

# Acoustic imaging with conventional frequency domain beamforming and generalized cross correlation: a comparison study

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## Abstract

Nowadays, acoustic cameras have become a standard tool for visualizing sound sources. The main post-processing techniques for generating acoustic images are associated with beamforming in the time or frequency domain. While many advanced techniques have been developed for the latter in order to achieve high resolution, such as deconvolution or inverse methods, the former have been less investigated. Recent studies have shown that time-domain beamforming can provide a narrow main lobe with low side lobe levels in different situations. However, a comparison of both techniques in terms of acoustic imaging does not exist. This paper provides a detailed comparison of conventional frequency-domain beamforming (CBF) and time-domain beamforming based on the generalized cross-correlation technique (GCC). First, numerical data are used for assessing both techniques. Then, experimental tests are carried out in a hemi-anechoic room using loudspeakers or power tools. As expected, acoustic images provided by CBF and GCC are found to be very similar but with different computation times. In order to improve the acoustic image, advanced techniques are considered (Clean-SC and the GCC based on geometric mean). When dealing with low frequency, Clean-SC is not able to detect both sources while the GCC based on geometric mean provides an accurate source separation.

**Keywords:** Acoustic imaging, beamforming, time domain, frequency domain, generalized cross-correlation, deconvolution technique

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## 1. Introduction

Nowadays, many acoustic manufacturers propose acoustic cameras for visualizing sound sources. These tools include a microphone array, a camera and

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4 a data acquisition system. The digitized microphone signals are combined in  
5 order to generate an acoustic image while the camera captures an image of the  
6 environment under study. Then, both images are overlaid. The result is usually  
7 a black and white image of the environment with filled colored contours. The  
8 peaks of the contours exhibit the source positions and the colors provide the  
9 sound levels. Acoustic imaging techniques have been used for a wide range of  
10 applications from cars [1, 2, 3] to aircraft [4, 5, 6] or even snowmobiles [7]. If  
11 the microphone array is set close to the object under study, typically less than  
12 a wavelength away, the common technique is the near-field acoustic hologra-  
13 phy [8, 9]. However, if the source is far from the microphone array, the classic  
14 technique used is beamforming [10, 11]. In this work, the focus is made on the  
15 comparison of the frequency and time domain beamforming.

16 The standard beamforming technique for generating an acoustic image, also  
17 known as conventional beamforming (CBF) [12] is applied in the frequency-  
18 domain. First, the Cross Spectral Matrix (CSM), obtained from a Fast Fourier  
19 Transform of the microphone signals, is computed. The self noise of the micro-  
20 phones may be removed by setting to 0 the diagonal elements of the CSM [13].  
21 Then, a virtual focusing plane containing the source is considered which allows  
22 for the computation of the steering vector [14, 15]. Finally, the CSM and the  
23 steering vector are combined in order to generate the acoustic image. The two  
24 main characteristics of an acoustic image are the main lobe, which indicates  
25 the source position, and the side lobes which represent spurious sources posi-  
26 tions [16]. The CBF is known as a fast and robust technique, but presents some  
27 drawbacks when dealing with low or high frequencies. Indeed, the main lobe  
28 width enlarges when frequency decreases, while the side lobe amplitude increases  
29 with the frequency [17]. In order to tackle these issues, improvements have been  
30 proposed such as optimized geometries [18, 19, 20, 21, 22, 23] or advanced post-

31 processing techniques [24, 25, 26]. In terms of geometry, the common tradeoff  
 32 seems to be now the spiral arrangements for planar array [27], while for the  
 33 post-processing, deconvolution techniques or inverse problems become standard  
 34 techniques. The growing interest in acoustic imaging with CBF has been re-  
 35 cently highlighted by the publication of three review papers dealing with ad-  
 36 vanced post-processing techniques, array geometry or applications [24, 25, 26].  
 37 As mentioned by the Ref. [26], the major steps forward in acoustic imaging have  
 38 been performed in the frequency-domain although some time-domain beamform-  
 39 ing techniques exist.

40 The time-domain beamforming consists in delaying and summing the micro-  
 41 phone signals (called D&S) [12, 28, 29]. Unlike the CBF which is a narrow-band  
 42 technique (an acoustic image is obtained for each frequency bin), the D&S is a  
 43 broadband technique. Indeed, the whole microphone signals are delayed, there-  
 44 fore no frequency band is selected. In 2004, Dougherty has investigated the  
 45 influence of a pre-filter operation on microphone signals in order to select the  
 46  $n^{th}$  octave band [30]. He also introduced a formulation for diagonal removal  
 47 and cross-shaped array for improving the acoustic image. However, the acous-  
 48 tic images provided by the time D&S and CBF were not compared. In 2006,  
 49 Jaeckel has discussed the strength and weakness of D&S beamforming [31].  
 50 The strength of D&S beamforming were the applicability to non-stationary and  
 51 strongly transient signals [32] as well as good resolution of the broadband sig-  
 52 nals. The weakness were the need for high sampling rate and the poor resolution  
 53 at low frequency.

