PERFORMANCE EVALUATION OF MIXED-MODEL TRANSFER LINES WITH ACYCLIC PRODUCT ARRIVALS AND RANDOM LOT SIZING

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ABSTRACT: *Transfer lines are more and more dedicated to producing a variety of similar products allowing some flexibility to this important class of manufacturing systems. For many decades, an important work has been done to evaluate the performance and specifically the capacity of transfer lines. However, most of these works are so far limited to the mono-product and homogeneous transfer lines. This paper analyses the performance of transfer lines composed of several machines with a serial configuration and dedicated to producing a variety of products in a batch production environment. The products are manufactured according to the real stochastic and dynamic demand characterized by random product arrivals and random lot sizing. The considered transfer lines have no intermediate buffers between adjacent machines. Machines have different deterministic processing times and are subject to random operation-dependent failures. The purpose of this paper is to propose analytical models to assess the steady-state availability and the overall throughput of such lines. A general simulation model was developed and statistical tests were carried out to prove the robustness and the exactness of the proposed analytical models.*

KEYWORDS: *Performance evaluation, Mixed-model transfer lines, operation-depended failures, acyclic product arrivals, random lot sizing, simulation modeling.*

1 INTRODUCTION

Transfer lines are product oriented automated manufacturing systems employed in industry for mass production. They were considered as the best solution to producing parts with the required high production rate at minimal cost. Sequential tasks are assigned to specific workstations along the line. Buffers between the workstations provide inventories to regulate product flow along the line. In spite of their lack of flexibility, transfer lines are considered as the most performing systems. In fact, they present superiority for several performance measures such as throughput, efficiency, work in process, easy managing, and cost effectiveness. Throughput is often considered as the main performance measure of transfer lines which is defined as the overall average long run production rate of the transfer line.

Transfer line throughput and efficiency are directly affected by station interferences: blocking and starvation (Yeralan and Muth, 1987). If a workstation finishes an operation when the downstream one has failed, then it cannot start a new operation, and it is said to be blocked. Similarly, if a workstation finishes an operation when the upstream one is down, then it cannot start with a new part, and it is said to be starved. Blocking and starvation phenomena depend not only on the occurrence of failures but also on the variability of processing times and their effects on the entire transfer line behavior (Dhouib *et al*., 2008-2009a).

Two types of machine failures have often been considered when dealing with manufacturing systems: Operation-depended failures and time-dependent failures (Papadopoulos and Heavy, 1996; Sherwin, 2000; Schneeweiss, 2005). Operation-dependent failures (ODF) can occur only when stations are in a processing state. However, time-dependent failures (TDF) can also occur when a manufacturing station is at the rest. Many authors agree that operation-dependent failures are the main failure mode for manufacturing systems (Buzacott, 1968; Sherwin, 2000; Schneeweiss, 2005; Dhouib *et al*., 2006).

Transfer lines are often designed to producing only one product type, named mono-product transfer lines. In order to improve the performance of mono-product transfer lines many research have studied the problem of transfer line balancing in order to equitably assign the work to all manufacturing stations and consequently generate pure homogeneous lines (Askin and Standrige, 1993; Scholl, 1999).

An important work has also been done to evaluate the throughput and the steady-state availability of mono-product homogeneous transfer lines assuming a perfect work balancing through all the manufacturing workstations (Buzacott and Shanthikumar, 1993; Gershwin, 1994; Papadopoulos and Heavy, 1996). However, in practice, it is very difficult to get a perfectly balanced transfer line. In fact, processing times differ from one station to the other and transfer lines are said to be non-homogeneous. Gershwin (1987) was the first to propose an analytical approach to deal with non-homogeneous transfer lines. He consider a disaggregation technique decomposing each manufacturing machine into two equivalent machines having the same production rate as the fastest machine of the entire production line. The first equivalent machine captures the processing time of the original machine and the second one represents its unreliable behavior. Dallery *et al*. (1989) propose a homogenization technique that consists in replacing each original machine by a single equivalent machine, with all equivalent machines having the same production rate. As was proposed by Gershwin (1987), Dallery *et al*. (1989) assign the fastest machine processing time of the original line to all equivalent machines. The equivalent machine parameters are evaluated by considering that the original machine and its equivalent undergo the same repair process, and the isolated throughputs of the original machine and its equivalent are equal. Liu and Buzacott (1990) also use the equivalent machine concept to homogenize non-homogeneous transfer lines. The processing time of the equivalent line is the same as that of the fastest machine of the original line. The failure and repair rates of the equivalent machine are estimated based on the proposal that 'the equivalent machine has the same second moment of inter-departure times'. Chen and Yuan (2004) analyzed non-homogeneous, mono-product transfer lines and proposed to consider the line somewhat as one whose machines have the smallest production rate among the original machines (the bottleneck). However, no modifications were introduced to the failure and the repair rates of the original transfer line machines. Dhouib *et al*. (2009a) have developed a simulation model to analyze the effectiveness of the four aforementioned proposals in assessing the throughput of unbuffered, non-homogeneous, mono-product transfer lines subject to random operation-dependent failures. They have shown that all proposed approaches underestimate the throughput of non-homogeneous, mono-product transfer lines subject to operation-dependent failures. In a recent paper, Dhouib *et al*. (2008) propose a homogenization approach and analytical formulae to assess the steady-state availability and the throughput of non-homogeneous, mono-product transfer lines subject to operation-dependent failures. Compared to simulation results generated for several transfer line configurations, the authors prove the exactness and the robustness of the proposed analytical formulae.

