Metrological Evaluation of a Novel Medical Robot and Its Kinematic Calibration

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Abstract
The vessels are twisted in a longitudinal 3D space in the lower limbs of humans. Thus, it is difficult to perform an ultrasound scanning examination in this area. In this paper, a new medical parallel robot is introduced to effectively diagnose vessel disease in the lower limbs. The robot’s position repeatability and accuracy are evaluated. Furthermore, the robot’s accuracy is improved through a calibration process in which the kinematic parameters are identified through a simple identification approach.

Keywords Medical Robot, Parallel Robot, Kinematic Calibration, Laser Tracker

1. Introduction
Ultrasound (US) scanning examination is one of the major diagnostic modalities in daily medicine. It shows advantages in low cost and non-radiation to the human body. However, a survey reveals that the repetitive strain of daily US examination over many hours causes musculoskeletal disorders to sonographers [1]. Thus, much research is engaged towards the design of medical robots to perform the US scanning examination. Furthermore, the US medical robot can collect position data during the examination process, which provides the essential information for 3D reconstruction of the scanned area.

Several works on US medical robots have been performed. A portable US medical robot was proposed in a tele-scanning robot project [2]. It has four degrees of freedom (DOF) and assists a doctor in controlling a US probe remotely. The robot prototype is extended to 6 DOF in the OTELO project [3]. It is agile and able to cover the large scan area. Nevertheless, the US scanning examination requires an assistant to hold the robot during the examination process. In general, the portable medical US robot does not reduce the workload of sonographers. Many US medical robot systems are developed based on industrial robots. The Hippocrate system employs a PA-10 robot arm from Mitsubishi Heavy Industry to scan the carotid artery [4]. An F3 industrial robot from CRS Robotics was used in [5] to diagnose breast cancer. A lightweight robot LWR from KUKA was used in [6] to assist the sonographer. However, the industrial robots are mostly designed for general use, and medical applications are limited due to the closed architecture of the controllers. Thus, some serial robots are designed for US medical implement, such as an abdominal US scanning robot in [7], and a self-balanced robot from the University of British Columbia [8, 9]. Serial robots, however, have relatively low stiffness and their position errors are accumulated and amplified from link to link. Besides, the motors are generally mounted on links successively. Thus, each link has to support the weight of all the subsequent links and actuators. Several medical US robots were designed with parallel structures. A parallel robot was...
developed to hold the US probe in [10]. It consists of three legs displaced on both sides of the patient and a probe gripper hanging over the scanned area. A cable robot was developed in the TER project [11]. A sliding mechanism was used in a parallel robot to perform echo-graphic diagnosis [12, 13]. The patient has to support the weight of the robot since the mechanism is placed on the scanning area. WTA-2R was designed to hold the US probe and perform an automated scanning based on US image feedback in [14].

The medical robots mentioned above are mainly designed for carotid or abdominal US scanning, and are not appropriate for examination in the lower limbs due to their limited workspace, dexterity, etc. MedRUE (for Medical Robot for vascular Ultrasound Examination) is a new parallel robot designed to perform US scanning examination in lower limbs, improved based on its first concept proposed in [15]. It has a longitudinal workspace aligned with the patient’s leg, but relatively small size and low weight [16]. Since MedRUE needs to know the position of its end-effector with high accuracy, in order to reconstruct a 3D volume from US images it needs to be calibrated.

The calibration of a US medical robot involves probe calibration and kinematic calibration. The probe calibration identifies the constant transformation between the probe body and US image [17, 18]. However, in this article we focus only on the kinematic calibration of the robot. The kinematic calibration methods for serial robots are mostly on identifying the Denavit-Hartenberg parameters [19, 20]. These parameters are widely used to develop the kinematic model of serial robots. However, they are not always the simplest way to model parallel robots. The calibration methods of parallel robots vary depending on the different geometric structures of parallel robots. There are many calibration studies regarding planar robots, the Gough-Stewart platform and the Delta robot [21-25].

In this paper, we present a new medical robot with its repeatability and accuracy assessment. The position accuracy is improved by a calibration method based on direct position measurements with a laser tracker. The method is easy to implement on the robot without elaborate knowledge of the advanced kinematic calibration. In addition, the proposed calibration method identifies kinematic parameters individually, and the nonlinear interferences between kinematic parameters are significantly reduced. Therefore, the identified kinematic parameters are more accurate, and are important for further research, such as temporal stiffness calibration. By contrast, other calibration methods using optimization identify all kinematic parameters simultaneously [26-28]. The optimization approach may achieve better end-effector position accuracy, but it sacrifices precision for individual parameters. The MedRUE robot is briefly described in section 2 and its kinematic model and calibrated parameters are presented. Section 3 discusses the assessment of repeatability and accuracy. Then, the proposed calibration method and the result are demonstrated in section 4. At the end, a conclusion is addressed in section 5.

2. Robot description and kinematics

In this section, the new medical robot is introduced and its intuitive kinematic model is discussed. The kinematic model considers the errors of link lengths and offsets which need to be calibrated, and thereafter the calibrated parameters are listed. The robot base reference frame (Φ_b) and the world reference frame (Φ_w) are also defined in this section. The parameters to describe these two frames are identified with a laser tracker.

2.1 Introduction of MedRUE

MedRUE (Medical Robot for vascular Ultrasound Examination) is a prototype of medical parallel robot designed for the diagnosis of the peripheral arterial disease in the lower limbs.