54 Advanced signal processing algorithms are mainly implemented in the fre-  
 55 quency domain, few techniques exist in the time-domain [33, 34] and are inspired  
 56 by the well known frequency-domain technique Clean-SC [35]. Recently, stud-  
 57 ies have taken the advantages of the generalized cross-correlation formulation

58 (GCC) to improve D&S beamforming [36, 37]. The GCC has been introduced by  
 59 Knapp in 1976 [38]; this technique allows a pre-filtering operation by a weight-  
 60 ing function before computing the cross-correlation. The most known weighting  
 61 function is the PHase Transform (PHAT) which divides the cross spectrum of a  
 62 microphone pair by its absolute value [38]. When a broadband signal is consid-  
 63 ered, this weighting function narrows the main lobe of the cross-correlation and  
 64 therefore improves the acoustic image. Other improvement techniques based on  
 65 GCC have been proposed such as a spatial weighting [39], the use of the gen-  
 66 eralized mean [40] or enhanced weighting functions [41]. Each technique allows  
 67 for narrowing the main lobe and decreasing the amplitude of the side lobes. In-  
 68 verse method based on sparsity constraints have also been proposed to achieve  
 69 acoustic imaging with high resolution [42, 43, 44].

70 In 2014, Hamid *et al.* have compared the performance of the time and fre-  
 71 quency domain beamforming in terms of computation time [45]. However, from  
 72 the author’s knowledge no study has provided a detailed comparison of acous-  
 73 tic images given by both techniques. The CBF is considered as the standard  
 74 technique and advanced techniques are mainly based on its formulation. The  
 75 D&S beamforming is rarely used but seems to be more appropriated for some  
 76 cases. Although, the CBF and D&S beamforming should provide similar results,  
 77 their formulations are different as well as the advanced techniques. Therefore,  
 78 a detailed comparison of both techniques would allow for selecting the more  
 79 appropriated one.

80 The objective of this work is to propose a detailed comparison of both tech-  
 81 niques in terms of acoustic images. First, the theoretical background is intro-  
 82 duced in Section 2. Then, numerical data are used to compare the acoustic  
 83 images provided by both techniques (Section 3). Finally, experimental data are  
 84 considered in Section 4. The acoustic images are discussed in the simple case



85 of two loudspeakers. Then power tools are considered, a leaf blower (aeroa-  
 86 coustic source) and a nail gun (impulsive source). A comparison of advanced  
 87 post-processing is also discussed.

## 88 2. Theoretical background

### 89 2.1. Microphone signals

90 A source signal  $s(\mathbf{r}_s, t)$  is generated by an omnidirectional acoustic point  
 91 source located at  $\mathbf{r}_s$  and recorded by a set of  $M$  microphones at locations  $\mathbf{r}_m$ .  
 92 The signal  $x_m$  recorded by the microphone  $m$  is given by

$$x_m(t) = s(\mathbf{r}_s, t - \Delta t_{ms}) + v_m(t), \quad (1)$$

93 where  $t$  represents time and  $v_m(t)$  is an uncorrelated additive noise due to  
 94 background or sensor noise. The term  $\Delta t_{ms}$  corresponds to the time of flight  
 95 (ToF) between the source and the microphone and is defined by the Euclidean  
 96 distance

$$\Delta t_{ms} = \frac{\mathbf{r}_{ms}}{c_0} = \frac{\|\mathbf{r}_m - \mathbf{r}_s\|_2}{c_0}, \quad (2)$$

97 where  $c_0$  is the sound speed and  $\|\cdot\|_p$  is the  $l_p$ -norm of a vector or a matrix.

98 The cross spectrum  $C_{x_m x_n}(f)$  between the microphone signals  $x_m$  and  $x_n$  is  
 99 given by

$$C_{x_m x_n}(f) = X_m(f)X_n(f)^H, \quad (3)$$

100 where  $X_m$  and  $X_n$  are the frequency domain microphone signals obtained with  
 101 the Fast Fourier Transform (FFT) and the superscript  $(\cdot)^H$  represents the Her-  
 102 mitian transpose.

103 With the CBF, the cross and auto spectra are gathered into the so-called  
 104 Cross Spectral Matrix (CSM), denoted  $\mathbf{C}$ , and are usually averaged using the  
 105 Welch's periodogram [46] for removing background noise.

106 With the GCC, the cross spectra are used to recover the cross-correlation  
 107 function using the Inverse Fast Fourier Transform (iFFT).

108 For each technique, a scan zone where the source positions is sought has to  
 109 be defined. In the case of a planar microphone array, the scan zone is usually a  
 110 plane regularly discretized with  $L$  points at position  $\mathbf{r}_l$ .

## 111 2.2. Frequency-domain beamforming: Conventional Beamforming (CBF)

112 In the frequency domain, the beamformer output power  $Z_l(f)$  for a frequency  
 113 bin is given by

$$Z_l(f) = \mathbf{h}_l(f)^H \mathbf{C}(f) \mathbf{h}_l(f), \quad (4)$$

114 where  $\mathbf{h}_l(f)$  is known as the steering vector and is a normalized Green's function  
 115 between the microphone located at  $\mathbf{r}_m$  and the scan point at  $\mathbf{r}_l$ . In order to  
 116 reduce the self-noise of the microphones, the diagonal elements of the CSM are  
 117 set to 0.

118 There are several formulations of the steering vector in the literature [14],  
 119 here one element of the steering-vector is

$$h_l(\mathbf{r}_m, f) = \frac{g_l(\mathbf{r}_m, f)}{\sqrt{M} \sqrt{\mathbf{g}_l(f)^H \mathbf{g}_l(f)}}, \quad (5)$$

120 which is referred as formulation IV in reference [14].

