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Effect of Variability in Microgeometry of Polyurethane Foams on their Macroscopic Acoustic Performance

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ABSTRACT

The numerical formulations used for the modeling and design of sound absorbing materials are constructed based on a set of physical parameters, known as the Biot's parameters (for isotropic materials these are comprised of 5 non-acoustical parameters and 4 mechanical parameters). These parameters are inter-correlated and are microstructure-dependent. There is in consequence a need for the development of links between the cellular structure of the foams and the Biot's parameters before realistically using these models for material-level optimization. In this sense, a microstructure-based model has been developed by Doutres *et al.* [J. Appl. Phys. 110, 064901 (2011)] to link the microstructure (thickness and length of struts and the closed windows content) of polyurethane (PU) foams to their non-acoustical parameters. In this study, this model is first extended to add the link between the microstructure and the mechanical properties of the foam. Next, a global sensitivity analysis using Fourier Amplitude Sensitivity Test (FAST) is performed to investigate the impact of the variability, associated with the irregularities in microstructure, on the sound absorption and transmission loss (TL) of the foam when combined with an elastic structure.

1 INTRODUCTION

Propagation of waves in elastic porous media, e.g. polymeric foams, is described by Biot-Allard's theory[1]. Two classes of characteristic parameters are needed to describe the porous media in Biot-Allard model. First, non-acoustic parameters: porosity (ϕ), thermal characteristic length (Λ'), viscous characteristic length (Λ), flow resistivity (σ), and tortuosity (α) which are used in Johnson-Champoux-Allard (JCA) semi-phenomenological model[1]. Second, effective mechanical parameters, wherein the case of isotropic material, are: bulk density (ρ_b^*) the effective Young's modulus (E^*), loss factor (η), and Poisson coefficient (ν). It is well known that the mechanical and non-acoustical properties are inherently dependent on the microstructure. Hence, a clear understanding of the correlation between the microstructure and acoustical behavior of the foam is of great importance in the design and optimization of such foams.

The internal structure of most porous material is too complicated to be studied quantitatively. Fig. 1a, and Fig. 1b give insight into a lattice of PU foams by using, respectively, a scanning electron microscope (SEM), and micro-computed tomography (μCT). The images show that the lattice of PU foams with low relative density (ρ_r) can be idealized by a tetrakaidecahedral, repeating unit cell (Fig. 1c). Modelling a lattice in finite element (FE) using solid elements is computationally expensive. Therefore, as shown in Fig. 1d, a unit cell will be considered in FE analysis of this study.

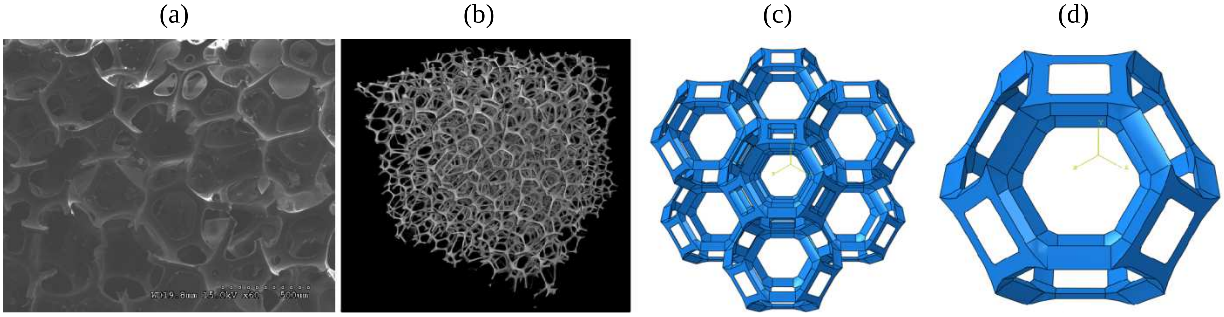

Figure 1: *Lattice of PU foams; (a) SEM photo. (b) X-Ray micro-tomography. (c) Idealized repeated unit cell. (d) Idealized unit cell.*

To enhance the vibro-acoustical performance of poroelastic materials, a poroelastic micro-macro model to predict vibro-acoustic performance of the foam is of the utmost importance. In this sense, a microstructure-based model has been developed by Doutres *et. al*[2,3] to link the microstructures (thickness and length of struts, and the closed windows content) of polyurethane foams to their non-acoustical parameters of Biot-Allard model. Gong *et. al*[4] and Jang *et. al*[5] developed analytically micro-macro correlations for mechanical properties of fully reticulated PU foams, considering all deformation mechanisms, non-uniformity in distributed mass along the strut with plateau border cross section, and taking into account the accumulated mass at the vertices. The effect of closed windows content on the effective Young's modulus of PU was studied by Hoang *et. al*[6]. But, this effect hasn't been integrated in the existing micro-macro correlations.