Several authors mention that the mono-product transfer line can be assimilated to an aggregation of various product types (Savsar and Biles, 1984; Kalir and Arzi, 1997; Dhouib *et al*., 2008-2009a)). Johri (1987) proposed a complex aggregation approach to deal with mixed-model transfer lines. He introduced the concept of the aggregate bottleneck workstation which is the workstation that requires the maximum time to process the mix of products. However, and due to complexity considerations, Johri's work has been limited to a transfer line composed of 2 workstations with intermediate buffer. Dhouib *et al*. (2009b) proposed a general homogenization and aggregation approach which estimates the throughput of mixed-model unbuffered transfer lines consisting of a series of machines. In order to analyze the exactness of the proposed model, the authors developed a general simulation model. Several transfer line configurations have been tested and the results show the robustness and the exactness of the proposed analytical formulae. In order to simplify the analytical models and to compare their approach to existing ones proposed by Johri (1987), by Gershwin (1987), Dalery *et al*. (1989), by Liu and Buzacott (1990), and by Chen and Yuan (2004) extended to the multi-product case, Dhouib *et al*. (2009b) assume that product types are processed in a cyclic manner and that the lot size associated with each product type has a constant and deterministic size. However, in practical situations dealing with mixed-model manufacturing systems, the arrival of products depends on demand requirements characterized by a stochastic behavior. Also, the lot size associated with each demand is a random variable depending on the customer requirements.

This paper extends the work of Dhouib *et al*. (2009b) in order to take into account the stochastic and the dynamic characteristics of the demand. The next section describes the characteristics of the studied mixed-model transfer line, and presents the assumptions and notations used in this work. Section 3 provides a homogenization and an aggregation approach and derives analytical formulae to assess the steady-state availability and throughput of non-homogeneous, unbuffered, mixed model transfer line with stochastic product arrivals and random lot sizing. A general simulation model imitating the real dynamic and stochastic operating behavior of these transfer lines is proposed in section 4 in order to analyze the robustness and the exactness of the proposed analytical formulae. Numerical results and comparison studies conducted on several production line configurations are given in section 5. Finally, section 6 contains a summary of the paper and some concluding remarks.

2 MIXED-MODEL TRANSFER LINES: DESCRIPTION, ASSUMPTIONS AND NOTATIONS

2.1 Manufacturing system description

Transfer lines being studied in this paper are sets of m machines ($M_1, M_2, \ldots, M_m$) arranged in a serial structure without intermediated buffers and dedicated to manufacturing p product types ($P_1, P_2, \ldots, P_p$).

The demand for producing a specific product type depends on demand arrival which is often considered as a discrete random variable. The product requirement is defined through the lot size associated to a specific product type demand and can be modeled through a discrete or a continuous random variable. Once demand requirements are known, parts flow from outside the production line to the first machine (M_1), then to the second one (M_2), and so forth until it reaches the last machine (M_m), after which they leave the production line (Fig.1).

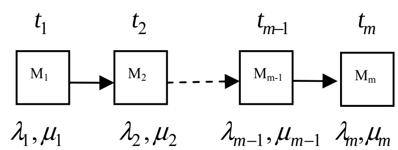

Figure 1: mixed-model transfer line with *m* machines

Machines are linked by an automatic transfer mechanism and are subject to random operation-dependant failures so that a failure can only occur while the machine is working (Dallery and Gershwin, 1992; Buzacott and Shanthikumar, 1993; Papadopoulos and Heavy, 1996; Sherwin, 2000; Dhouib *et al*., 2006). Each workstation is assigned the task of manufacturing, assembling or inspecting parts and each part is processed on each workstation during a fixed amount of time which is said processing time.

2.2 Notations

The following notations are considered during the next sections:

i : index identifying the machine number i (M_i); (i = 1, 2, ..., m).

j : index identifying the product type number j (P_j). (j = 1, 2, ..., p).

t_{ji} : Processing time of one item product type j on machine i.

t_{jbot} : Processing time of one item product type j on the bottleneck machine.