121 Note that the vector  $\mathbf{g}_l(f)$  gathers elements from matrix  $g_l(\mathbf{r}_m, f)$  that cor-  
 122 respond to each microphone location  $\mathbf{r}_m$ . The vector  $g_l(\mathbf{r}_m, f)$  is based on the  
 123 free-field Green's function, thus frequency-domain beamforming makes the as-  
 124 sumption of monopolar propagation,

$$g_l(\mathbf{r}_m, f) = \frac{r_{0l}}{r_{ml}} \exp(-jk(r_{ml} - r_{0l})), \quad (6)$$

125 where  $r_{0l}$  is the same as  $r_{ml}$  but using a reference microphone, usually the center  
 126 microphone of the array and  $k = 2\pi f/c_0$  represents the wavenumber. In the

127 following, the acoustic images provided by the frequency domain beamforming  
 128 are referred as CBF.

### 129 2.3. Time-domain beamforming: Generalized Cross-Correlation (GCC)

130 When the speed of sound is known, the ToF between a microphone and a scan  
 131 point can be computed. Each microphone signal is delayed by the corresponding  
 132 ToF, which corresponds to steer the microphone array into the direction of the  
 133 scan point. When the sum of the delayed microphone signals is maximized the  
 134 source is localized. This technique is known as delay-and-sum beamformer and  
 135 its output  $y_l(t)$  can be expressed for the scan point  $l$  as

$$y_l(t) = \sum_{m=1}^M x_m(t + \Delta t_{ml}), \quad (7)$$

136 where  $\Delta t_{ml}$  is the Tof between microphone  $m$  and scan point  $l$ .

137 The beamformer output power  $Y_l(t)$  is given by

$$Y_l(t) = E\{y_l^2(t)\} = \sum_{m=1}^M \sum_{n=1}^M R_{x_m, x_n}(\Delta t_{ml} - \Delta t_{nl}), \quad (8)$$

138 where  $E\{\cdot\}$  is the mathematical expectation and  $R_{x_m, x_n}$  the cross-correlation  
 139 function between the microphone signals  $x_m$  and  $x_n$  defined by

$$R_{x_m, x_n}(\tau) = E\{x_m(t)x_n(t + \tau)\}, \quad (9)$$

140 where  $\tau$  is a time lag.

141 The cross-correlation function  $R_{x_m, x_n}$  is typically recovered from the iFFT  
 142 of the cross spectrum  $C_{x_m x_n}$  (Eq. 3) between microphone signals

$$R_{x_m, x_n}(\tau) = \sum_{f=0}^{N_f-1} W(f) C_{x_m x_n}(f) \exp\left(j2\pi \frac{f}{N_f} \tau\right), \quad (10)$$

143 where  $f$  is the frequency bin,  $N_f$  the number of frequency bins and  $j = \sqrt{-1}$ .  
 144 A weighting function  $W(f)$  can be used for improving the cross-correlation  
 145 estimation which leads to the GCC [38, 47].

146 As the scan zone has a finite size, a peak time delay  $\tau_{max}$  can be defined  
 147 from all scan points which allows for selecting the values of the cross-correlation  
 148 between  $[-\tau_{max} : \tau_{max}]$ . The peak time delay is given by

$$\tau_{max} = \max \left( \frac{1}{c_0} |\mathbf{r}_{ml} - \mathbf{r}_{nl}| \right), \quad (11)$$

149 with  $|\cdot|$  the absolute value.

150 The projection of the cross-correlation function for a single microphone pair  
 151 over the scan zone is called Spatial Likelihood Function [48] (SLF)

$$SLF = R_{x_m, x_n}(-\tau_{max} : \tau_{max}). \quad (12)$$

152 Commonly, the computation of the beamformer output power Eq. 8 is per-  
 153 formed over the  $M_p = M(M - 1)/2$  microphone pairs due to the symmetry of  
 154 the cross-correlation matrix (redundant information) [49]. Moreover, the auto  
 155 correlation terms (auto correlation of microphone signals) are removed because  
 156 they do not bring any information to the acoustic image. These terms are known  
 157 as "DC" component or self-noise [49]. Finally, the acoustic image is provided by  
 158 the arithmetic mean of the SLF. In the following, the acoustic images provided  
 159 by the time domain beamforming are referred as the GCC.

#### 160 2.4. Detailed computation of CBF and GCC

161 For both acoustic imaging techniques, the first step is to gather the micro-  
 162 phone signals into a matrix. Then, the scan zone is defined. With the CBF, the  
 163 cross spectral matrix and the steering vector are computed. Then, the acoustic  
 164 images are given by Eq. 4 or are averaged over the frequency band if required.

165 With the GCC, the microphone signals are filtered in the frequency band re-  
166 quired. The SLF are computed for each microphone pair and averaged in order  
167 to get the acoustic image. In the following, both techniques (CBF and GCC)  
168 are compared with numerical and experimental data. The Figure 1 provides the  
169 detailed computation of both techniques.