The objective of this study is to add the effect of closed windows content into the micro-macro model presented by Gong and Jang. Then, one combined with the model of Doutres *et. al*[2] perform a global sensitivity analysis to study the effect of Biot-Allard parameters on the sound absorption, and a transmission loss problem. Investigate how the variability in the microstructure of the PU foams affects first, the Biot-Allard parameters, and second the vibro-acoustic behavior of the foam using the FAST method. The FAST method is one of the global sensitivity analysis (SA) techniques that is known as variance-based methods[7,8]. What makes FAST method computationally efficient is exploring the multidimensional space of inputs by a search curve. The FAST method is an efficient technique to study not only the main effect (also named first order term (SI)), but also to compute the so-called total sensitivity index (TSI) after the extended FAST presented by Saltelli[9]. Note that the FAST method already was used to define the indexes for acoustic and poroelastic application[10,11]. This paper is structured as follows. In Section 2 the effect of membrane closing the windows of unit cell is added numerically into micro-macro model presented for foams with fully opened windows. Then, in Section 3 the already developed[2] micro-macro relations associated to non-acoustical properties are represented. The FAST method is presented in Section 4. Finally results are discussed in Section 5.

2 IMPACT OF CLOSED PORE CONTENT ON THE EFFECTIVE YOUNG'S MODULUS

As mentioned before, the analytical model presented by Gong and Jang[4,5] is selected here as a basis for adding, using a numerical analysis, the effect of closed windows content. This model is selected since, in addition to adding the effect of non-uniformity in the thickness of struts, and accumulated mass at the vertices, all types of deformations are considered. This is performed in three steps: in the first step, the numerical model of a fully reticulated tetrakaidecahedral unit cell is validated by the Gong and Jang's model for a foam with $\phi = 94\ \%$. In this study, reticulation rate is defined as the ratio of the area of the open windows to the area of all windows of a 3-dimensional unit cell used for numerical modeling. In this work, we consider two ways of closing the windows: (1) a binary closure as shown in Fig. 2a (the windows are either fully open or fully closed)[2,3], (2) a partial closure as shown in Fig. 2b (the reticulation rate is defined by an opening ratio as done by Hoang *et. al* [6]. Binary reticulation may cause anisotropy in the cell. Therefore, the effective Young's modulus should be averaged over different directions, which is computationally expensive. In the second step, the membranes closing the windows are added in the numerical model of unit cell. Periodic boundary conditions (PBC) are imposed to the unit cell which is subjected to compression. In the final step, the effective Young's modulus is determined by averaging over principal directions.

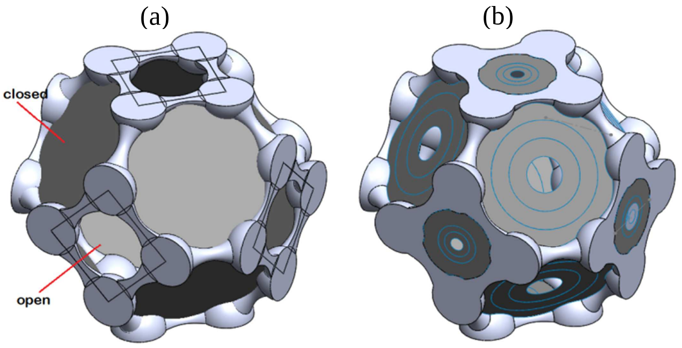

Figure 2: *Methods used to create different reticulation rate. (a) Binary reticulated window* [2,3]*, (b) Removing the same ratio of the surface of all membranes* [6]*.*

