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USING TRANSFER PATH ANALYSIS METHODS FOR ASSESSING THE VELOCITY RESPONSE ON A PLATE GENERATED BY A DUMMY VIBRATORY SOURCE.

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Multiple vibratory sources are integrated in the aircraft and rotorcraft. The vibrational power of these sources is injected to the receiving structure through their connection points resulting in annoying acoustic levels in the cabin. This noise, referred to as structure borne noise, could be mitigated if vibrating systems, receiving structures and interfaces between them are well designed. Methods such as Reception Plate Method (RPM) and Component-Based Transfer Path Analysis (CB-TPA) have been developed to specify proper design guidelines related to noise mitigation during the design phase. This paper focuses on the CB-TPA. The main advantage of this method is to predict the vibratory behavior of an assembly (source and receiving structure) from the intrinsic properties of its subsystems. Nevertheless, it is not straightforward to assess these properties because of several experimental difficulties such as the completeness of the mobility matrices, a passive property required for both subsystems. The objective of this work is to assess the experimental applicability of two CB-TPA methods on a small scale laboratory setup comprising of a controlled vibratory source mounted onto a flat panel. Two controlled vibratory sources with more or less complex vibratory behaviors have been designed in order to assess the CB-TPA methods sensitivity to matrices incompleteness. Passive and active properties of subsystems are assessed, taking into account 3 translational degrees of freedom. The vibratory response of the assembly generated by the source coupled to the plate is estimated and the impact of the completeness of the subsystem's characterization is discussed.

Keywords: transfer path analysis, structure-borne noise, component-based TPA, dummy source

1. Introduction

Multiple vibratory sources are integrated in the aircraft. The vibrational power of these sources is injected to the receiving structure through their connections points resulting in annoying acoustic levels in the cabin. This noise, referred to as structure borne noise (SBN), could be mitigated if vibrating systems, receiving structures and interfaces between them are well designed. Methods such as Reception Plate Method (RPM) [1], Inverse Force Method (IFM) [2] and Component-Based Transfer Path Analysis (CB-TPA) [3] have been developed to specify proper design guidelines related to noise mitigation during the design phase. The objective of this work is to assess the experimental applicability of two CB-TPA methods on a small scale laboratory setup comprising of a controlled vibratory source mounted onto a flat panel. Unlike classical TPA methods, CB-TPA methods allow for the estimation of interface forces of an assembly from the intrinsic properties of its subsystems (source and receiving structure); subsystems which may not exist or cannot be coupled yet. Once the interface forces are known, it is possible to assess the SBN in the aircraft cabin from experimental approaches (e.g., measured vibro-acoustic transfer function) or numerical approaches (e.g., finite element model of the aircraft).

Two types of intrinsic properties are required for assessing SBN following CB-TPA methods: (1) active properties of the source subsystem such as the free velocity or the blocked force and (2) passive properties of the two subsystems such as mobility matrices at the interface points. These methods encounter several experimental difficulties [5] such as the completeness of the mobility matrices including quantification of rotational degrees of freedom (RDOFs). Indeed, in order to (i) avoid a cumbersome and time-consuming measurement procedure and (ii) limit the introduction of measurement noise, only translational degrees of freedom (TDOFs) are generally characterized; the contributions of moment and angular velocity are neglected and the mobility matrix is 25% full. Furthermore, because triaxial accelerometers are not always available, the experimental procedure can be greatly simplified by neglecting the couplings between the three TDOFs. In this case, the mobility matrix only accounts for diagonal terms and is 8% full. Finally, the mobility matrix can also account for only one TDOF. This corresponds to the most common measurement configuration where both source and receiving structure behaviors are characterized according to one TDOF only (usually the one in the direction perpendicular to the receiving structure). The mobility matrices are only 3% full in this latter case.

In this work, two CB-TPA methods are investigated using a laboratory test bench used both as a final receiving structure (i.e., considered as a dummy aircraft structure) and as a host structure for characterizing the source active properties. Interface forces are assessed for two custom-made dummy vibratory sources having different vibratory behaviors. The validity of the two CB-TPA methods is assessed from the vibratory response on a point of the test bench either measured directly or estimated from the interface forces derived from the two CB-TPA methods of interest. Furthermore, using the test bench as final receiving structure (i.e., a dummy aircraft structure) allows the use of the classical Inverse Matrix TPA method for assessing the operational forces. The latter is considered as a reference and also used for assessing CB-TPA validity. Only TDOFs are considered for all measurements and computations. The influence of the number of TDOFs and the couplings between them are investigated.