As shown in Figure 1, MedRUE is a 6-DOF parallel robot consisting of a robot base with a linear guide, two five-bar mechanisms and the tool part to carry the US probe. The U-shaped robot base is mounted on a linear guide (driven by actuator Q_1). The two five-bar mechanisms are assembled symmetrically on the robot’s base, and are driven by actuators Q_2, Q_3, Q_4, and Q_5. Considering the first five-bar mechanism as an example, the two links driven by actuators are called proximal links, and the two links farther from the robot base are called distal links. The distal links connect at the shaft of the tool part. The tool part consists of a force/torque sensor and a dummy probe at the end. It is driven to rotate along the shaft by a small actuator Q_6 mounted on a distal link.

Since most actuators of MedRUE are located at the robot’s base, the links of the five-bar mechanisms do not need to bear the heavy load of motors. Thus, the robot is relatively
lightweight and agile. The linear guide extends the workspace along the length of the patient’s leg and the curved distal links avoid mechanical interferences between the robot arms and the patient leg during the examination.

Frame \( \Phi_o \), also referred to as the robot frame, is defined on the robot base plate. The top surface of the robot base plate is defined as the \( xy \) plane, and its normal is the \( z \) axis. The linear guide determines the direction of the \( x \) axis, and then the front side of the robot is the direction of \( y \) axis. The origin of \( \Phi_o \) is located in the front centre hole of the bottom plate.

### 2.2 Kinematic model of MedRUE

The kinematic model of MedRUE and its kinematic parameters are illustrated in Figure 2. The two five-bar mechanisms are symmetrically assembled on the robot’s base, and can therefore be modelled in the same way. As shown in Figure 2(a), the links of the \( i^\text{th} \) five-bar mechanism are named \( L_i \) where \( i = 1, 2 \) and \( j = 0, \ldots, 4 \). The corresponding link lengths are denoted by \( l_i \). Link \( L_0 \) is fixed on the robot’s base with an angle \( \theta \) offset. Four actuators \( Q_{abc} \) are mounted on the robot’s base, and the corresponding active joint variables are \( q_{abc} \) and \( q_{0abc} \) at \( A \) and \( C \), respectively. The other joints \( q_{0a}, q_{0b}, \) and \( q_{0bc} \) are passive. The two five-bar mechanisms are parallel to the \( yz \) plane. In Figure 2(b), the tool part connects the two five-bar mechanisms at \( E_i \) (with an offset \( d_i \)). Two universal joints are located at \( F_i \) (with an offset \( d_i \)) to provide orientation of the probe-support. The synchronized motion (i.e., same speed and direction) of two five-bar mechanisms provide translation motions to the tool part, while the unsynchronized motion provides rotation motions. During the rotational motion, the distance variation between the two universal joints \( F_i \) is compensated by a passive translation joint located between the probe-support and \( F_2 \). Actuator \( Q_2 \) is attached on \( L_2 \) to provide a rotation motion of the tool part along \( x_2 \). The probe-support is considered as the wrist of MedRUE, and a reference frame \( \Phi_6 \) is located at its geometric centre. Axis \( x_6 \) is collinear with vector \( F_1 F_2 \) and \( z_6 \) is pointing to the probe tip.

Given the joint values \( (q_{0}, \ldots, q_{6}) \), the forward kinematic solution of MedRUE can be obtained as follows:

Since \( A \) and \( C \) are static w.r.t. \( \Phi_6 \), for the \( i^\text{th} \) \((i=1,2)\) five-bar mechanism, the coordinates of \( A \) and \( C \) are represented as:

\[
a_i = \begin{bmatrix} x_0 & y_0 & z_0 & \frac{l_0}{2} \sin \theta & \frac{l_0}{2} \cos \theta \end{bmatrix},
\]

\[
c_i = \begin{bmatrix} x_0 & y_0 & z_0 & -\frac{l_0}{2} \sin \theta & -\frac{l_0}{2} \cos \theta \end{bmatrix}^T
\]

where \( O \) is the midpoint of \( L_{0i} \) with

\[
x_i = d_i + q_i + \tilde{q}_i,
\]

and \( \tilde{q}_i \) is the offset error of \( k^\text{th} \) active joint. Then the coordinates of \( B_i \) and \( D_i \) are

\[
b_i = a_i + l_1 \Psi(q_{2i} + \tilde{q}_{2i}),
\]

\[
d_i = c_i + l_3 \Psi(q_{2i+1} + \tilde{q}_{2i+1}),
\]

where \( \Psi(q) = \begin{bmatrix} -\sin(q) & \cos(q) \end{bmatrix}^T \).

With the coordinates of \( B \) and \( D \), all side lengths of the triangle \( B_i D E_i \) are known. Then, the coordinates of \( E_i \) are obtained as

\[
e_i = d_i + \frac{D_i H_i}{2} + \frac{H_i E_i}{2} u_{B_i E_i}^T,
\]

where

\[
D_i H_i = \sqrt{D_i B_i^2 - \left[ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right] u_{B_i E_i}^T},
\]

\[
H_i E_i = \sqrt{E_i H_i^2 - \left[ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right] u_{B_i E_i}^T},
\]

and \( u \) represents a unit vector. As shown in Figure 2(a), \( F \) has an offset \( d_{1,2} \) w.r.t. \( E_i \) along \( x_0 \). Thus, the coordinates of \( F_i \) are

\[
f_i = D(d_{1,2} - d_{1,2} 0 0 e_1),
\]

where \( D(x,y,z) \) is the translation operation.