To modify the micro-macro model presented by Gong and Jang, with the impact of the reticulation rate, the tetrakaidecahedral unit cells with different reticulation rate (from $R_w = 14\%$ to $R_w = 100\%$) were considered. The FE models are solved in Abaqus 6.14-2-(Dassault Systèmes). The windows are reticulated partially and binary. The membranes are modeled by using shell elements (linear quadrilateral S4R), and the struts and vertices modeled using solid elements (quadratic tetrahedron, type C3D10). To impose the PBC, the mesh patterns on the left, bottom, and front sides are copied to the right, top, and rear sides of the unit cell respectively. This study is limited to the mechanical properties of constituent material measured by Gong[4], (density $\rho_s = 1190\, kg/m^3$, Elastic modulus $E_s = 69 \times 10^6\ (Pa)$, and Poisson's ratio $\nu = 0.49$). The effective Young's modulus is calculated for different reticulation rates using either binary and proportional reticulation method. The averaged thickness of the membrane is estimated as $t_m = 2.1 \pm 0.25\ (\mu\, m)$ using SEM photos. Numerical results show that fully reticulated ($R_w = 1$) PU foams have the minimum relative Young's modulus in agreement with Gong and Jang's micro-macro model $(E^*/E_s)_{Rw=1}$. The effects of reticulation rate and membrane thickness are added to this model by fitting a curve to numerical tests where $0.15 \leq R_w \leq 1$:

$$\left(\frac{E^*}{E_s}\right)_{Rw} = C_E(1-R_w)^{1.4} + \left(\frac{E^*}{E_s}\right)_{Rw=1}, \tag{1}$$

where $C_E = 0.0017$.

3 IMPACT OF CLOSED PORE CONTENT ON NON-ACOUSTICAL PARAMETER

As mentioned before, the 3-parameter micro-macro model presented by Doutres *et. al*[2] is used in this work to correlate the microstructure of flexible PU foams to the non-acoustic properties of JCA's model[1]. The two geometrical parameters (porosity and thermal length) are:

$$\phi = 1 - \mathrm{C}_{\mathrm{t}}^{\rho}\left(\frac{\mathrm{t}}{\mathrm{l}}\right)^2, \tag{2}$$

$$\Lambda' = 8\mathrm{l}\frac{\sqrt{2}}{3}\,\frac{1-\dfrac{\mathrm{t}^2\left(2\sqrt{3}-\pi\right)}{\mathrm{l}^2\sqrt{2}}}{1+2\sqrt{3}-\mathrm{R_w}\left(1+2\sqrt{3}-4\pi\dfrac{\mathrm{t}}{\mathrm{l}\sqrt{3}}\right)}, \tag{3}$$

where $C_t^{\rho} = \left(2\sqrt{3}-\frac{\pi}{\sqrt{2}}\right)$ for struts with plateau border cross section shape. The generalized expression for flow resistivity is defined as[2]:

$$\sigma = \mathrm{C}^{\beta}\left(\mathrm{C}_{\mathrm{r}}^{\rho}\frac{\mathrm{t}}{\mathrm{l}^2}\right)^2\left(\frac{1}{\mathrm{R_w}}\right)^{1.1166}, \tag{4}$$

where, $C_r^{\rho} = 3\pi/8\sqrt{2}$, $C^{\beta} = 128\eta$, and (η) is dynamic air viscosity ($\eta = 1.85\times10^{-5}\ Pa\ s$). Finally the viscous characteristic length and the tortuosity are, respectively:

$$\Lambda = \frac{\Lambda}{1.55}\left(\frac{1}{\mathrm{R_w}}\right)^{-0.6367}, \tag{5}$$

$$\alpha_{\infty} = 1.05\left(\frac{1}{\mathrm{R_w}}\right)^{0.3802}. \tag{6}$$

These micro-macro relations together with the modified model presented in Equation 1 are used, in the next sections, to investigate the effect of variability in microstructure of PU foams on the Biot-Allard parameters, and the impact on the sound absorption and TL problem.

4 GLOBAL SENSITIVITY ANALYSIS: THE FAST METHOD

The contribution of input parameters on the output, in variance based techniques, is investigated by quantifying the impact of input variation on the output variance. A global sensitivity analysis method is used to determine the output sensitivity when the inputs vary over wide ranges. FAST is an efficient technique to explore the n-dimensional space of inputs using a searching curve, and to avoid the evaluation of n-dimensional integrals for the computation of functions (f_i) involved in the decomposition[8]. FAST is used to estimate the $SI(i)$ (which is the main effect of parameter i), and to calculate the TSI which measures the total effect of input Xi and is defined as the sum of all SIs. For example, the total effect of parameter A on the output in a 3-inputs $(A, B$ and $C)$ model is: $TSI(A) = SI(A) + SI(AB) + SI(AC) + SI(ABC)$, where the $SI(AX)$ is the second order SI for parameter A and X when $A \neq X$. For more information the readers are referred to references[8,9]. The ratio of standard deviation to the mean of output, namely Normalized Standard Deviation (NSD) is used to normalize the indexes[11].