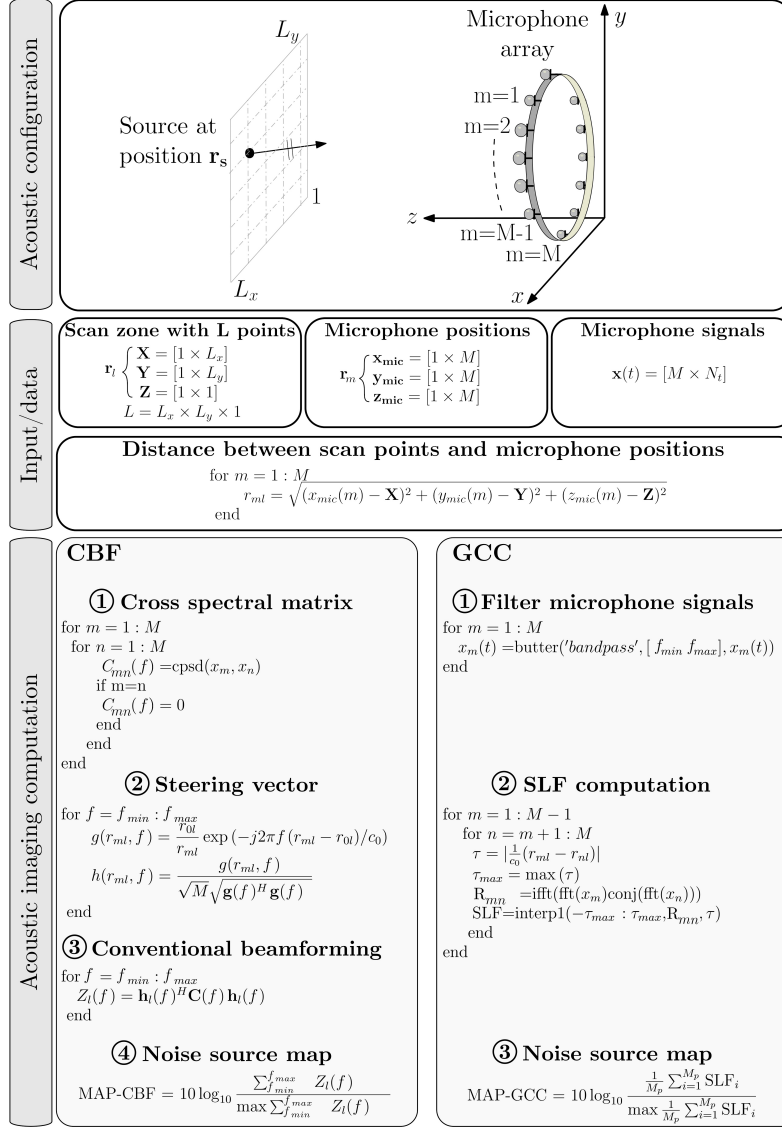


Figure 1: Detailed computation of the CBF and GCC.

## 2.5. Criteria for assessing an acoustic image

The result provided by both techniques is an acoustic image where a main lobe should be located at the source position. Unfortunately, in addition to

173 the main lobe, the acoustic image also contains several side lobes (Figure 2.a).  
174 Moreover, the size of the main lobe and the amplitude of the side lobes are  
175 directly linked to the microphone array geometry, the source-array distance and  
176 the source characteristics [16, 17].

177 In the past, criteria have been developed to assess the acoustic image. The  
178 first criterion is the Main Lobe Width (MLW) at -3 dB which assesses the  
179 ability to separate two sources [16]. The MLW can be expressed in centimeters  
180 or degrees (Figure 2.b). In the case of a 2D acoustic image, this criterion is  
181 obtained by slicing the map along the main directions. If the distance between  
182 two sources is smaller than the MLW, the main lobes will be merged and the  
183 two sources can not be separated. Therefore, the smaller the MLW, the better  
184 the source localization. This criterion is also known as spatial resolution.

185 The second criterion is the Maximum Side lobe Level (MSL) which is given  
186 by the amplitude difference between the main lobe and the nearest side lobe.  
187 The MSL is expressed in decibel (dB). Again, in the case of a 2D acoustic image,  
188 this criterion is obtained by slicing the map along the main directions. If the  
189 side lobes amplitude is high, they could be interpreted as sources with lower  
190 amplitude. Therefore, the higher the MSL, the better the source localization.  
191 This criterion is also known as dynamic resolution.

192 An example of 2D acoustic image (for an arbitrary configuration with a single  
193 source) is given in Figure 2.a where the main lobe indicates the source position.  
194 This main lobe is surrounded by a side lobe. Secondary side lobes are also  
195 present. To make the difference with the main side lobe, these secondary side  
196 lobes are called spurious lobes in the following. To assess the acoustic image,  
197 slices along the lines  $x = 0$ ,  $y = 0$  and  $x = y$  are shown in Figure 2.b. In this  
198 case, the MLW and MSL are independent of the slicing directions. However,  
199 both criteria do not take into account the spurious lobe which have a higher

200 amplitude and are different for each direction. Therefore, although both criteria  
 201 provide a useful information, they do not completely characterize the acoustic  
 202 image especially the 2D aspect.