Assuming the orientation of the wrist reference frame \( \Phi_6 \) is represented in the XYZ Euler angles, then

\[
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = u_{\Phi_6} = \begin{bmatrix} u_5 \\ u_6 \\ u_7 \end{bmatrix},
\]

where \( R(\gamma, \beta, \alpha) \) is the rotation matrix. Since actuator \( Q_3 \) is fixed on \( L_{a,c} \) the first Euler angle is obtained as

\[
\gamma = q_{DE_i} + \tilde{q}_6 + \tilde{q}_6
\]
where $q_{DE1} = \text{atan2}(-(y_{E1} - y_{D1}), z_{E1} - z_{D1})$. The other two Euler angles are obtained by substituting Eq. (11) into Eq. (10):

$$
\alpha = \sin^{-1} \left( u_x \cos \gamma + u_z \sin \gamma \right),
$$

(12)

$$
\beta = \sin^{-1} \left( \frac{u_x \sin \gamma - u_z \cos \gamma}{\cos \alpha} \right).
$$

(13)

Thus, the coordinates of $O_w$ w.r.t. $\Phi_0$ are

$$
p_w = f + R(\gamma, \beta, \alpha)d_{j1} + d_w 0 \begin{bmatrix} x_w \ y_w \ z_w \end{bmatrix}^T.
$$

(14)

In our calibration process, a spherically mounted retroreflector (SMR) is attached to the probe. The coordinates of the SMR centre w.r.t. $\Phi_w$ are $x_S, y_S, z_S$. Thus the coordinates of SMR w.r.t. $\Phi_0$ are

$$
p_S = p_w +\begin{bmatrix} x_S \ y_S \ z_S \end{bmatrix}.
$$

(15)

Assuming the transformation matrix from the world reference frame $\Phi_w$ to $\Phi_b$ is represented by six parameters $x_W, y_W, z_W, \gamma_W, \beta_W, \alpha_W$. Then $p_S$ is represented w.r.t. $\Phi_w$ as:

$$
p_S^W = p_w^W.
$$

(16)
3. Repeatability and Accuracy Assessment

In this section, robot repeatability and accuracy are assessed before calibration. Our assessment method is an adaptation of the international standard on robot performance and test method (ISO-9283) [29]. The nominal kinematic model (i.e., the model before calibration) is assessed in this section, and the calibrated kinematic model is validated in section 4.

In the nominal kinematic model [16], the corresponding parameters of the two five-bar mechanisms in Figure 2 are identical (e.g., $l_{i1} = l_{i2}$, $j = 0, ..., 4$). Moreover, each five-bar mechanism is symmetrically designed. In other words, the proximal links are identical ($l_{i1} = l_{i2}$, $i = 1, 2$), and so are the distal links ($l_{i3} = l_{i4}$, $i = 1, 2$). The offset joint values are considered to be equal to zero.

3.1 Robot path design and measurement points

In our implementation, the position accuracy has the priority since it is required both for safety reasons and 3D reconstruction. Orientation errors can be compensated in this section, and the calibrated kinematic model is validated in section 4.

In this method, $P_1$ is visited from eight different directions, while all other measurement points are visited in a unidirectional approach (direction from $P_1$). Thus, the experiment design is used to estimate multidirectional repeatability at position $P_1$, and unidirectional repeatability at positions $P_2$ to $P_9$.

3.2 Accuracy and repeatability definition

At each measurement point $P_i$ ($i = 1, ..., 9$), the position of the end-effector is measured $n$ times, where $n = 8$ directions × 30 cycles = 240 at $P_1$, and $n = 30$ at each of the other eight measurement points. A set of $n$ measurements on a measurement point $P_i$ is called a cluster of $P_i$. For any cluster, the barycentre is defined as a virtual point whose coordinates $[\bar{x}, \bar{y}, \bar{z}]$ are the mean values of all the $n$ measurements:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i,$$
$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i,$$
$$\bar{z} = \frac{1}{n} \sum_{i=1}^{n} z_i.$$

The distance between the $\eta^{th}$ cycle measurement at $P_i$ and the barycentre of the cluster of $P_i$ is

$$l_{i\eta} = \sqrt{(x_{i\eta} - \bar{x})^2 + (y_{i\eta} - \bar{y})^2 + (z_{i\eta} - \bar{z})^2}.$$

Then the repeatability at $P_i$ is defined as follows:

$$RP_i = \bar{l} + 3S_{l_i}.$$
The absolute position accuracy of $P_i$ is defined by

$$APA_i = \sqrt{(x_i - \bar{x}_i)^2 + (y_i - \bar{y}_i)^2 + (z_i - \bar{z}_i)^2}, \quad (22)$$

where $\bar{x}_i$, $\bar{y}_i$ and $\bar{z}_i$ are the command positions (reference positions) of $P_i$, and $x_i$, $y_i$ and $z_i$ are barycentre coordinates defined in Eqs. (17) to (19).

MeRUE will be used to take US images at prescribed intervals. These US images will then be used to reconstruct the 3D model of the blood vessel. For the purposes of medical examination, the accuracy in measuring the position of a given US image is important w.r.t. the neighbouring images. In other words, the relative accuracy is important for our robot.

A relative position accuracy of a point can be defined as the accuracy of the distance between adjacent points (e.g., the distance accuracy between $P_i$ and $P_2$). Since each point $P_i$ ($i = 2,...,9$) is reached starting from $P_1$ in Figure 3(b), the nominal relative displacement of $P_i$ ($i = 2,...,9$) is calculated as follows:

$$\delta_i = \hat{p}_i - \hat{p}_{i-1}, \quad i = 2,...,9 \quad (23)$$

where $\hat{p}_i$ is the nominal coordinates vector. Then Eq. (22) is modified to compute the relative position accuracy as

$$RPA_i = ||\delta_i - \hat{\delta}_i||, \quad i = 2,...,9 \quad (24)$$

where $\delta_i = p_i - p_{i-1}$ is the measured relative displacement based on laser tracker. The notation $\hat{p}_i$ is the measurement value of $P_i$ before moving towards to each $P_i$ ($i = 2,...,9$).