5 RESULTS

5.1 Impact of Microstructure Variability on Macroscopic Parameters

The impact of microstructure variability associated with the irregularities of microstructure on the effective Young's modulus and non-acoustical parameters are first investigated using the micro-macro models, and the FAST method introduced in previous sections. Two foams, representative of real PU foams, are studied. The ranges of variation of their micro-structural properties are given in Table 1. The variability in the microstructure of the first and the second foam is set to 10%, V1 is considered as highly reticulated foams, while V2 has low content of opened windows. The results are shown in Fig. 3. It is observed that the length of strut has more impact than thickness of strut on flow resistivity, viscous and thermal characteristic lengths of all materials. However, length and thickness of struts have the same effect on mechanical properties, porosity, and effective density. In the case of foam with high close pore contents, the reticulation rate is a key parameter for the air flow resistivity, and viscous characteristic length.

Table 1: Microstructure properties for two materials

Foam	$t \times 10^{-6}(m)$		$l \times 10^{-6}(m)$		$R_w(\%)$	
	Inferior limit	Superior limit	Inferior limit	Superior limit	Inferior limit	Superior limit
V1	51.3	62.7	157.9	192.5	70	90
V2	51.3	62.7	157.9	192.5	10	30

In the next sections, first the impact of Biot-Allard parameters, for a wide range of PU foams, on two vibro-acoustic performance indicators is studied. Then, the effect of variability in microstructures of two PU foams on the sound absorption and the transmission loss of the foam when combined with an elastic structure are investigated.

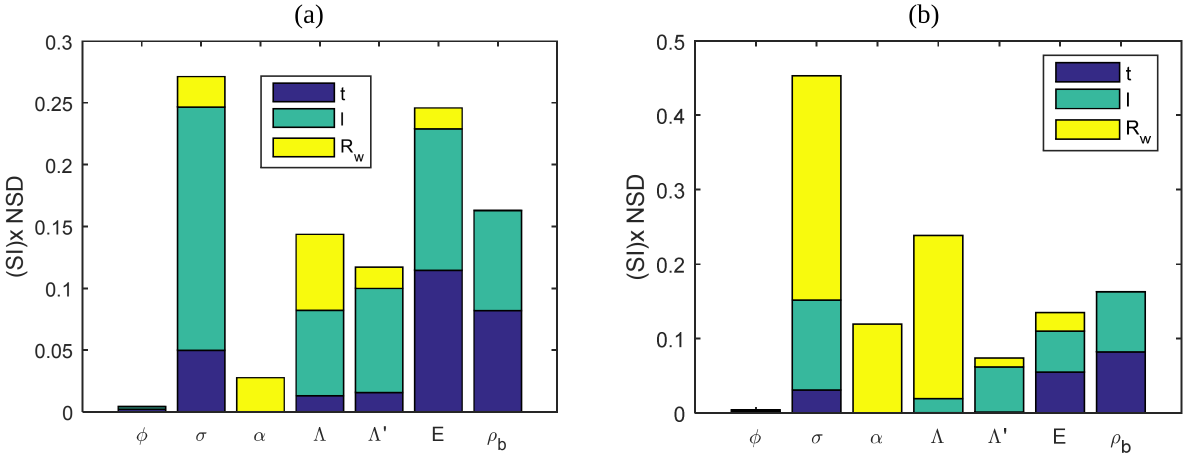

Figure 3: *The effect of microstructure variability on macroscopic parameters for: (a) Materia V1, and (b) Material V2 (Table 1).*