$\Pr_j$: Arrival probability associated with product type j.

L_j : Random variable identifying the lot size of the product type j.

f_{L_j} : Density function associated with the random variable L_j.

$\overline{L_j}$: Average lot size associated with product type j.

t_{bot} : One part processing time of the aggregated product manufactured on an equivalent one-product, homogeneous transfer line machine.

λ_i : Failure rate of machine i.

μ_i : Repair rate of machine i.

$MTTF_i$: Mean time to failure of machine i ($MTTF_i = 1/\lambda_i$).

$MTTR_i$: Mean time to repair of machine i ($MTTR_i = 1/\mu_i$).

UR_{ji} : Utilization rate of machine i when manufacturing product j.

UR_i : Utilization rate of machine i.

MUR_i : Mean utilization rate of machine i.

CF_i : Failure rate correction factor corresponding to machine i.

λ_i^e : Failure rate of the equivalent machine i.

UTR_i : Steady-state availability of machine i.

UTR : Steady-state availability of the entire transfer line evaluated by analytical model.

UTR_s : Steady-state availability of the entire transfer line evaluated by simulation model.

Th : Throughput of the entire transfer line evaluated by analytical model.

Th_s : Throughput of the entire transfer line evaluated by simulation model.

ε_{UTR} : Steady-state availability relative error.

ε_{Th} : Overall throughput relative error.

2.3 Working assumptions

It is assumed that the mixed-model transfer line satisfies the following conditions:

1- Failure times and repair times are exponentially distributed random variables.
2- Repair actions are done perfectly so the machines are restored to the 'as good as new' state.
3- On failure, parts remain at machines and processing resumes after repair completion.
4- The line operates under saturation: the first station is never starved and the last one is never blocked.
5- Transfer times between machines are considered negligible.
6- Setup times to switch from one product to another are considered negligible.
7- No parts are scrapped.

3 STEADY-STATE AVAILABILITY AND THROUGHPUT OF MIXED-MODEL TRANSFER LINES

Throughput is the principal performance measure for transfer lines subject to random failures and repairs. It is mainly affected by its steady-state availability. In mono-product homogeneous cases, the transfer line throughput is simply obtained by multiplying the line steady-state availability by its processing rate (Eq. 1) (Buzacott, 1968); where the transfer line processing rate is equal to 1/t. If the transfer line is subject to operation-dependent failures, its steady-state availability is given by equation (2) (Dallery and Gershwin, 1992; Papadopoulos and Heavy, 1996; Sherwin, 2000; Schneeweiss, 2005; Dhouib *et al*., 2006).

$$Th = UTR \cdot 1/t \quad (1)$$

$$UTR = \frac{1}{1+\sum_{i=1}^{m}\frac{\lambda_i}{\mu_i}} = \frac{1}{1+\sum_{i=1}^{m}\frac{MTTR_i}{MTTF_i}} \quad (2)$$

Analyzing mixed-model transfer lines is a complex venture since processing times from one product type to another and from one machine to another are different implying a non-homogeneous character of the transfer line affecting reliability characteristics of each machine composing the transfer line (Gershwin, 1987; Dalery *et al*., 1989; Liu and Buzacott, 1990; Dhouib *et al*., 2008-2009a). Also, mixed-model transfer lines imply producing various product types having different operational characteristics (specific product manufacturing times, random arrival of products, and random lot sizing).

In order to analyze the overall throughput of mixed-model transfer lines, Dhouib *et al*. (2009b) propose a combined homogenization and aggregation approach allowing to convert the original mixed-model transfer line into a homogeneous mono-product equivalent one. The authors propose two approaches to assess the overall throughput of these flexible transfer lines: the weighted mean of individual throughputs by product type (WMTPT) and the weighted mean of bottleneck processing times (WMBPT). They have also compared exact simulation results to their proposed techniques (WMTPT and WMBPT), to the proposal by Johri (1987), and to the proposals by Gershwin (1987), Dallery *et al*. (1989), Liu and Buzacott (1990), and Chen and Yuan (2004) extended to the multi-product case.

Results carried out on numerous transfer line configurations with random generated parameters show that the proposed formulae based on the WMTPT and the WMBPT techniques produce a negligible error. Furthermore, simulation results show that the WMBPT technique slightly outperforms the WMTPT technique.

The weighted mean of bottleneck processing times technique (WMBPT) will be extended in this section to take into consideration the random aspects associated with the mixed-model demand requirements. The WMBPT technique is based on converting the discrete operational character of each manufacturing machine composing the transfer line (Fig. 2) into a steady-state and a continuous operating mode (Fig. 3). Accordingly, the original mixed-model transfer line is assimilated to a homogeneous equivalent one manufacturing only one product type, and operating in a continuous manner with steady capacity.