Section 2 describes the theory of TPA methods. The experimental set-up is detailed in Section 3. The TPA methods are finally assessed in Section 4.

2. Theory

2.1 Mobility concept

The mobility is a characteristic of the dynamical behavior of a mechanical system and characterizes its "ease of motion" according to three TDOFs and three RDOFs. It is defined by:

$$Y_{X,jk} = \frac{u_j}{F_k}, \tag{1}$$

where $Y_{X,jk}$ is the mobility matrix of the system X linking points k and j (belonging to the system), F_k is the force vector (force and torque) applied at point k and u_j is the resulting velocity vector measured at point j. The subscript X indicates the system on which the mobility is measured: A for the active subsystem alone, P for the aircraft passive subsystem alone, AP for the real assembly (active subsystem attached to the aircraft structure), B for a test bench passive subsystem alone and AB for the lab assembly (active subsystem attached to the test bench). All terms of Eq. (1) are frequency dependent but (ω) is omitted to lighten notation.

2.2 Interface forces assessment

An active subsystem A and an aircraft passive subsystem P are schematically presented in Figures 1.a) and 1.b). They are rigidly connected together at interface points c thanks to rigid and massless connections (soft connections are not considered in this work). The origins of vibrations are due to the internal dynamic excitation of the active subsystem (therefore difficult to assess in practice) [3] which is schematically represented by point i in Figure 1.a). Vibrations are transmitted to interface points between both subsystems and transmitted to the entire passive subsystem, thus inducing SBN.

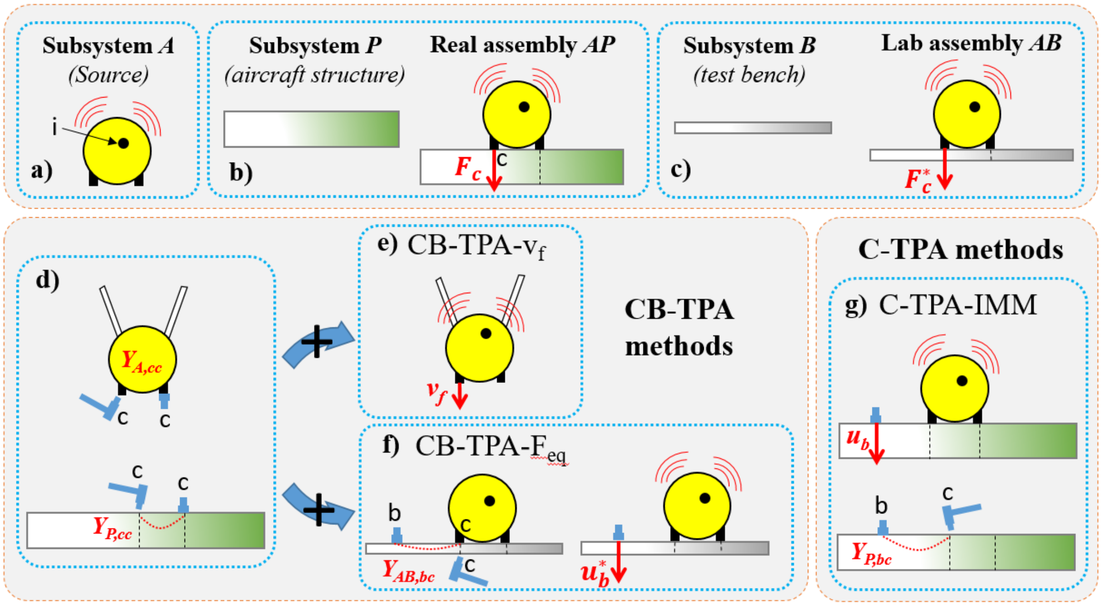

Figure 1: a) Source; b) Source / aircraft structure assembly; c) Source / test bench assembly; d) passive properties of the subsystems; (e) free velocity v_f of the active subsystem freely suspended; f) « *in situ method* » for equivalent force assessment and g) classical TPA Inverse Matrix Method.

When the active subsystem is turned on, forces are generated at the interface between both subsystems, represented by vector F_c in Figure 1.b). These forces are called interface forces (or sometimes operational forces) and depend on the dynamic behavior of both subsystems. They are thus referred to as F_c^* when the source is mounted onto the test bench (subsystem B) as shown in Figure 1.c).