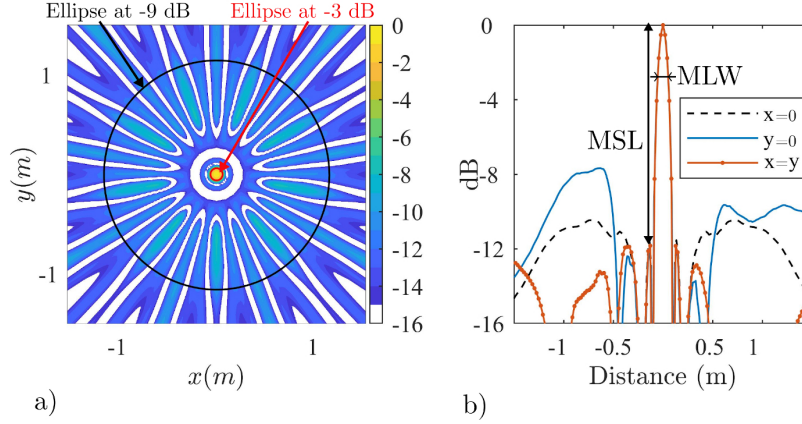


Figure 2: a) Example of acoustic image with ellipses at -3 dB (red) and -9 dB (black). b) Slices of the acoustic image along the lines  $x = 0$ ,  $y = 0$  and  $x = y$ . (color online)

203 To fully characterize the acoustic image, a criterion based on the surface of  
 204 an ellipse is proposed. The goal is to surround the acoustic image values higher  
 205 than a threshold. For instance, if the goal is to determine the spatial resolution,  
 206 the threshold is set to  $-3$  dB and the ellipse which surrounds all the higher values  
 207 is searched. The main advantage is that the 2D aspect is taken into account.  
 208 Again, the smaller the ellipse's surface, the better the source localization. If  
 209 the threshold value is decreased, for instance up to  $-9$  dB, the ellipse's surface  
 210 will increase because the side lobes will be taken into account too. Finally, the  
 211 ellipse's surface is also divided by the microphone array surface which provides  
 212 a dimensionless criterion, called Ellipse Array Ratio (EAR). Low value of this  
 213 criterion means a small surface ellipse and is expected for an efficient source  
 214 localization. The EAR criterion has the advantage to provide an unique value  
 215 to characterize either the main lobe or the side lobes no matter which direction



216 is considered.

217 Many methods exist to define an ellipse surrounding a set of data. In this  
218 work the confidence ellipse is considered: first a threshold value has to be de-  
219 fined, then the acoustic image values below this threshold are discarded. The  
220 covariance matrix of this new data set is computed and the eigenvalues and  
221 vectors are searched. The minor and major axes of the ellipse are given by the  
222 eigenvalues and the orientation by the eigenvectors [50]. The 90% confidence  
223 ellipses are shown in Figure 2.a with thresholds equal to  $-3$  dB and  $-9$  dB. The  
224 value of 90% has been chosen to avoid influence of outliers. The first threshold  
225 allows for surrounding the main lobe only while the second takes also into ac-  
226 count the side lobes. This value has been chosen because beamforming is not  
227 able to localize two sources with an amplitude difference larger than 9 dB (it is  
228 typically the side lobe level). Finally, the EAR criterion is given by

$$EAR = \frac{S_{ellipse}}{S_{array}} \quad (13)$$

229 where  $S_{ellipse}$  and  $S_{array}$  are the ellipse and microphone array surfaces, respec-  
230 tively.

### 231 3. Numerical comparison of the CBF and GCC

#### 232 3.1. Single source

233 The EAR criterion is now used to compare both techniques the CBF and  
234 GCC in the case of a single source in front of a 16-microphones circular array.  
235 The microphone array radius and the source-array distance range from 0.4 m  
236 to 1.4 m and from 0.2 m to 1.6 m respectively. The source signal is a white  
237 noise and the microphone signals are filtered in the 1000 Hz octave band. The  
238 frequency sampling is 44,100 Hz and 1 s of source signal is used. The scan zone  
239 is a square with 3 m sides and 40,401 points ( $201 \times 201$  grid). The EAR criterion

240 values at -3 dB and -9 dB obtained with the CBF and GCC are shown in Figure 3  
 241 with respect to source-array distance and array radius. Both techniques provide  
 242 similar trends. For instance, if the array radius is small, increasing the source-  
 243 array distance increases the EAR criterion. Both techniques yield similar EAR  
 244 criterion values which means that both techniques provide similar main lobe  
 245 and side lobe levels.

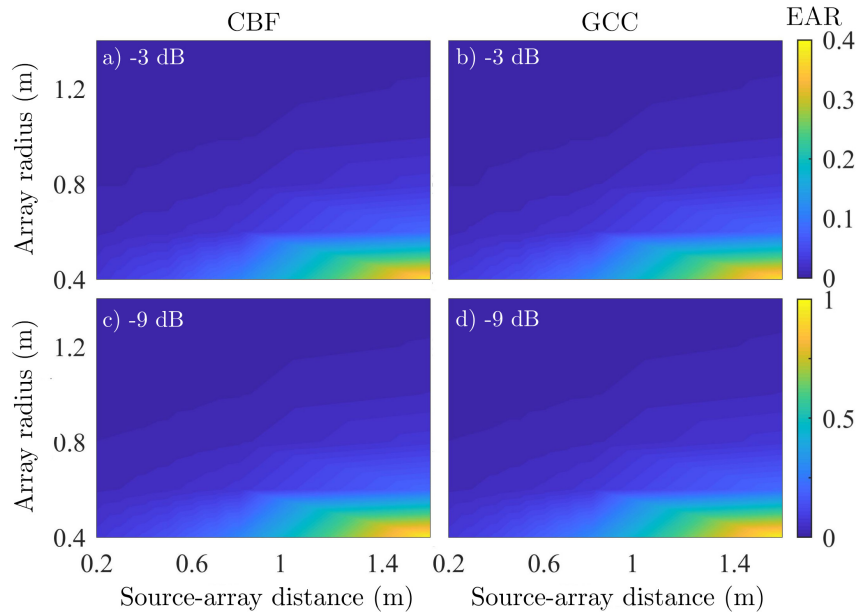


Figure 3: EAR criterion at -3 dB (a-b) and -9 dB (c-d) obtained with the CBF (left) and GCC (right). A 16-microphones circular array is used. The source signal is a white noise and the microphone signals are filtered in the 1000 Hz octave band. (color online)

