h_a}\left(v^h(a+h_a,x,\alpha)-v^h(a,x,\alpha)\right)g(\alpha,x,\cdot)\\ +\sum_{\alpha\neq\beta}q_{\alpha\beta}(\cdot)[v^h(a,x,\beta)-v^h(a,x,\alpha)]\end{array}\right\}, \forall\alpha,\beta\in A$$

with

$q_{\alpha\alpha}(\cdot) = -\sum_{\beta\neq\alpha} q_{\alpha\beta}(\cdot)$, where $\alpha, \beta \in$ A.

We then obtain:

$$v(a,x,\alpha)^h\left(\rho+|q_{\alpha\alpha}|+\frac{u-d}{h_x}+\frac{f(u)}{h_a}\right)= \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(\alpha)}\left\{\begin{array}{l}(u-d)\frac{1}{h_x}\left(v^h(a,x+h_x,\alpha)\right)\\ +f(u)\frac{1}{h_a}v^h(a+h_a,x,\alpha)g(\alpha,x,\cdot)\\ +\sum_{\alpha\neq\beta}q_{\alpha\beta}(\cdot)v^h(a,x,\beta)\end{array}\right\}, \forall\alpha,\beta\in A \qquad \text{(A.2.14)}$$

For the case with $(u-d)<0$ if we replace A.2.9 with A.2.12 and A.2.13, we have:

$$\rho v^h(a,x,\alpha)= \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(x,\alpha)}\left\{\begin{array}{l}(u-d)\frac{1}{h_x}\left(v^h(a,x,\alpha)-v^h(a,x\text{-}h_x,\alpha)\right)\\ +f(u)\frac{1}{h_a}\left(v^h(a+h_a,x,\alpha)-v^h(a,x,\alpha)\right)g(\alpha,x,\cdot)\\ +\sum_{\alpha\neq\beta}q_{\alpha\beta}(\cdot)[v^h(a,x,\beta)-v^h(a,x,\alpha)]\end{array}\right\}, \forall\alpha,\beta\in A$$

We then obtain:

$$v^h(a,x,\alpha)\left(\rho+|q_{\alpha\alpha}|+\frac{|u-d|}{h_x}+\frac{f(u)}{h_a}\right)= \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma(\alpha)}\left\{\begin{array}{l}|u-d|\frac{1}{h_x}v^h(a,x\text{-}h_x,\alpha)\\ +f(u)\frac{1}{h_a}v^h(a+h_a,x,\alpha)g(\alpha,x,\cdot)\\ +\sum_{\alpha\neq\beta}q_{\alpha\beta}(\cdot)v^h(a,x,\beta)\end{array}\right\}, \forall\alpha,\beta\in A \quad \text{(A.2.15)}$$

Grouping equations A.2.14 and A.2.15, the HJB equation can be expressed follows:

$$v^h(a,x,\alpha)= \min_{(u,k_{13},k_{21},k_{35},k_{41})\in\Gamma^h(\alpha)}\left\{\left(\rho+|q_{\alpha\alpha}|+\frac{|u-d|}{h_x}+\frac{f(u)}{h_a}\right)^{-1}\times\left[\frac{|u-d|}{h_x}\left(v^h(a,x+h_x,\alpha)K^+\right.\right.\right.$$
$$\left.\left.\left.+v^h(a,x-h_x,\alpha)K^-\right)+\frac{f(u)}{h_a}v^h(a,x+h_a,\alpha)+\text{g}(a,x,\alpha)+\sum_{\alpha\neq\beta}q_{\alpha\beta}(\cdot)v^h(a,x,\beta)\right]\right\}, \qquad \forall\alpha,\beta\in A \quad \text{(A.2.16)}$$

Where $\Gamma^k(\alpha)$ is the discrete feasible control space and $K^+$ and $K^-$ are defined as follows:

$$K^{+} = \begin{cases} 1 & if\ (u-d) \geq 0 \\ 0 & otherwise \end{cases}$$

$$K^{-} = \begin{cases} 1 & if\ (u-d) < 0 \\ 0 & otherwise \end{cases}$$

For a discrete-time, discrete-state decision processes, equation A.2.16 may be considered as the infinite-horizon dynamic programming equation that takes into account problems faced in production optimization as well as maintenance control [55].The following theorem demonstrates that value function $v^{h}(a,x,\alpha)$ is an approximation to $v(a,x,\alpha)$ for small increments $h_x$ and $h_a$ .

**Theorem 2**

Let $v^{h}(a,x,\alpha)$ be a solution to HJB equation A.2.16 and $C_g$ and $K_g$ be positive constants satisfying the Lipschitz property (see [48] for details):

$0 \leq v(a,x,\alpha) \leq \mathrm{C}_g (1+|x|^{K_g} + |a|^{K_g})$, then

$$\lim_{h \to 0} v^{h}(a,x,\alpha) = v(a,x,\alpha) \tag{A.2.17}$$

Knowing that $g(\alpha,x,a,\cdot)$ is non-negative and has a growth rate not exceeding $1+|x|^{K_g} + |a|^{K_g}$ associated with assumptions and lemmas A.1. Furthermore, controls $k_{13}, k_{21}, k_{35}, k_{41}$ are all bounded variables. This means that the value function $v(a,x,\alpha)$ is also non-negative and has a growth rate lower than a multiple of $1+|x|^{K_g} + |a|^{K_g}$ . Then we can conclude that $\lim_{h \to 0} v^{h}(a,x,\alpha) = v(a,x,\alpha)$ , for small increments of *h*.

**Proof**

We refer the reader to the published literature [18,21] for proof of theorem 2.

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