The CB-TPA methods allow for assessing the interface forces F_c from passive intrinsic properties of both decoupled subsystems (i.e., mobility matrices $Y_{A,cc}$ and $Y_{P,cc}$ measured at the contact points c as shown in Figure 1.d)) and from an intrinsic property of the active subsystem. Two CB-TPA methods are investigated here depending on the required source active property. The first CB-TPA method is based on the active subsystem free velocity v_f, (see Figure 1.e)) and is referred to as CB-TPA-v_f:

$$F_c = \left(Y_{A,cc} + Y_{P,cc}\right)^{-1} v_f \,. \tag{2}$$

The active subsystem free velocity can be measured at the interface points *c* when "freely" suspended as shown in Figure 1.e). The second CB-TPA method is based on the equivalent force F_{eq} (the reader is referred to reference [3] for a detailed presentation of this concept), and is referred to as CB-TPA-$\mathrm{F_{eq}}$:

$$F_c = \left(Y_{A,cc} + Y_{P,cc}\right)^{-1} Y_{A,cc} F_{eq} \,. \tag{3}$$

The equivalent force can be estimated from « *in situ* method » [2][3], where the active subsystem is coupled to a test bench *B* as passive subsystem as shown in Figures 1.c) and 1.f). Accelerometers are set onto the passive subsystem close to the interface, as represented by point *b* in Figure 1.f). Equivalent forces are computed thanks to the pseudo-inversion of the lab assembly mobility matrix $Y_{AB,bc}$ and the velocity u_b^* given by accelerometers:

$$F_{eq} = \left(Y_{AB,bc}\right)^{-1^*} u_b^* \,. \tag{4}$$

In this work, the aforementioned CB-TPA methods are assessed using a dedicated laboratory test bench (subsystem *B)* (see sec. 3.2), also considered as a final receiving structure (which means that subsystem *B* = subsystem *P*, see Figures 1.b) and 1.c)). Thus, only F_c^* are estimated but are considered equal to F_c since the test bench is used as a "dummy" aircraft structure.

Considering the test bench as a final receiving structure allows for the use of classical TPA methods for estimating the operational forces on the same test bench ($F_c = F_c^*$). In this work, the classical TPA Inverse Matrix Method (referred to as C-TPA-IMM here) [2][3] is used (see Figure 1.g)). From this method, the operational forces are reconstructed from:

$$F_c = \left(Y_{P,bc}\right)^{-1^*} u_b \,, \tag{5}$$

where $(.)^{-1^*}$ means the pseudo inversion, $Y_{P,bc}$ is the receiving structure mobility (i.e., the force is applied at the contact points *c* and the structure velocity is measured at points *b* (see Figure 1.g)). The velocity u_b ($u_b = u_b^*$) is measured at various locations *b* on the receiving structure when the vibratory source is attached and operating (see Figure 1.g)). The C-TPA-IMM method is considered as a reference in this work since it is much more direct compared to CB-TPA methods. Note that it is also used in the companion paper [4] for assessing the vibratory power injected by an active system into the test bench structure using the Inverse Force Method (IFM)[2].

2.3 Response assessment of the assembly

The normal vibration velocity at an arbitrary location on the test bench (i.e., referred to as "reference point" *m*) induced by the source vibration is used as objective indicator for CB-TPA methods assessment. A reference value can be easily obtained from direct measurements using a simple accelerometer. This property can also be estimated from the interface forces reconstructed from the TPA methods and a transfer matrix H_X such as:

$$u_m = H_B F_c \,. \tag{6}$$

The matrix H_B is obtained by applying an impulse excitation on the 4 contact points of the test bench alone (subsystem B) along the three TDOFs and by measuring the vibration velocity at the reference point *m* on the test bench along the *z*-TDOF.