### 3.3 Experiment setup and results

The measurement setup is shown in Figure 4. The measurements are taken with a Faro Laser Tracker ION having a distance accuracy of $8 \mu m + 0.4 \mu m/m$, and angular accuracy of $10 \mu m + 2.5 \mu m/m$. The emitted laser is reflected by an SMR, which is magnetically attached to a nest. In our experiment, the target measurement is a 1.5 inch SMR mounted on the tool part (Fig. 4). The measured positions are expressed w.r.t. the laser reference frame $(\Phi_W)$ and transformed w.r.t. $(\Phi_{Wc})$.

We note that the measurement accuracy might be influenced by many aspects, such as environment, duration of operation, and the distance between the target and the laser tracker. Therefore, we evaluated the laser tracker accuracy for our own setup. A calibrated bar with a known length was measured ten times, and the distance error was found to be $26 \mu m \pm 14 \mu m$ with 95.4% confidence interval of uncertainty. However, the measurement uncertainty was reduced by taking several measurements at each position.

The results for the position repeatability at the nine measurement points are shown in Table 1. The first row shows the composed repeatability defined in Eq. (21), and the other three rows list the repeatability according to the $x$, $y$ and $z$ axes. Naturally, the position repeatability at $P_i$ is worse, which is mainly because the arrivals at $P_i$ are from eight different directions. Furthermore, the position repeatability along $x$ at $P_i$ is lower than along $y$ and $z$. This is caused by the fact that the motion along the $x$ axis is dominated by the linear guide, which has a backlash of 76 $\mu m$ according to its manufacturer.

<table>
<thead>
<tr>
<th>$P_i$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$P_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>APA</td>
<td>4.264</td>
<td>1.271</td>
<td>3.849</td>
<td>5.770</td>
<td>0.885</td>
<td>3.788</td>
<td>2.183</td>
<td>2.155</td>
<td>4.792</td>
</tr>
<tr>
<td>APA1</td>
<td>2.217</td>
<td>0.432</td>
<td>1.386</td>
<td>1.823</td>
<td>0.103</td>
<td>1.463</td>
<td>0.568</td>
<td>0.118</td>
<td>2.022</td>
</tr>
<tr>
<td>APA2</td>
<td>1.127</td>
<td>0.129</td>
<td>1.586</td>
<td>2.978</td>
<td>0.677</td>
<td>0.157</td>
<td>1.877</td>
<td>2.124</td>
<td>0.192</td>
</tr>
<tr>
<td>APA3</td>
<td>3.463</td>
<td>1.188</td>
<td>3.222</td>
<td>4.594</td>
<td>0.560</td>
<td>3.491</td>
<td>0.960</td>
<td>0.346</td>
<td>4.340</td>
</tr>
</tbody>
</table>

Table 1. Position repeatability (in $\mu m$)

<table>
<thead>
<tr>
<th>$P_i$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$P_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>APA</td>
<td>2.058</td>
<td>0.677</td>
<td>0.427</td>
<td>0.871</td>
<td>0.977</td>
<td>0.569</td>
<td>0.157</td>
<td>1.829</td>
<td>2.022</td>
</tr>
<tr>
<td>APA1</td>
<td>2.155</td>
<td>0.427</td>
<td>0.886</td>
<td>0.377</td>
<td>1.074</td>
<td>0.427</td>
<td>0.977</td>
<td>0.569</td>
<td>0.157</td>
</tr>
<tr>
<td>APA2</td>
<td>2.124</td>
<td>0.346</td>
<td>0.445</td>
<td>2.273</td>
<td>0.871</td>
<td>0.565</td>
<td>0.896</td>
<td>0.506</td>
<td>0.157</td>
</tr>
<tr>
<td>APA3</td>
<td>1.822</td>
<td>1.165</td>
<td>3.018</td>
<td>1.656</td>
<td>2.068</td>
<td>2.058</td>
<td>2.042</td>
<td>1.829</td>
<td>2.022</td>
</tr>
</tbody>
</table>

Table 2. Absolute position accuracy (APA) and relative position accuracy (RPA) before calibration (in mm)
The results for the repeatability at $P_1$ in the two measurement planes (i.e., planes $C_1C_2C_7C_8$ and $C_3C_4C_5C_6$) are illustrated in Figure 5. On each plane, there are four groups ($G_1$, $G_2$, $G_3$, $G_4$) of data in different colors indicating four different directions to reach $P_1$. The measured data are illustrated with an asterisk marker, while the barycentre of each group is a solid circle. The cluster groups are clearly separated from each other. The divergence of measurement on $P_1$ shows the imperfection of the multidirectional movement caused mainly by the backlash of the mechanical parts of our robot.

The position absolute accuracy and relative accuracy before calibration is given in Table 2. As an early prototype, the robot’s poor accuracy is mainly due to its manufacture and assembling errors. The worst case of the absolute accuracy before calibration is about 3 mm, while the worst relative accuracy before calibration is about 3 mm.

4. Kinematic Calibration Experiment

In this section the proposed calibration approach is explained. The actual values of the robot parameters are identified to improve robot accuracy.