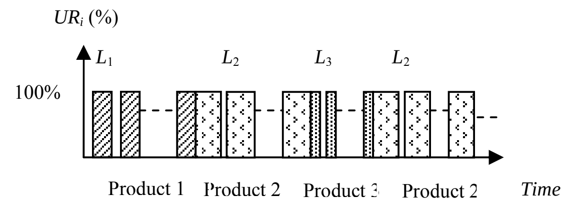

Figure 2: Workload of a machine *i* operating the product mix.

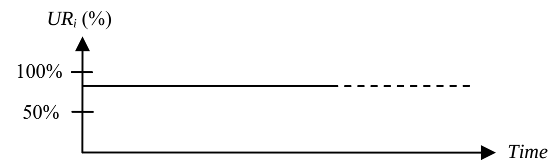

Figure 3: Equivalent workload of machine *i* operating under a continuous and steady-state mode

Each equivalent one-product transfer line machine (*i*) has a reduced failure rate (λ_i^e), and a repair rate equal to that of the corresponding original mixed-model machine (μ_i). Equation (3) gives the reduced failure rate of any equivalent one-product transfer line machine (*i*) where CF_i is the failure rate correction factor associated to machine (*i*).

$$\lambda_i^e = \lambda_i \cdot CF_i \quad (3)$$

The failure rate correction factor can be estimated by the expected utilization rate of the original machine when operating without failures (Eq. 4).

$$CF_i = MUR_i = \frac{\sum_{j=1}^{p}\Pr_j \cdot t_{ji} \cdot \overline{L_j}}{\sum_{j=1}^{p}\Pr_j \cdot t_{jbot} \cdot \overline{L_j}} \quad (4)$$

Therefore, the equivalent failure rate (λ_i^e) can be reduced to equation (5).

$$\lambda_i^e = \lambda_i \cdot \frac{\sum_{j=1}^{p} \Pr_j \cdot t_{ji} \cdot \overline{L_j}}{\sum_{j=1}^{p} \Pr_j \cdot t_{jbot} \cdot \overline{L_j}} \quad (5)$$

The expected lot size of a specific product *j* is given by equation (6) in the case where L_j is a discrete random variable and by equation (7) in the case where L_j is a continuous random variable.

$$\overline{L_j} = \sum_{k=1}^{N_j} \Pr\left(L_j = L_{kj}\right) \cdot L_{kj} \quad (6)$$

where L_j takes its values in the set $\Omega = \left\{L_{1j}, L_{2j}, .., L_{N_j j}\right\}$, N_j is the possible occurrences of the r. v. L_j.

$$\overline{L_j} = \int_{-\infty}^{+\infty} L_j \cdot f_{L_j}(L_j) \cdot dL_j \quad (7)$$

Consequently, the mixed-model transfer line steady-state availability will be estimated by equation (8) after replacing λ_i by λ_i^e in equation (2).

$$UTR = \frac{1}{1 + \sum_{i=1}^{m} \frac{\lambda_i}{\mu_i} \cdot \frac{\sum_{j=1}^{p} \Pr_j \cdot t_{ji} \cdot \overline{L_j}}{\sum_{j=1}^{p} \Pr_j \cdot t_{jbot} \cdot \overline{L_j}}} \quad (8)$$

Based on the weighted mean of bottleneck processing times technique (WMBPT) (Dhouib *et al*., 2009b), each working machine is converted into a steady-state and a continuous operating equivalent machine manufacturing a single aggregate product type with homogeneous processing time denoted (t_{bot}). The homogeneous processing time of the aggregate product is equal to the weighted expected time required to process all product mix at their corresponding bottleneck machines (Eq. 9). Equation 9 takes into account the random character of each product type demand requirements.

$$t_{bot} = \frac{\sum_{j=1}^{p} \Pr_j \cdot t_{jbot} \cdot \overline{L_j}}{\sum_{j=1}^{p} \Pr_j \cdot \overline{L_j}} \quad (9)$$

Accordingly, the mixed-model transfer line overall throughput can be evaluated using equation (10) after replacing t by t_{bot} in equation (1) and *UTR* by its corresponding value given by equation (8).

$$Th = \frac{\sum_{j=1}^{p} \Pr_j \cdot \overline{L_j}}{\left(\sum_{j=1}^{p} \Pr_j \cdot t_{jbot} \cdot \overline{L_j}\right) \cdot \left(1 + \sum_{i=1}^{m} \frac{\lambda_i}{\mu_i} \cdot \frac{\sum_{j=1}^{p} \Pr_j \cdot t_{ji} \cdot \overline{L_j}}{\sum_{j=1}^{p} \Pr_j \cdot t_{jbot} \cdot \overline{L_j}}\right)} \quad (10)$$

The next section proposes a general simulation model imitating the real operating behavior of mixed-model, unbuffered transfer lines subject to operation-dependent failures with considerations of random product type demand requirements.