246 As the EAR criterion is similar for both techniques at the 1000 Hz octave  
 247 band, the acoustic images for octave bands ranging from 250 Hz to 4000 Hz are  
 248 displayed in Figure 4 in order to visually assess the likely differences. Again, the  
 249 acoustic images are very similar for both techniques, which provide the same  
 250 EAR criterion for all the octave bands. The only difference is the spurious  
 251 lobes below -13 dB which appear due to spatial aliasing [16]. For example, the

252 GCC acoustic image obtained for the 2000 Hz octave band (Figure 4.i) exhibits  
 253 thinner spurious lobes than the CBF acoustic image (Figure 4.d). This first  
 254 investigation allows for demonstrating that CBF and GCC similarly perform in  
 255 terms of acoustic imaging in the case of a single source in front a microphone  
 256 array.

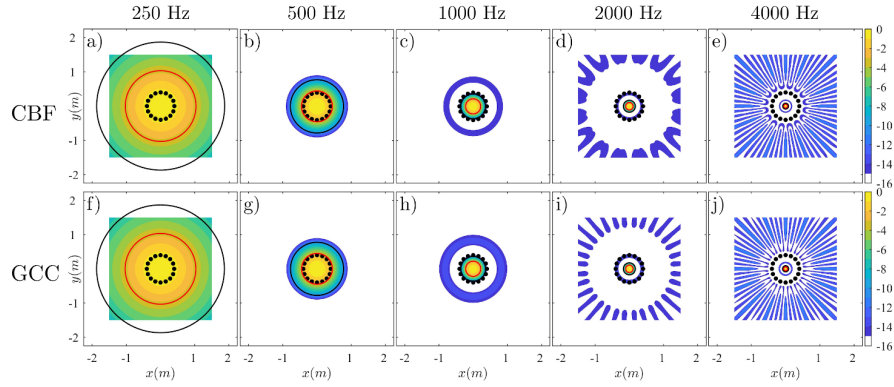


Figure 4: Acoustic images obtained with CBF (top) and GCC (bottom). The source signal is a white noise and the microphone signals are filtered in the 250 Hz (a-f), 500 Hz (b-g), 1000 Hz (c-h), 2000 Hz (d-i) and 4000 Hz (e-j) octave bands. A 16-microphones circular array is used. The red and black lines are the EAR criterion at -3 dB and -9 dB. The black dots denote the microphone positions. (color online)

### 257 3.2. Two sources with different amplitudes

258 In this section, the performance of both techniques is investigated in the case  
 259 of two sources with different amplitudes. A 16-microphones circular array with  
 260 0.4 m radius is used. The source array distance is 1.2 m. The source amplitude  
 261 difference is 6 dB and the source positions are  $x = -0.3$  m and  $x = 0.3$  m (with  
 262  $y = 0$ ). The signal duration is 1 s and the octave bands 1000 Hz and 2000 Hz  
 263 are considered. The acoustic images obtained and the slices along  $x$ -axis are  
 264 presented in Figure 5. The EAR criterion is not considered here because the  
 265 main interest is the amplitude difference. Both techniques correctly identify

266 the two sources no matter the frequency. The side lobes amplitude below -  
 267 12 dB are more important with the GCC but do not prevent an efficient source  
 268 localization. The slices along the  $x$ -axis, Figure 5.c-f show that both techniques  
 269 estimate similar source amplitude difference in both cases (octave bands 1000 Hz  
 270 and 2000 Hz). In the octave band 1000 Hz, the source amplitude difference is  
 271 slightly overestimated by both techniques, probably due to the presence of the  
 272 side lobes. In the octave band 2000 Hz, the source amplitude difference is of  
 273 6.2 dB for both techniques which is close to the initial value. Therefore, both  
 274 techniques provide similar source amplitude difference estimation.

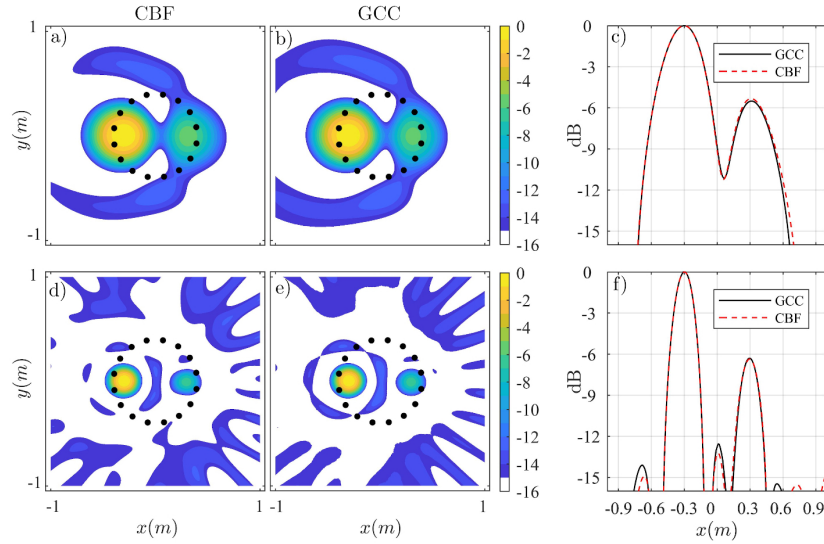


Figure 5: Acoustic images obtained with CBF (left) and GCC (center) for microphone signals  
 filtered in the 1000 Hz (a-b) and 2000 Hz (d-e) octave bands. c-f) Slices along the line  $x = 0$ .  
 A 16-microphones circular array is used. The black dots denote the microphone positions.  
 (color online)