<table>
<thead>
<tr>
<th>Group</th>
<th>Description</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>The world reference frame</td>
<td>$y_W, y_W, y_W, y_W, p_W, a_W$</td>
</tr>
<tr>
<td>(2)</td>
<td>Active joint offset error</td>
<td>$\tilde{\gamma}_3, \tilde{\gamma}_4, \tilde{\gamma}_5$</td>
</tr>
<tr>
<td>(3)</td>
<td>Assembling of five-bar mechanisms</td>
<td>$y_1, z_0, y_1, z_0, z_0$</td>
</tr>
<tr>
<td>(4)</td>
<td>Link lengths of five-bar mechanisms</td>
<td>$l_{10}, l_{10}, l_{10}, l_{10}, l_{10}$</td>
</tr>
</tbody>
</table>

Table 3. Kinematic parameters

The parameters expected to be identified are summarized in four groups, as shown in Table 3. Parameters in Group 1 define $\Phi_L$ in Eq. (30). Group 2 represents the offset of active joints at the home position. Group 3 is named assembling parameters, which specify how the two five-bar mechanisms are assembled on the robot base, by illustrating the position and angle of link $L_0$ on the robot base (Figure 2(a)). Group 4 englobes the lengths of the links of the two five-bar mechanisms.

4.1 The world reference frame parameters

The world reference frame $\Phi_L$ is defined in the robot’s workspace by three SMR nests $P_{W1}$, $P_{W2}$ and $P_{W3}$ as shown in Figure 4. Assuming $P_{W1}$ is the origin of $\Phi_L$, and $P_{W1}$ is a point on the $x_W$ axis, then the three unit vectors of $\Phi_L$ are defined w.r.t. the $\Phi_L$. The unit vector $x_w$ is determined by the normalized vector $\frac{P_{W1} - P_{W1}}{|P_{W1} - P_{W1}|}$, where $P_{W1}$ is a vector from $P_{W1}$ to $P_{W1}$ in $\Phi_L$. The $z_W$ axis of $\Phi_L$ is defined as the normal of the plane defined by $P_{W1}, P_{W2}$ and $P_{W3}$. The rotation matrix of $\Phi_L$ w.r.t. $\Phi_L$ is written as:

$$^W_R = \begin{bmatrix} x_W, y_W, z_W \end{bmatrix}.$$  

The transformation matrix of $\Phi_L$ w.r.t. $\Phi_L$ is defined as follows:

$$^W_T = \begin{bmatrix} R^W_W \end{bmatrix}.$$  

where vector $o_W$ is the composed by the coordinates of $P_{W1}$. All measured position data can be represented w.r.t. $\Phi_L$ by using the transformation matrix $^W_T$. A point $^W_0P$ is derived from the laser measured data $\bar{p}$ by the following equation:

$$\bar{p}_n^W = \bar{p}_n^W T^{-1} R^W_W \bar{p}^W_1.$$  

For convenience, all the further measured positions, in this paper, are implicitly expressed w.r.t. $\Phi_L$.

Frame $\Phi_L$ is defined on the robot base plate, as shown in Figure 1. The top surface of the plate is considered as the $x_{yd}$ plane. By placing the SMR in $n$ positions on the $x_{yd}$ plane, a reference plane is fitted from the laser tracker measurement data ($3n$ matrix $P_n = [p_n, ..., p_n]$, where vector $p_n$ is the $n$th measurement). The barycentre of $P_n$ is $\bar{p}_n = \frac{1}{n} \sum_{i=1}^{n} p_i$. For the smallest eigenvalue of $\bar{p}_n^T \bar{p}_n$ (where $\bar{p}_n = [p_1 - \bar{p}, p_2 - \bar{p}, ..., p_n - \bar{p}]$), its corresponding eigenvector approximates the normal vector $z_0$ of the fitted plane $x_{yd}$. Finally, the $x_{yd}$ plane is determined by a point $\bar{p}$ and the normal vector $z_0$. The $x_0$ axis is aligned with the direction of the linear guide and is estimated by measuring an SMR on the robot base, while actuating the linear guide. The method to fit a line from 3D cluster points is similar to the method to estimate the normal vector $z_0$, except that the vector $x_0$ is estimated as the eigenvector with largest eigenvalue. Then, the rotation matrix of $\Phi_y$ w.r.t. $\Phi_L$ is found as:

$$^W_R = \begin{bmatrix} x_0, y_0, z_0 \end{bmatrix}.$$  

The origin $O_0$ of $\Phi_y$ is located on the robot base plate as shown in Figure 2(a). Its coordinates w.r.t. $\Phi_y$ are obtained by placing an SMR above the robot base plate. The projection of the SMR on the $x_{yd}$ plane provides the coordinates of $O_0$ as follows:

$$o_0 = o_0' - ((o_0' - \bar{p}) \cdot z_0) z_0.$$  

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where \( o'_0 \) is the vector of measured coordinates of the SMR. Then the transformation matrix of \( \Phi_0 \) w.r.t. \( \Phi_W \) is obtained as:

\[
_{W}^{W}T(x_W, y_W, z_W, \gamma_W, \beta_W, \alpha_W) = \begin{bmatrix} _{W}^{W}R & _{0}^{W}O \\ 0 & 1 \end{bmatrix} \tag{30}
\]

where \( x_W, y_W, z_W, \gamma_W, \beta_W, \alpha_W \) are the \( \Phi_W \) parameters to be identified: \( x_W, y_W, z_W \) are the coordinates of \( \alpha_W \) and Euler angles \( \gamma_W, \beta_W, \alpha_W \) are obtained from the rotation matrix \( _{W}^{W}R \).

### 4.2 Active joint offset errors

The objective of this subsection is to evaluate the difference between the nominal and the real joint offsets. Active joints \( q_4, q_5, q_6 \) and \( q_4 \) are considered. As mentioned earlier, the five-bar mechanisms are symmetrically assembled, and therefore are calibrated with the same method. For simplicity, only the calibration of the second five-bar mechanism is demonstrated.