4 SIMULATION MODEL

A general discrete event simulation model was developed with the AweSim/VisualSlam system (Pritsker and O'Reilly, 1999) to determine the steady-state availability and the throughput of unbuffered mixed-model transfer lines subject to operation-dependent failures. It describes the real dynamic behaviour of such lines and takes into account the random character of product type demand requirements. Figure (4) gives the flow chart of the simulation model with the following description of the principal modules:

- INITIALIZATION module: sets for each experiment the number of line machines, the number of product types, the processing times for each product type, the distribution of product arrivals, the product specific distribution of lot sizing, the mean time to failure, and the mean time to repair for each machine. The simulation horizon and the warm up period after which statistics are cleared are also assigned at this step.
- PRODUCTION module: controls the flow of parts through each machine in the transfer line and manages interference situations.
- FAILURE and REPAIR module: samples times to failure and repair durations for each machine from their respective probability distributions.
- PERFORMANCE module: saves the number of produced parts during simulation horizon. This allows the evaluation of the transfer line steady-state availability and its overall throughput.

For each line configuration, simulation program was run for 10 replications in order to obtain 10 incurred steady-state availability and throughput observations which will be next compared to analytical results generated by the proposed models. The simulation model has also been run for long time and a warm-up period has been considered to guarantee the stability of performance measures. For each randomly generated configuration, we have also compared simulation results with those obtained analytically with the proposed approach through the Student's t test.

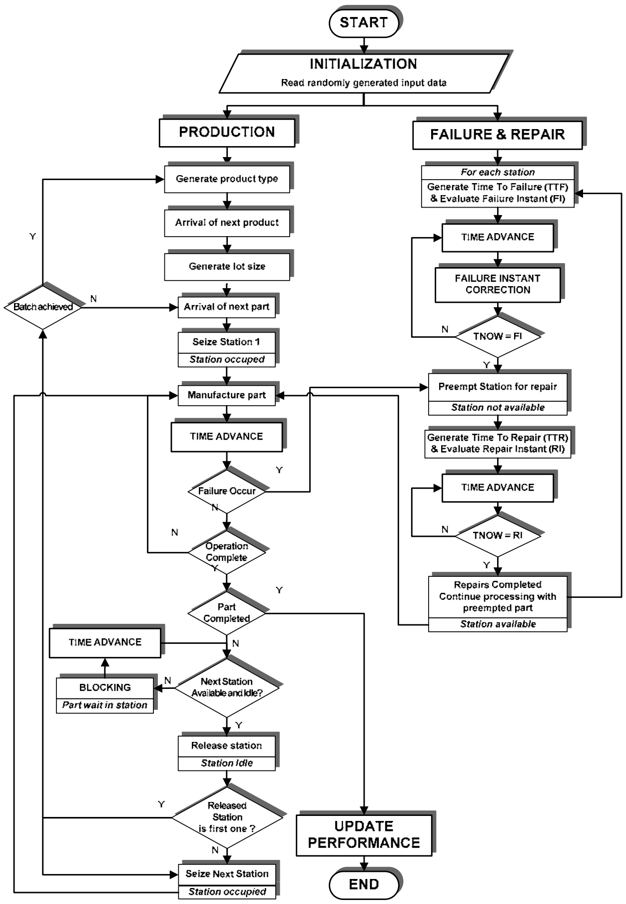

Figure 4. Flow chart of the simulation model

5 NUMERICAL RESULTS

In order to analyze and to confirm the performance of the different formulations considered in this study, thousands of experiments on several production line configurations were carried out. This study has considered lines with 3, 5 and 9 workstations and processing 3, 5 or 9 product types. For each configuration of the mixed-model transfer line, product and workstation parameters have been generated randomly. Two ranges of machine availability are considered ([70, 75%], and [95, 100%]), in order to analyze the effect of individual machine availability on the production line performance.

A program with has been developed Visual Basic programming language in order to randomly generate all parameters characterizing a specific mixed-model transfer line. First, the program randomly selects a bottleneck machine for each product manufactured by the transfer line (bot = 1, 2, …, m) and generates its processing time ($t_{jbot} \in$ [10, 25] time units). For each product and for each machine, the program evaluates the processing time (Eq. 11) based on the randomly generated utilization rate value of the considered machine (i = 1, 2, …, m, $i \neq bot$, and $UR_{ji} \in$ [70, 100%]).

$$t_{ji} = UR_{ji} \cdot t_{jbot} \qquad (11)$$

The program generates subsequently the product type arrival frequency ($\Pr_j$), and the distribution parameters of the product type lot size random variable (L_j). In this study, the lot size distributions of the different product are assumed to be uniformly distributed with minimum value L_{jmin} and maximum value L_{jmax}.