3. Experimental setup

3.1 Dummy sources

In this preliminary work, the objective is to design two dummy sources with two different vibratory behaviors. These custom-made sources have a common design based on three aluminum beams (H-shape) and two miniature inertial electrodynamic actuators (Modal Shop model 2002E) as shown in Figure 2. The input signal of both actuators is a sine wave generated by a BK precision 4052 signal generator. The dummy sources has four feet used as contact points to be attached to the receiving structure. In order to create the two dummy sources, two main design variables are considered: (1) the position of the actuators on the H-structure either on the center of the H-structure (see Figure 2.a)) or at the opposite of each branch of the H-structure (see Figure 2.b)) and (2) the actuators input signals on the two actuators being in phase or counter phase. Identification of the two sources is based on source free velocity measurements carried out on the 4 feet along the 6 directions (3 TDOFs and 3 RDOFs) using four triaxial accelerometers (PCB model 356A03) and one angular rate sensor (Kistler 8840). The actuators input signals frequency is 500 Hz. This operating frequency ensured that the source characterization, when freely suspended (as required in CB-TPA methods as shown in Figure 1), is performed well above the "spring/mass" frequency of the "source/suspension" coupled system. Note that the dummy sources can also be operated at higher frequencies as shown in the companion paper [4].

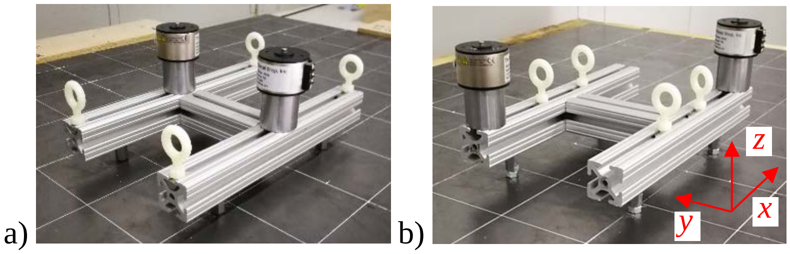

Figure 2: Dummy sources coupled to the test bench: a) dummy source S1 with actuators at the center of the H-structure, b) dummy source S2 with actuators at the opposite of each branch of the H-structure.

3.2 Test bench

The test bench is a simply supported stainless-steel plate with dimensions of 965.2 × 1371.6 × 4.8 mm (38 × 54 × 3/16 inches) (Figure 3.a)). A set of 8 accelerometers (PCB model 356A45) are placed onto the plate. 7 accelerometers are used to apply the C-TPA-IMM and the CB-TPA-F_{eq} method (locations b in Figure 1). These points are referred to as "response points" (red cross in Figure 3.b)). Then, interface forces F_c and the transfer matrix H_B are used to assess the vibration velocity response at the "reference point" u_m (see blue cross in Figure 3.b)) according to Eq. (6).

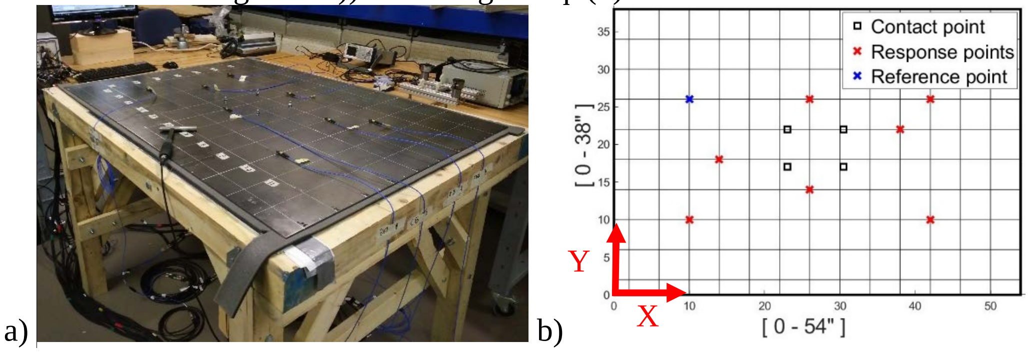

Figure 3: a) Picture of the test bench; b) Position of the accelerometers onto the plate.

4. Results

4.1 Dummy sources

Based on free velocity measurements (not shown here) with dummy source actuators operating at 500 Hz, the first dummy source, referred to as S1 is obtained when the two actuators are operated in phase and located at the center of the H-structure (Figure 2.a)) and the second one, referred to as S2, is obtained when the two actuators are also operated in phase but located at the opposite of each branch of the H-structure (Figure 2.b)). S1 and S2 generate similar translational free vibrations along *z* but S2 shows much more important translational vibrations along *x* and *y*. Regarding the rotational free vibrations, both sources show similar behaviors along *x* and *y*, the maximum rotational vibrations being along the *y*-axis for both configurations. However, S2 generates more important rotational free vibrations along the *z*-axis compared to S1. From the previous observations, it is considered that (i) the source S1 is a “translational source” and (ii) the vibratory behavior of S2 is more complex than the one of S1.