5.2 Impact of Biot-Allard parameters variability on the sound absorption and TL

The first acoustic performance indicator of interest is the sound absorption coefficient at normal incidence. The laterally infinite poroelastic material of thickness $(0.05\ (m))$ is backed by an impervious rigid wall (see Fig. 4). The sound absorption is derived from reflection coefficient (r):

$$\alpha(\omega) = 1 - |r(\omega)|^2, \quad r = \frac{(Z_s(\omega) - \rho_0\, c_0)}{(Z_s(\omega) + \rho_0\, c_0)}\,, \tag{7}$$

where (Z_s) is the impedance at the surface, and c_0 is sound speed in air. The surface impedance is calculated using transfer matrix method (TMM)[1]. First, a global sensitivity analysis is done to investigate the impact of Biot-Allard parameters on the sound absorption. Hence, a set of PU foams, compatible with the presented micro-macro model, is considered. Their properties are listed in Table 2. Figure 4a presents the sensitivity results, while Fig. 4b presents the sound absorption curve with its minimum, maximum, and standard deviation envelops. As expected the flow resistivity has more effect on the absorption at low frequencies, while mechanical properties are key parameters at the frame-born resonances (where one-quarter of acoustic wave length is equal to the foam thickness). The effect of tortuosity increases as reaching to the maximum sound absorption. These results are in agreement with the results presented by Ouisse[11].

Table 2: Input parameters for Bio-Allard model (wide range)

Parameters	**Variable**	**Inferior limit**	**Superior limit**
Porosity	$\phi\ (-)$	0.95	0.98
Flow resistivity	$\sigma\ (N\,s\,m^{-4})$	5,000	40,000
Tortuosity	$\alpha_\infty(-)$	1.05	2.00
Viscous characteristic length	$\Lambda(m)$	0.041×10^{-3}	0.205×10^{-3}
Thermal characteristic length	$\Lambda'(m)$	0.190×10^{-3}	0.319×10^{-3}
Elastic modulus	$E\ (kPa)$	320	480
Poisson ratio	$\nu(-)$	0.3	0.4
Density	$\rho_s(kgm^{-3})$	22	27
Loss factor	$\eta\ (-)$	0.10	0.15

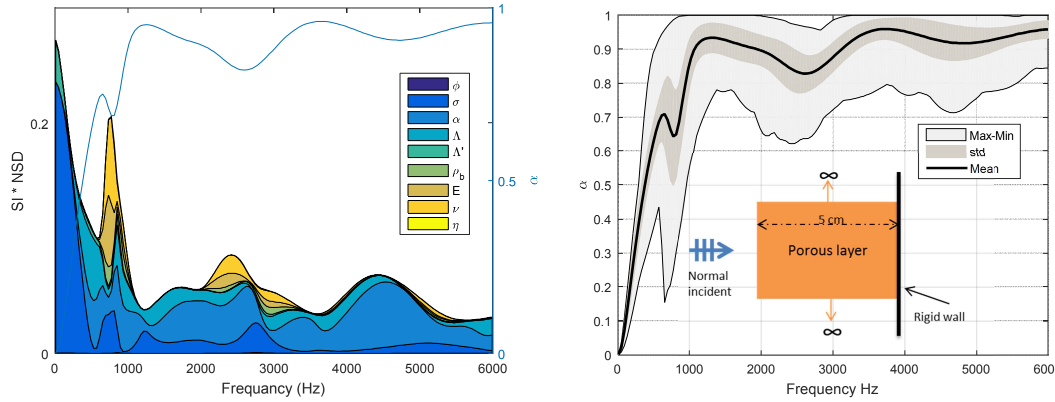
(a) (b)

Figure 4: *(a) The impact of Biot-Allard parameter variations on the sound absorption coefficient. (b)* Minimum, maximum, and standard deviation for the sound absorption coefficient.

Then, a global sensitivity analysis is done on the TL when the foam is attached to an elastic structure. As shown in Fig. 5b, it is assumed that the porous layer is laterally infinite, and is placed on an isotropic thin $(0.004(m))$ Aluminum (Al) plate. Thickness of the porous layer is 0.05 (m), and the properties are the same as listed in Table 2. An oblique $(\theta = \pi/3\ )$ plane wave is impinging on the plate. Figure 5a shows the impact of macroscopic Biot-Allard parameters on the TL. Results show that the effects of mechanical properties of the foam are dominant in the vicinity of frame-born resonance. Since the plate is transparent at the coincidence frequency, the effect of sound absorption of the foam is considerable. At this frequency, parameters that have more impact on the sound absorption are dominant.