Finally, and for each machine composing the mixed-model transfer line, the program randomly generates the individual machine availability according to the afore-mentioned two availability ranges (machines with low availability ($UTR_i \in [70, 75\%]$) or machines with high availability ($UTR_i \in [95, 100\%]$)). The program also randomly generates the mean time to repair for each machine ($MTTR_i \in [60, 360]$ time units) and evaluates its mean time to failure using equation (12).

$$MTTF_i = MTTR_i \cdot (UTR_i / (1 - UTR_i)) \qquad (12)$$

Table 1 presents ten random generated configurations for a mixed-model transfer line manufacturing three products and having three workstations with low machine availability. Table 1 gives the characteristics of manufactured products and transfer line workstations. According to experiments of table 1, table 2 resumes, consequently, the steady-state availability and the throughput generated by the simulation model and by the proposed technique. It also gives the relative errors and the student's t-test acceptance by comparing analytical results to simulation.

In order to demonstrate the exactness and the robustness of the proposed models, tables 3 and 4 show, for each p-product, m-machine transfer line (p = 3, 5 or 9 and m = 3, 5 or 9), the absolute values of the availability and the overall throughput mean relative errors for 100 randomly generated configurations by comparing simulation results to analytical ones, respectively when $UTR_i \in$ [70, 75%] and $UTR_i \in$ [95, 100%] (Eq. 13-14).

$$\varepsilon_{UTR}(\%) = \frac{\sum_{c=1}^{100} \frac{\left|\overline{UTR}_{S\_c} - UTR\right|}{\overline{UTR}_{S\_c}}}{100} \cdot 100\% \qquad (13)$$

$$\varepsilon_{Th}(\%) = \frac{\sum_{c=1}^{100} \frac{\left|\overline{Th}_{S\_c} - Th\right|}{\overline{Th}_{S\_c}}}{100} \cdot 100\% \qquad (14)$$

were c is the c^{th} randomly generated configuration of a p-product, m-machine transfer line. For a specific configuration c of the mixed-model transfer line, $\overline{UTR}_{S\_c}$ and $\overline{Th}_{S\_c}$ are the respective mean steady-state availability and overall throughput values generated from 10 executed replications.

	Product 1						Product 2						Product 3					
Line config.	t_{11}	t_{12}	t_{13}	Pr_1	L_{1min}	L_{1max}	t_{21}	t_{22}	t_{23}	Pr_2	L_{2min}	L_{2max}	t_{31}	t_{32}	t_{33}	Pr_3	L_{3min}	L_{3max}
1	16.4	12.8	18.0	0.27	516	630	14.5	11.8	14.3	0.36	517	631	21.4	21.0	17.3	0.37	455	556
2	12.6	13.9	10.7	0.26	456	558	16.3	22.3	22.3	0.54	363	443	20.6	17.2	23.6	0.20	504	616
3	12.7	14.2	16.0	0.17	393	481	12.4	09.0	10.4	0.39	523	639	12.6	16.1	13.1	0.44	412	504
4	20.2	19.8	23.7	0.37	433	529	18.1	13.0	14.0	0.41	536	656	14.4	20.1	16.3	0.22	448	548
5	18.8	13.9	19.8	0.27	374	457	08.4	11.6	08.5	0.42	360	440	18.3	21.3	18.3	0.31	509	622
6	19.1	17.4	23.9	0.47	391	477	21.3	15.5	17.8	0.35	410	501	16.7	17.3	13.5	0.18	482	589
7	17.2	16.8	13.7	0.19	458	560	07.7	10.6	08.7	0.36	533	651	22.9	22.4	20.3	0.45	423	517
8	18.2	18.2	15.4	0.37	435	531	14.6	16.1	13.3	0.34	458	560	18.9	17.3	20.0	0.29	452	552
9	14.3	13.3	12.7	0.49	500	612	18.7	17.6	14.2	0.34	532	650	12.3	12.3	17.1	0.17	428	524
10	13.3	18.4	16.8	0.13	363	443	17.5	21.2	16.5	0.44	537	657	22.1	23.5	19.4	0.43	492	602
…	…	…	…	…	…	…	…	…	…	…		…	…	…	…	…	…	…