### 275 3.3. Influence of the signal-to-noise ratio

276 In this section, the performance of both techniques is investigated in the  
277 case of different Signal-to-Noise Ratio (SNR). A 16-microphones circular array  
278 with 0.4 m radius is used. The source array distance is 1.2 m. The signal  
279 duration is 1 s and the octave band 2000 Hz is considered. The SNR ranges from  
280 5 dB to -15 dB. The acoustic images obtained are presented in Figure 6. With  
281 SNR=5 dB, the GCC acoustic image does not seem to be affected by the additive  
282 noise while some spurious lobes appear around the main lobe with the CBF.  
283 When the SNR decreases to -5 dB, the CBF acoustic image is contaminated by  
284 the presence of random spurious lobes. On the other hand, the GCC acoustic  
285 image is less affected. Decreasing the SNR to -15 dB, increases the spurious  
286 lobe amplitude for both techniques. However, the spurious lobes amplitude is  
287 larger with the CBF with values reaching -6 dB (as shown by the EAR criterion  
288 at -9 dB).

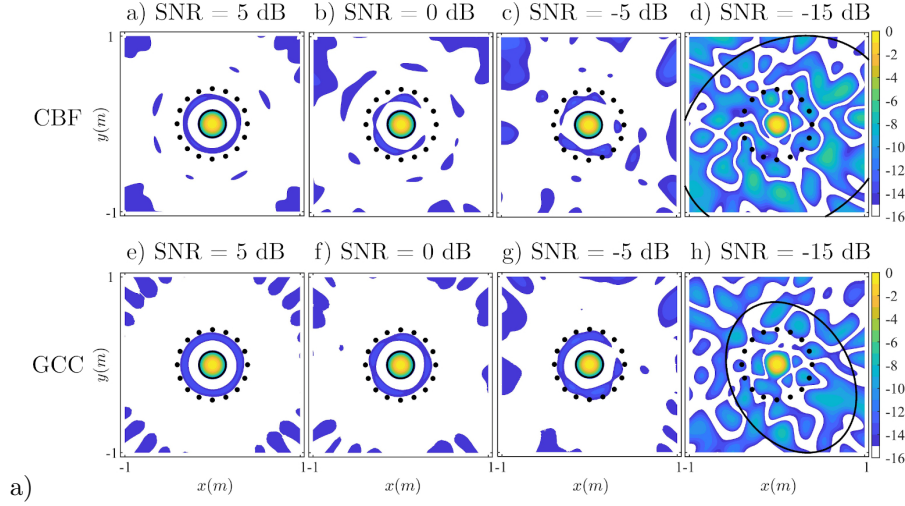


Figure 6: Acoustic images obtained with CBF (top) and GCC (bottom). The SNR are 5 dB (a-e), 0 dB (b-f), -5 dB (c-g) and -15 dB(d-h). The source signal is a white noise and the microphone signals are filtered in the 2000 Hz octave band. A 16-microphones circular array is used. The black lines are the EAR criterion at -9 dB. The black dots denote the microphone positions. (color online)

### 3.4. Computation time

In this section, the computation time of both techniques is compared. The algorithms are coded with Matlab using classical functions, no optimization toolbox such as parallel computing are used. The codes, partially described in Figure 1, were run onto a personal laptop (Intel i7-7600 at 2.8 GHz, 16 Go Ram).

First the influence of the number of scan points on the computation time is investigated. The array is circular with 16 microphones and only one frequency bin is used to compute the CBF (in order to avoid frequency averaging). The number of scan points ranges from 10 to  $10^6$  and the computation time is displayed in Figure 7.a. In between 10 and  $10^4$ , the GCC is four times faster than the CBF. When the number of scan point is equal to  $4 \cdot 10^5$ , the computation

time is similar for both techniques. Above  $4.10^5$  points, the CBF becomes faster than GCC; however the time difference is only 1 sec at  $10^6$  points.

Now, the number of scan points is set to  $4.10^4$  ( $200 \times 200$  grid size) and the number of microphones ranges from 4 to 128. The computation time is shown in Figure 7.b. The computation times are similar when the number of microphones is equal to 10 but become largely different when the number of microphones increases. For instance, with 32 microphones the CBF is four times longer than the GCC. This fact can be easily explained by the computation process of both techniques. Indeed, the GCC requires only the microphone pairs ( $M_p = (M \times (M - 1))/2$ ), while the CBF requires the square of the number of microphones ( $M^2 > M_p$ ).

Finally, the computation time of both techniques is compared when octave bands are considered (from 250 Hz to 8000 Hz). In this case, the number of microphones and scan points are 16 and  $4.10^4$ , respectively. The frequency resolution (of the FFT) is an important parameter in this case because it determines the number of averages required in the CBF calculation. An arbitrary frequency resolution of 10 Hz is chosen and the result is displayed in Figure 7.c. As expected, the CBF computation time increases with the octave band because the number of frequency bins, required for the average, increases. The GCC computation time is the same for each octave band. Therefore, the GCC is much more efficient than the CBF when acoustic images have to be displayed for octave bands.