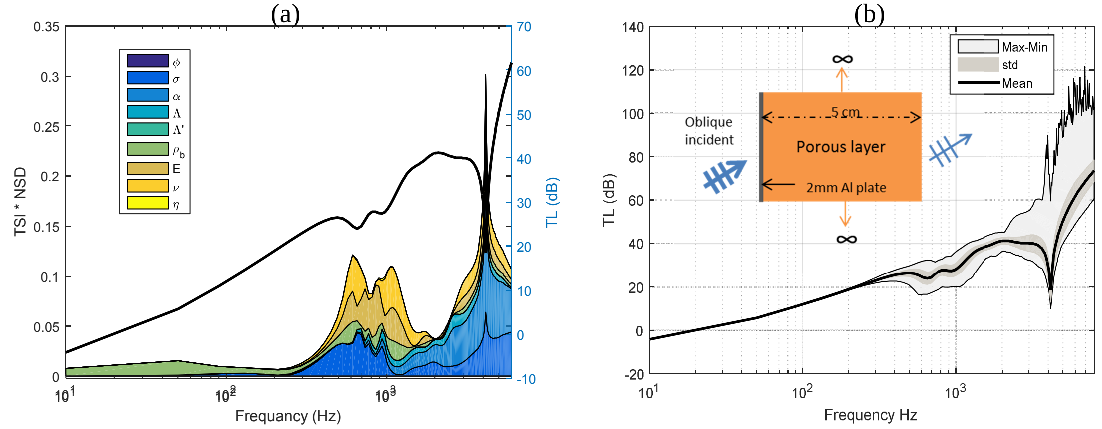

Figure 5: *(a) The effect of Biot-Allard parameters on the TL. (b)* Minimum, maximum, and standard deviation for the transmission loss.

5.3 Impact of microstructure variability on the sound absorption

The macroscopic properties of Biot-Allard model are now estimated from microstructure properties (Table 1) using the presented micro-macro models. The sound absorption curve and the contribution of each parameter on the output are shown in Fig. 6. Results show that the strut's length has lightly more impact than strut's thickness on the frame-born resonance frequency where, as shown in Fig. 4, mechanical properties, flow resistivity, and tortuosity are dominant parameters. These results are observed in Fig. 3, where thickness and length of struts have the same impact on mechanical properties, tortuosity is controlled by reticulation rate, and the effect of strut's length is dominant in flow resistivity. The same contribution is seen in Fig. 6. The effect of reticulation rate is dominant at low frequencies where flow resistivity plays important role in the absorption problems.

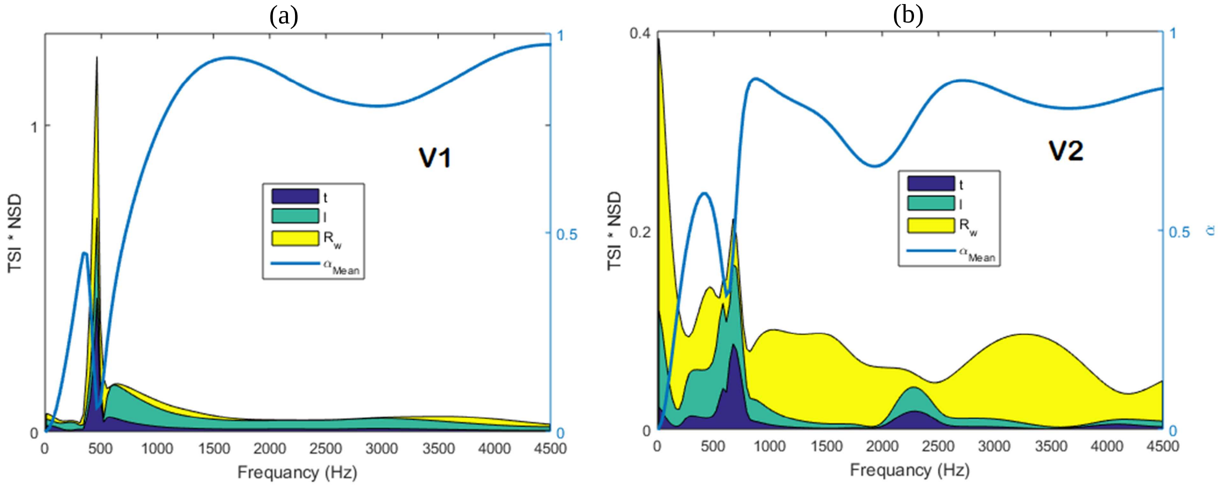

Figure 6: *Effect of microstructure variability on the sound absorption for (a) V1 and, (b) V2.*