(a) Product characteristics

	Machine 1		Machine 2		Machine 3	
Line config.	$MTTR_1$	$MTTF_1$	$MTTR_2$	$MTTF_2$	$MTTR_3$	$MTTF_3$
1	76.05	214.67	200.61	541.29	246.81	618.88
2	73.69	184.16	174.60	437.61	344.57	864.45
3	249.52	732.85	188.54	513.39	228.31	545.36
4	202.34	490.13	248.63	617.36	106.89	284.79
5	117.58	279.78	196.26	541.29	104.99	267.19
6	86.97	215.85	156.82	368.53	149.18	423.48
7	203.99	493.42	358.12	881.05	68.67	165.29
8	245.76	605.79	264.25	694.90	171.15	498.20
9	95.86	250.46	112.13	269.01	274.44	647.75
10	166.98	416.65	343.49	890.76	253.79	609.72
…	…	…	…	…	…	…

(b) machine characteristics

Table 1: Randomly generated configurations for a 3-product, 3-machine flexible transfer line

	UTR (%)				*Th*			
Line config.	Simulation	Analytical Model	$\varepsilon_{UTR}(\%)$	Accepted Student test	Simulation	Analytical Model	$\varepsilon_{Th}(\%)$	Accepted Student test
1	49.29	49.29	0.0060	1	0.0278	0.0276	0.2431	1
2	48.28	48.27	0.0174	1	0.0237	0.0237	0.0821	1
3	50.68	50.66	0.0307	1	0.0350	0.0350	0.0860	1
4	49.49	49.44	0.1006	1	0.0243	0.0242	0.2745	1
5	49.17	49.16	0.0274	1	0.0282	0.0282	0.0840	1
6	49.75	49.76	0.0153	1	0.0230	0.0230	0.0159	1
7	46.77	46.78	0.0125	1	0.0277	0.0278	0.2991	1
8	48.65	48.64	0.0258	1	0.0270	0.0270	0.0164	1
9	47.60	47.59	0.0204	1	0.0290	0.0291	0.2777	1
10	48.20	49.29	0.0419	1	0.0219	0.0220	0.2666	1
…	…	…	…	…	…	…	…	…

Table 2: Comparison of simulation results with analytical results for production line configurations of Table 1

The analysis of tables 3 and 4 show that the proposed models assessing the steady-state availability and the overall throughput of mixed-model, non-homogeneous, unbuffered transfer lines generate a negligible error compared with simulation results for all transfer line configurations irrespective of the number of workstations, the number of products, and the availability range of individual machines. Student's t-tests also show that the analytical proposed models reflect the real stochastic behavior of the mixed-model transfer lines, which confirms the exactness and the robustness of the proposed analytical models.

		Number of Machines	*UTR*		*Th*	
			ε_{UTR}(%)	Accepted Student's t-tests (%)	ε_{Th}(%)	Accepted Student's t-tests (%)
Number of Products	3	3	0.0448	100	0.1393	100
		5	0.0783	100	0.2118	100
		9	0.1312	100	0.2762	100
	5	3	0.0496	100	0.1504	100
		5	0.0784	100	0.2180	100
		9	0.0962	100	0.2543	100
	9	3	0.1205	100	0.2065	100
		5	0.0837	100	0.2112	100
		9	0.1044	100	0.2722	100

Table 3: *UTR* and *Th* mean relative errors and accepted Student's t-tests resulting from Comparing simulation results with proposed analytical models ($UTR_i \in$ [70, 75%])

		Number of Machines	*UTR*		*Th*	
			ε_{UTR}(%)	Accepted Student's t-tests (%)	ε_{Th}(%)	Accepted Student's t-tests (%)
Number of Products	3	3	0,0153	100	0,0880	100
		5	0,0271	100	0,1023	100
		9	0,0198	100	0,1762	100
	5	3	0,0151	100	0,0969	100
		5	0,0266	100	0,1259	100
		9	0,0589	100	0,2666	100
	9	3	0,0159	100	0,1217	100
		5	0,0245	100	0,1093	100
		9	0,0367	100	0,1601	100

Table 4: *UTR* and *Th* mean relative errors and accepted Student's t-tests resulting from Comparing simulation results with proposed analytical models ($UTR_i \in$ [95, 100%])

6 CONCLUSIONS

In this paper the steady-state availability and the overall throughput of unbuffered mixed-model transfer lines subject to operation-dependent failures with acyclic random product arrivals and random lot sizing were studied. A homogenization and an aggregation approach was proposed based on transforming the discrete intermittent operating behavior of the flexible transfer line into an equivalent homogeneous transfer line manufacturing a single aggregate product.

A general discrete event simulation model for mixed-model transfer lines was also developed. The simulation model imitates the real dynamic and stochastic behavior of such transfer lines and the random character of product demand requirements. It allows evaluation of the steady-state performance measures of such transfer lines: the overall throughput and the steady-state availability.