4.2 Assessment of the CB-TPA methods

Figure 4 illustrates the plate vibration velocity along *z* at the reference point (u_m, referred to as the “plate reference response” in the following) resulting from the vibration of the dummy source S1 (first line of Figure 4) or S2 (second line of Figure 4). As mentioned in sec. 2.3, the plate reference response is measured directly (solid purple lines) or assessed from Eq. (6) with the interface forces F_c determined from the C-TPA-IMM (Figures 4.a) and 4.d)), CB-TPA-v_f (Figures 4.b) and 4.e)) and CB-TPA-F_{eq} (Figures 4.c) and 4.f)). For each TPA method, the calculations account either for (i) all TDOFs, (ii) all TDOFs but neglecting the couplings between the 3 TDOFs and (iii) only the *z*-TDOF and neglecting the couplings between the 3 TDOFs.

Figures 4.a) and 4.d) show that the C-TPA-IMM method, considered as a reference, correctly estimates the plate reference response due to S1 or S2 when all TDOFs and couplings between them are accounted for. This method seems sensitive to TDOFs couplings in the case of S2 since it overestimates (around 3 dB) the plate reference response when they are neglected. Conversely, the C-TPA-IMM method underestimates (around 6 dB for S1 and 4 dB for S2) the plate reference response when only the z-TDOF is accounted for.

Figures 4.b) and 4.e) show that the CB-TPA-v_f method greatly overestimates the plate reference response, and especially in the case of the dummy source S2. The overestimation is reduced as the matrix’ completeness decreases and this method almost correctly predicts the reference response in the case of the dummy source S1 when only the *z*-TDOF is taken into account (see Fig. 4.b)). This method thus seems highly affected by matrix incompleteness and the large discrepancies could be attributed to the source active property characterization being based on “free” conditions and thus very different from the source boundary conditions when attached to the receiving structure.

Finally, Figure 4.c) shows that the CB-TPA-F_{eq} method correctly estimates the plate reference response due to S1 when all TDOFs are accounted for (with or without couplings between TDOFs). The method underestimates by 3 dB the plate reference response due to S1 when only z-TDOF is considered; which remains acceptable. On the contrary, this method highly underestimates the plate reference response due to S2 even when all TDOFs with couplings between them are taken into account (see Figure 4.f)).