As shown in Figure 6, ten nests \((N_i, i = 1, ..., 10)\) are attached on the five-bar mechanism for measurement purpose. Four nests \((N_i, i = 3, 4, 7, 10)\) are used in this experiment, and the remaining nests are used to identify other parameters.

Figure 5. Measurements at \( P_a \) for arrivals from multiple directions: (a) on measurement plane \( C_6C_5C_7 \) view in \( xy \) plane; (b) on measurement plane \( C_6C_5C_7 \) view in \( xz \) plane; (c) on measurement plane \( C_6C_5C_7 \) view in \( xy \) plane; (d) on measurement plane \( C_6C_5C_7 \) view in \( xz \) plane.

Figure 6. Experiment setup for active joint offset value estimation (nests in orange) and link length estimation (nests in orange and blue) on the second five-bar mechanism. 

Active joint \( q_4 \) directly drives the rotation motion of \( L_{21} \). Thus, the joint offset error of \( q_4 \) is assessed via the evaluation of the orientation error of link \( L_{21} \) (Figure 2(a)). The orientation of \( L_{21} \) is determined by the coordinates of \( A_1 \) and \( B_2 \). Similarly, the joint offset error of \( q_5 \) is assessed via the
orientation of link \( L_{23} \), which is determined by the coordinates of \( C_2 \) and \( D_2 \). Two experiments are designed to obtain the coordinates of \( A_2, B_2, C_2 \) and \( D_2 \) at the robot home position. The joint offset errors are then evaluated.

The first experiment is designed to determine the coordinates of \( A_2 \) and \( D_2 \) (Figure 7(a)) as follows:

a. Two nests denoted \( N_i \) and \( N_{10} \) are attached to links \( L_{21} \) and \( L_{34} \) respectively, as shown in Figure 6 and Figure 7(a).

b. The robot starts from its home position, which is illustrated with the highest transparent image in Figure 7(a). The angle of joint \( \theta_i \) is increased gradually with a step of 2°, within a range of 130°. Meanwhile, link \( L_{23} \) is kept in its home position throughout the experiment process. At each motion step the robot motion is halted, and measurements of \( N_i \) and \( N_{10} \) are taken.

c. The measured positions of \( N_i \) and \( N_{10} \) — which follow circular curves centred at \( A_i \) and \( D_i \) respectively — are fitted to two circles [31] in order to determine the coordinate of \( A_i \) and \( D_i \).

For illustration purposes, an intermediate position of MedRUE is illustrated with intermediate transparency, and the opaque image shows the final position of MedRUE. The nests \( N_i \) and \( N_{10} \) are demonstrated with a dashed circle, solid circle and filled circle in these positions, respectively.

To determine the coordinates of \( B_i \) and \( C_i \) we used the same approach as for \( A_i \) and \( D_i \). The used nests are denoted \( N_4 \) and \( N_7 \) and are attached to links \( L_{20} \) and \( L_{35} \) respectively, as shown in Figure 6. The experiment is illustrated in Figure 7(b).

As shown in Figure 2(a), the coordinates of \( A_i \) and \( B_i \) evaluate the orientation of links \( L_{34} \) in the \( y_{\theta_i}z_{\theta_i} \) plane. The orientation of the link \( L_{23} \) is driven directly by active joint \( q_5 \). Thus, the joint offset error of \( q_5 \) is evaluated by the coordinates of \( C_i \) and \( D_i \).

\[
\hat{\theta}_i = \text{atan2}\left(-\left(y_{B_i} - y_{A_i}\right),\left(z_{B_i} - z_{A_i}\right)\right) - \hat{\theta}_i, \quad (31)
\]

where \( y_{B_i} \) and \( z_{B_i} \) are the \( y \) and \( z \) coordinates of \( A_i \) w.r.t. \( \Phi_{\theta_i} \). Similarly, the joint offset error of \( q_4 \) is evaluated by the coordinates of \( C_i \) and \( D_i \).

\[
\hat{\theta}_i = \text{atan2}\left(-\left(y_{D_i} - y_{C_i}\right),\left(z_{D_i} - z_{C_i}\right)\right) - \hat{\theta}_i, \quad (32)
\]

The active joint offsets of \( q_2 \) and \( q_3 \), related to the first five-bar mechanism, are assessed similarly to \( q_4 \) and \( q_5 \) by using the coordinates of \( A_i, B_i, C_i \) and \( D_i \).

### 4.3 Assembling parameters

The assembling parameters are \( y_{CM}, z_{CM}, y_{DO}, z_{DO}, \theta_{DO}, \theta_{BO}, \) and these describe the assembly of the two five-bar mechanisms on the robot base. The \( i^{th} \) \((i = 1, 2)\) five-bar mechanism is assembled on the robot base joints \( A_i \) and \( C_i \). As shown in Figure 2(a), \( O_i \) is the middle point of link \( L_{cd} \) defined by \( A_i \) and \( C_i \). Angle \( \theta_i \) illustrates the orientation of link \( L_{23} \) in the \( y_{\theta_i}z_{\theta_i} \) plane. Knowing that the coordinates of \( A_i \) and \( C_i \) are obtained in the previous subsection, the assembling parameters are calculated as follows:

\[
y_M = \frac{y_{A_i} + y_{C_i}}{2}, \quad (33)
\]

\[
z_M = \frac{z_{A_i} + z_{C_i}}{2}, \quad (34)
\]

\[
\theta_i = \text{atan2}\left(-\left(y_{C_i} - y_{A_i}\right),\left(z_{C_i} - z_{A_i}\right)\right). \quad (35)
\]
4.4 Link length parameters

The assessment of the link length parameters is demonstrated using the second five-bar mechanism (i.e., $l_{24}$, $l_{23}$, $l_{12}$, $l_{13}$, $l_{14}$, $l_{15}$), while those for the first mechanism (i.e., $l_{10}$, $l_{11}$, $l_{12}$, $l_{13}$, $l_{14}$, $l_{15}$) are obtained similarly.