Several mixed-model transfer lines manufacturing 3, 5, or 9 product types and including 3, 5, or 9 workstations with randomly generated parameters were analyzed in order to compare the exact performances given by the simulator to those evaluated using the proposed analytical models.

The results show that the proposed models produce a negligible relative error for all randomly generated configurations, no matter the transfer line length is, the number of products being processed, and the availability range of individual machines. This confirms that the proposed analytical models are exact and robust to assess the steady-state availability and the overall throughput of mixed-model, unbuffered, non-homogeneous transfer lines subject to operation-dependent failures with acyclic product arrivals and random lot sizing.

This research can eventually be extended to develop analytical models for the analysis of the performance of more complex systems such as transfer lines with intermediate buffers, cellular and flexible manufacturing systems.

REFERENCES

Askin R.G. and C.R. Standridge, 1993. *Modeling and Analysis of Manufacturing Systems*. John Wiley & Sons, Inc., New York, USA.

Buzacott J.A., 1968. Prediction of the Efficiency of Production Systems Without Internal Storage. *International Journal of Production Research*, 6, p. 173–188.

Buzacott J.A. and J.G. Shanthikumar, 1993. *Stochastic Models of Manufacturing Systems*. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Chen C.-T. and J. Yuan, 2004. Transient Throughput Analysis for a Series Type System of Machines in Terms of Alternating Renewal Processes. *European Journal of Operational Research*, 155, p. 178–197.

Dallery Y., R. David, and X.L. Xie, 1989. Approximate Analysis of Transfer Lines with Unreliable Machines and Finite Buffers. *IEEE Transactions on Automatic Control*, 34, p. 943–953.

Dallery Y. and S.B. Gershwin, 1992. Manufacturing Flow Line Systems: A Review of Models and Analytical Results. *Queueing Systems*, 12, p. 3–94.

Dhouib K., A. Gharbi, and S. Ayed, 2006. Disponibilité et Taux de Production des Lignes de Transfert Homogènes, Sans Stocks Intermédiaires. *International Conference JTEA'06*, Hammamet, Tunisisa, p. 1145–1152.

Dhouib K., A. Gharbi, and S. Ayed, 2008. Availability and Throughput of Unreliable, Unbuffered Production Lines with Non-Homogeneous Deterministic Processing Times. *International Journal of Production Research*, 46, p. 5651–5677.

Dhouib K., A. Gharbi, and S. Ayed, 2009a. Simulation Based Throughput Assessment of Non-Homogeneous Transfer Lines. *International Journal of Simulation Modeling*, 8, p. 5–15.

Dhouib K., A. Gharbi, and N. Landolsi, 2009b. Throughput Assessment of Mixed-Model Flexible Transfer Lines with Unreliable Machines. *International Journal of Production Economics*, 122, p. 619–627.

Gershwin S.B., 1994. *Manufacturing Systems Engineering*. Prentice-Hall, New-York.

Gershwin S.B., 1987. Representation and Analysis of Transfer Lines With Machines that Have Different Processing Rates. *Annals of Operations Research*, 9, p. 511–530.

Johri P.K., 1987. A Linear Programming Approach to Capacity Estimation of Automated Production Lines With Finite Buffers. *International Journal of Production Researches*, 25, p. 851–866.

Kalir A. and Y. Arzi, 1997. Automated Production Line Design With Flexible Unreliable Machines for Profit Maximization. *International Journal of Production Research*, 35, p. 1651–1664.

Liu X.-G. and J.A. Buzacott, 1990. Approximate Models of Assembly Systems With Finite Inventory Banks. *European Journal of Operational Research*, 45, p. 143–154.

Papadopoulos H.T. and C. Heavey, 1996. Queueing Theory in Manufacturing Systems Analysis and Design: A Classification of Models for Production and Transfer Lines. *European Journal of Operational Research*, 92, p. 1–27.

Pritsker A.A. and J.J. O'Reilly, 1999. *Simulation with Visual SLAM and AweSim*. John Wiley & Sons, New York.

Savsar M. and W.E. Biles, 1984. Two-Stage Production Lines With a Single Repair Crew. *International Journal of Production Research*, 22, p. 499–514.

Schneeweiss W.G., 2005. Toward a Deeper Understanding of the Availability of Series-Systems Without Aging During Repairs. *IEEE Transactions on Reliability*, 54, p. 98–99.

Scholl A., 1999. *Balancing and Sequencing of Assembly Lines*. Physica-Verlag, New York.

Sherwin D.J., 2000. Steady-State Series Availability. *IEEE Transactions on Reliability*, 49, p. 146–147.

Yeralan S. and E.J. Muth, 1987. A General Model of a Production Line With Intermediate Buffers and Station Breakdowns. *IIE Transactions*, 19, p. 130–139.