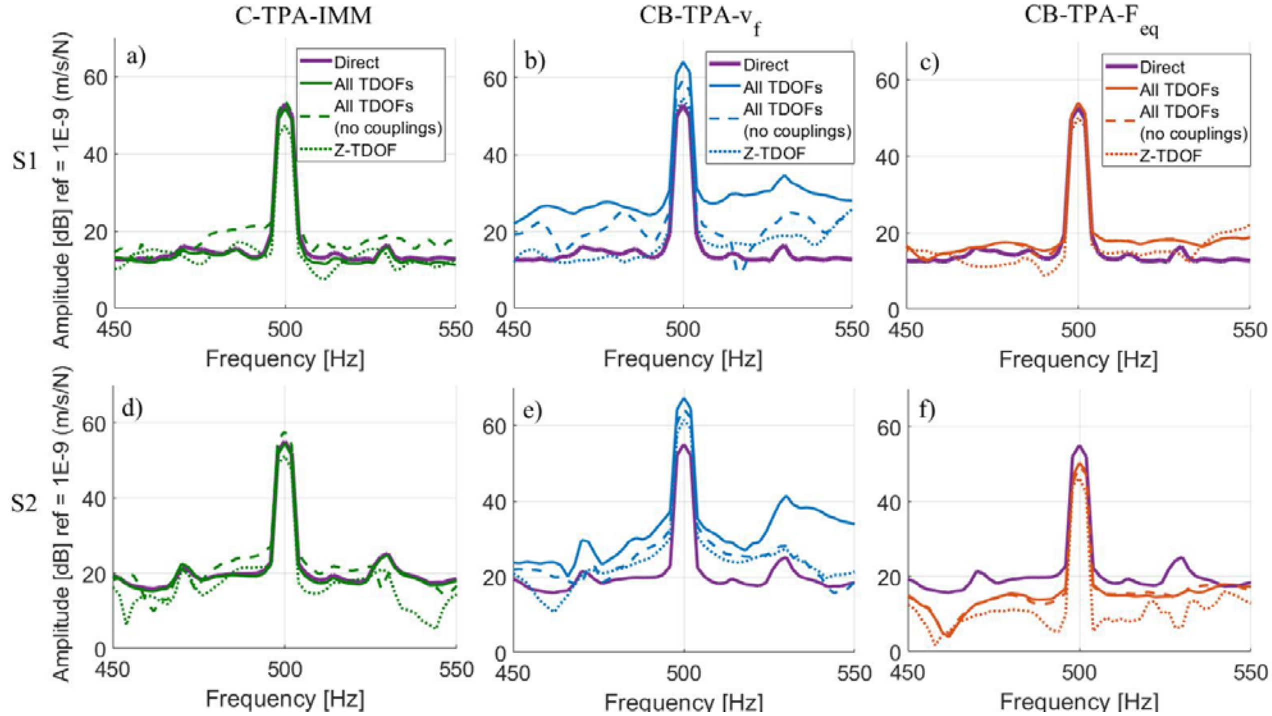

Figure 4: Velocity power spectrum at the reference point on the plate due to the dummy source S1 (first line) or S2 (second line) and measured directly (purple line) or assessed from the C-TPA-IMM (first column), the CB-TPA-v$_f$ (second column) and the CB-TPA-F$_{eq}$ (third column).

More general comments can be drawn from these observations. First, TPA methods based on measurements carried out on the coupled subsystems (i.e., C-TPA-IMM and CB-TPA-Feq) including a receiving structure having the mobility close to the one of the final receiving structure (it is the case here since subsystem *P* = subsystem *B*) seem working properly for sources having a preferred translational axis of vibration (case of source S1). The use of sources with more complex vibratory behavior (case of source S2) seems to highlight some limitations of the CB-TPA method even if based on properties of the coupled systems (e.g., Y_{AP} and u_b for the CB-TPA-F$_{eq}$). This could be attributed to the fact that (i) the RDOFs have not been taken into account and (ii) both CB-TPA methods require passive property of the source alone (i.e., Y_A) which is difficult to measure in practice. This latter observation is supported by the fact that the CB-TPA method mainly based on "free" source properties (i.e., Y_A and v_f for the CB-TPA-v$_f$) is unable to predict correctly the plate vibration for both dummy sources S1 and S2 and even when all TDOFs are taken into account.

5. Conclusion

In this work, the potential of two CB-TPA methods was investigated using a laboratory test bench and two custom-made dummy vibratory sources having a more or less complex vibratory behavior when freely suspended. Both CB-TPA methods require the passive intrinsic properties of the two decoupled subsystems (i.e., the mobility matrices of the source Y_A and the one of the receiving structure Y_P) but differ from the required active subsystem intrinsic property: the free velocity v_f for the CB-TPA-v$_f$ method and the equivalent force F_{eq} for the CB-TPA-F$_{eq}$ method. The validity of the two CB-TPA methods was assessed from their ability to correctly predict the vibratory response at a reference point of the test bench. This reference response (i.e., the plate vibration velocity) was either measured directly or estimated from the interface forces derived from three TPA methods: the classical TPA Inverse Matrix Method (referred to as C-TPA-IMM and also used as reference), the CB-TPA-v_f and the CB-TPA-F_{eq} methods. All the aforementioned methods require the characterization of the passive property of coupled or decoupled subsystems through mobility matrices. This work thus evaluated the completeness of the mobility matrices when only TDOFs are taken into account while degrading it progressively in three steps (i.e., 3 TDOFs with or without couplings between TDOFs and 1 TDOF only without couplings between TDOFs). It is shown that the TPA methods based on measurements carried out on the assembled subsystems (i.e., C-TPA-IMM and CB-TPA-F_{eq}) including a receiving structure having the mobility close to the one of the final receiving structure (which is the case here since subsystem P = subsystem B) seem working properly for sources having a preferred translational axis of vibration. However, the use of sources with more complex vibratory behavior highlighted some limitations of the CB-TPA-F_{eq} method; most probably because this method requires passive property of the active component in "free" conditions. In line with this observation, it is shown that the CB-TPA-v_f method (which is mainly based on active source properties in "free" conditions) is unable to predict correctly the plate vibration even when all TDOFs are accounted for. Finally, it is shown that (i) all TDOFs including couplings between TDOFs should preferably be taken into account since (as expected) the impact of matrices incompleteness depends on the active source vibratory behavior and (ii) the sensitivity to TDOFs couplings depends on the used TPA method.

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