The link length of $L_{20}$ is determined by using the coordinates of $A_2$ and $C_2$ w.r.t. $\Phi_{20}$, which were obtained in subsection 4.2:

$$L_{20} = \sqrt{(x_{c_2} - x_{a_2})^2 + (y_{c_2} - y_{a_2})^2 + (z_{c_2} - z_{a_2})^2}.$$  \hspace{1cm} (36)

The identification of $l_{21}$, $l_{22}$, $l_{23}$ and $l_{24}$ is carried out by moving simultaneously joints $q_3$ and $q_4$ inside their limit ranges, as shown in Figure 8(a).

Note that link $L_{20}$ has both endpoints $A_2$ and $C_2$ fixed w.r.t. $\Phi_{20}$. However, for the other links, one or both endpoints change position during the experiment. To find the position of endpoints $B_2$, $D_2$, and $E_2$, reference frames are attached to the corresponding links. For example, Figure 8(a) shows a link frame $\Phi_{23}$, which is associated with link $L_{23}$ and defined by $N_2$, $N_3$ and $N_4$. Therefore, the movement of the nest $N_{20}$ seen from $\Phi_{23}$ as shown in Figure 8(b), makes a circle centred at $D_2$. The coordinates of $D_2$ w.r.t. $\Phi_{23}$ are then transformed to be w.r.t. $\Phi_{0}$.

The coordinates of $B_2$ and $E_2$ w.r.t. $\Phi_{0}$ are determined in similar ways as $D_2$. Point $B_2$ is determined by observing nest $N_2$ in link reference frame $\Phi_{21}$ (built by nests $N_1$, $N_2$ and $N_3$). Then, $E_2$ is determined by observing nest $N_4$ in link reference frame $\Phi_{24}$ (built by nests $N_3$, $N_4$ and $N_3$). With all coordinates of $A_2$, $B_2$, $C_2$, $D_2$ and $E_2$ given w.r.t. $\Phi_{0}$, the length of the links are estimated as follows:

$$l_{21} = \sqrt{(x_{b_2} - x_{a_2})^2 + (y_{b_2} - y_{a_2})^2 + (z_{b_2} - z_{a_2})^2}$$
$$l_{22} = \sqrt{(x_{b_2} - x_{b_2})^2 + (y_{b_2} - y_{b_2})^2 + (z_{b_2} - z_{b_2})^2}$$
$$l_{23} = \sqrt{(x_{d_3} - x_{c_3})^2 + (y_{d_3} - y_{c_3})^2 + (z_{d_3} - z_{c_3})^2}$$
$$l_{24} = \sqrt{(x_{e_4} - x_{d_4})^2 + (y_{e_4} - y_{d_4})^2 + (z_{e_4} - z_{d_4})^2}$$

4.5 Parameter calibration results and validation

To validate the proposed calibration method, we perform the same experiment with nominal and calibrated parameters. Both the nominal and identified values of calibrated parameters are listed in Table 4.

<table>
<thead>
<tr>
<th>Link length of Five-bar mechanisms (unit)</th>
<th>Nominal value</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{20}$ (mm)</td>
<td>150</td>
<td>151.580</td>
</tr>
<tr>
<td>$l_{21}$ (mm)</td>
<td>400</td>
<td>400.510</td>
</tr>
<tr>
<td>$l_{22}$ (mm)</td>
<td>520</td>
<td>518.605</td>
</tr>
<tr>
<td>$l_{23}$ (mm)</td>
<td>400</td>
<td>400.656</td>
</tr>
<tr>
<td>$l_{24}$ (mm)</td>
<td>520</td>
<td>523.003</td>
</tr>
<tr>
<td>$l_{25}$ (mm)</td>
<td>150</td>
<td>151.007</td>
</tr>
<tr>
<td>$l_{26}$ (mm)</td>
<td>400</td>
<td>400.401</td>
</tr>
<tr>
<td>$l_{27}$ (mm)</td>
<td>520</td>
<td>523.075</td>
</tr>
<tr>
<td>$l_{28}$ (mm)</td>
<td>400</td>
<td>400.926</td>
</tr>
<tr>
<td>$l_{29}$ (mm)</td>
<td>520</td>
<td>526.285</td>
</tr>
</tbody>
</table>

Assembling of five-bar mechanisms (unit)

| $y_{c_0}$ (mm)                           | -158          | -153.714         |
| $z_{c_0}$ (mm)                           | 308           | 305.442          |
| $y_{c_0}$ (mm)                           | -158          | -156.169         |
| $z_{c_0}$ (mm)                           | 308           | 309.001          |
| $\theta_1$ (°)                           | 150           | 148.906          |
| $\theta_2$ (°)                           | 150           | 150.583          |

Active joint offset error (unit)

| $q_3$ (°)                                | 0             | 1.445            |
| $q_4$ (°)                                | 0             | -0.730           |
| $q_4$ (°)                                | 0             | 1.321            |
| $q_5$ (°)                                | 0             | -0.993           |

The world reference frame (unit)

| $x_{w}$ (mm)                             | -110          | -115.587         |
| $y_{w}$ (mm)                             | -136          | -140.868         |
| $z_{w}$ (mm)                             | 30            | 27.452           |
| $\gamma_{w}$ (°)                        | 0             | 0.292            |
| $\beta_{w}$ (°)                          | 0             | -0.057           |
| $\phi_{w}$ (°)                           | 0             | 0.286            |

Table 4. Nominal and identified parameter values
MedRUE is an early prototype of a medical US robot, and there are many imperfections in its manufacture and assembling. As we can see in Table 4, there are noticeable differences between the nominal parameter values and the calibrated parameter values. The majority of errors come from the assembling of the two five-bar mechanisms on the robot base. Furthermore, the manual nest setup (as in Figure 4) for $\Phi_5$ brings noticeable errors as well. These errors are the main cause of the low accuracy before calibration, as demonstrated in Table 2.