5.4 Impact of microstructure variability on transmission loss

The effect of microstructure variability on the TL of the plate-foam system of section 5.2 is presented in Fig. 7 for the two materials of Table 1. This figure shows the mean of the outputs and the sensitivity of the TL to each of the three microstructure parameters. Results at the frame-born resonance are the same as mentioned for the absorption problem. Strut's length is the dominant parameter at the coincidence frequency when reticulation rate is low (V1). While the reticulation rate plays the key role at the coincidence frequency for the second material (V2).

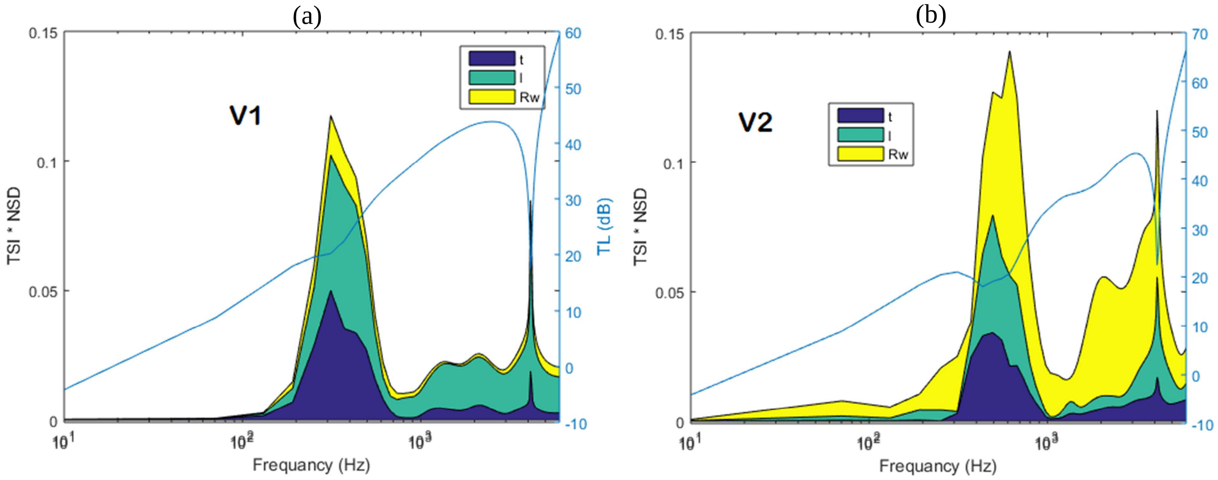
(a) (b)

Figure 7: *Effect of microstructure variability on the TL for (a) V1 and, (b) V2.*

6 CONCLUSION

To enhance the acoustical behavior of cellular materials, developing correlations between microstructure and Biot-Allard parameters is needed. In this study, an augmented micro-macro model is presented to correlate the effect of reticulation rate on effective Young's modulus of PU foams. The model is based on the analytical micro-macro model, presented for fully reticulated PU foams by Gong and Jang[4,5]. The modified micro-macro model together with micro-macro model developed by Doutres *et. al* is used to investigate the impact of microstructure irregularities on: first, macroscopic Biot-Allard parameters, and then, the acoustical performance indicators. Before that, a global sensitivity analysis is done to show the contribution of the Biot-Allard parameters, at each frequency, on a sound absorption, and a transmission loss problem. Results show that the irregularities in length and thickness have the same impact on the effective Young's modulus, while in the case of high reticulated PU foams the strut's length is important on non-acoustical properties. But, in the case of low reticulated PU foams, reticulation rate is dominant microstructure parameter in the flow resistivity, tortuosity and viscous length. The sound absorption of the foam is studied as the first acoustical performance indicator. Results show that length and thickness of strut have almost the same effect where mechanical properties are more important, while the length effect gets the lead where non-acoustical properties are dominant parameters. Study on the impact of the irregularities in microstructure of PU foams on the TL shows that the mechanical properties are dominant at the frame-born resonances, while the parameters which are controlling the sound absorption get lead at the coincidence frequency.

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