The robot’s position accuracy assessment after calibration obtained using the ISO 9283 evaluation approach is shown in Table 5. The maximum absolute position error (i.e., absolute accuracy) has been improved from 5.770 mm before calibration to 0.764 mm after calibration. The relative accuracy is also important in medical application, and its accuracy is satisfactory after calibration. The maximum relative position error was improved from 3.018 mm before calibration to 0.489 mm after calibration, as shown in Table 5.

Figure 9 illustrates the improvement of accuracy when the robot is tracking a reference command line, which is marked in a blue solid line. As shown in Figure 9(a) and Figure 9(b), which represent the trajectory projection on planes $xy$ and $xz$, respectively, the trajectory before calibration has a significant error (poor absolute accuracy), and it is greatly improved after the calibration.

Figure 9(c) and Figure 9(d) illustrate the relative position offset between adjacent points. Since the reference command line is created by a set of points with constant offsets, the relative position offsets are represented as a fixed point (illustrated by an asterisk). The obtained offsets before calibration are illustrated by green squares: ideally all these squares should coincide with the reference offset (i.e., the blue asterisk mark). However, they scattered around the reference offset because of the robot parameter residuals. The obtained results (offsets) after calibration are illustrated in red triangles and it clearly demonstrates the improvement of the robot relative accuracy: i.e., the obtained relative position offsets converge towards to the reference offset.

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
</tr>
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<tbody>
<tr>
<td>APA</td>
<td>0.612</td>
<td>0.497</td>
<td>0.449</td>
<td>0.351</td>
<td>0.764</td>
<td>0.341</td>
<td>0.343</td>
</tr>
<tr>
<td>APA$_x$</td>
<td>0.527</td>
<td>0.469</td>
<td>0.353</td>
<td>0.309</td>
<td>0.752</td>
<td>0.284</td>
<td>0.341</td>
</tr>
<tr>
<td>APA$_y$</td>
<td>0.235</td>
<td>0.152</td>
<td>0.228</td>
<td>0.158</td>
<td>0.053</td>
<td>0.122</td>
<td>0.040</td>
</tr>
<tr>
<td>APA$_z$</td>
<td>0.202</td>
<td>0.054</td>
<td>0.155</td>
<td>0.047</td>
<td>0.123</td>
<td>0.144</td>
<td>0.003</td>
</tr>
<tr>
<td>RPA</td>
<td>0.259</td>
<td>0.382</td>
<td>0.253</td>
<td>0.276</td>
<td>0.489</td>
<td>0.396</td>
<td>0.294</td>
</tr>
<tr>
<td>RPA$_x$</td>
<td>0.066</td>
<td>0.132</td>
<td>0.156</td>
<td>0.157</td>
<td>0.313</td>
<td>0.249</td>
<td>0.254</td>
</tr>
<tr>
<td>RPA$_y$</td>
<td>0.107</td>
<td>0.029</td>
<td>0.058</td>
<td>0.212</td>
<td>0.103</td>
<td>0.235</td>
<td>0.070</td>
</tr>
<tr>
<td>RPA$_z$</td>
<td>0.227</td>
<td>0.357</td>
<td>0.191</td>
<td>0.082</td>
<td>0.361</td>
<td>0.198</td>
<td>0.130</td>
</tr>
</tbody>
</table>

Table 5. Absolute position accuracy (APA) and relative position accuracy (RPA) after calibration (mm)

The calibration method demonstrated in this section can be used for many other kinds of serial or parallel robots as well. As it is based on direct measurements, it provides more accurate parameter identification than conventional standard calibration methods based on optimization (e.g., forward calibration and reverse calibration method). The proposed method also requires no complex computation.
(e.g., identification Jacobian matrix, observability analysis) or advanced optimization knowledge in calibration. The comparison between proposed calibration method and standard calibration method are summarized in Table 6.

<table>
<thead>
<tr>
<th>Kinematic parameter identification</th>
<th>Proposed method</th>
<th>Standard calibration method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individually, more accurate</td>
<td>All parameters identified simultaneously, less accurate</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>End-effector accuracy</th>
<th>Good</th>
<th>Optimized for end-effector accuracy</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Complex computation</th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robotic calibration knowledge</td>
<td>Basic</td>
<td>Advanced</td>
</tr>
<tr>
<td>Time consumption</td>
<td>More time for experiment</td>
<td>Less time for experiment, more time in method developing and computation</td>
</tr>
</tbody>
</table>

Table 6. Comparison of proposed calibration method and standard calibration method

5. Conclusion

An assessment method of the repeatability and the accuracy of a new medical robot were presented. The complete kinematic model of the robot was introduced, and the corresponding parameters were calibrated by a direct measurement method. The proposed method is very easy to implement and requires minimum knowledge of advanced calibration techniques. This approach was validated through experiments which demonstrated a significant improvement of the position accuracy from about 6 mm before calibration to less than 1 mm after calibration. Thus, the presented method has great potential value in robot calibration when advance techniques are not available or not necessary.

6. Acknowledgements

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7. References