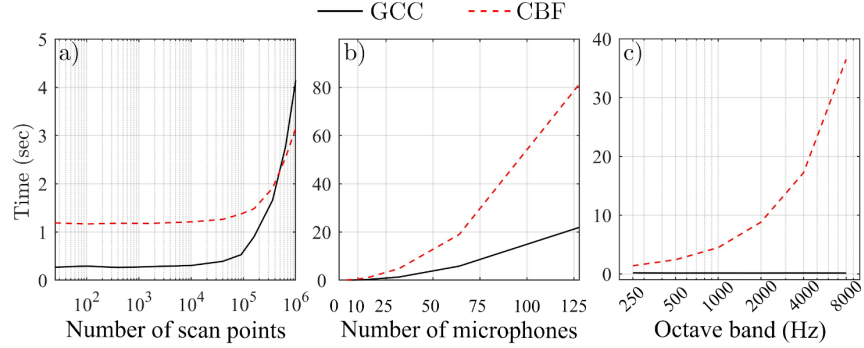


Figure 7: Computation times of the CBF (dashed red line) and the GCC (black line) versus a) the number of scan points, b) the number of microphones and c) the octave bands considered.

#### 4. Experimental tests

The acoustic images provided by both techniques are now compared with experimental data obtained in a hemi-anechoic chamber. Various microphone array geometries and sources are considered. The aim is not to characterize the source behavior but to assess the performance of each technique for acoustic imaging. The colorbar is set to -10 dB to focus on acoustic imaging performance.

##### 4.1. Two loudspeakers in front of a circular microphone array

During the first experiment, two loudspeakers (Eris E5 PreSonus), spaced by 50 cm were set in front of a 16-microphones circular array located at 1.2 m away. The loudspeakers signals were uncorrelated white noises with the same level and were generated with a PXI-4461 Sound and Vibration Module. The loudspeaker crossover frequency, which separates the signal of the tweeter (top) and the woofer (bottom), was of 3000 Hz. The acoustic signals were recorded with Brüel&Kjaer microphones type 4935 and 12-Ch input module type 3038B and sampled at 65,536 Hz. The microphone array aperture was 86 cm in both directions (see black dots in Figure 8.a). The scan zone, where the source



positions were sought, is a square with side equal to 1 m and containing 160,801  
points ( $401 \times 401$ ). The octave band 1000 Hz and 4000 Hz were considered.

The acoustic images obtained with the CBF and GCC are shown in Figure 8  
for both octave bands investigated. Overall, the acoustic images are similar and  
detect the woofer when the 1000 Hz octave band is considered or the tweeter  
when the 4000 Hz octave band is considered. The main lobe width and the  
side lobes level are also equivalent. In this case, both techniques yield similar  
results.

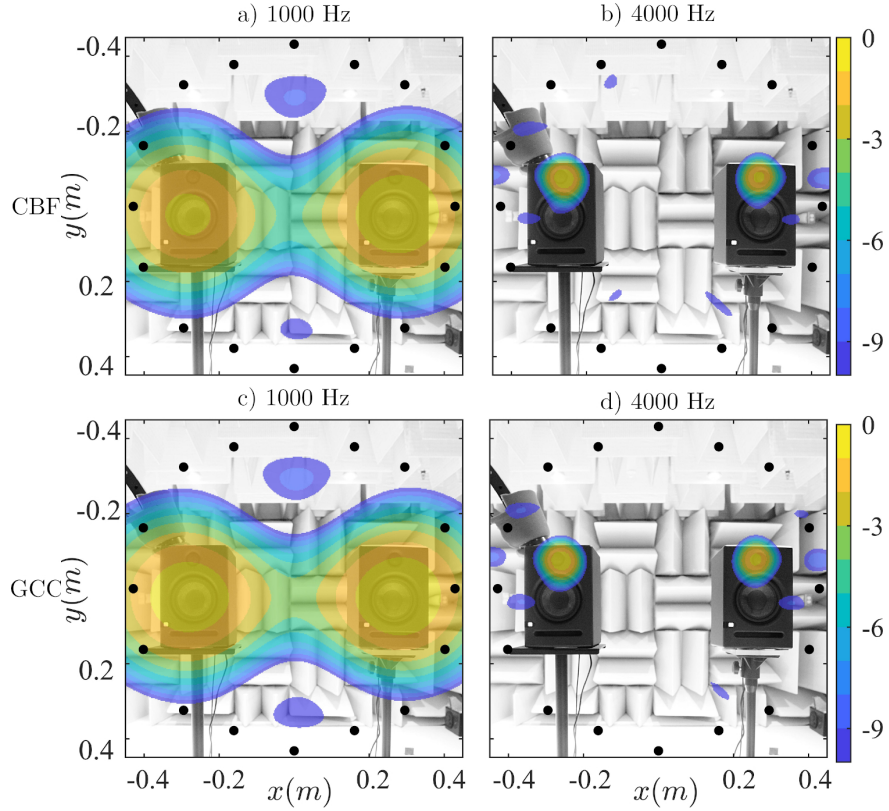


Figure 8: Acoustic images obtained with the CBF (top) and GCC (bottom). The source signals were white noises and the microphone signals were filtered in the 1000 Hz (a-c) and 4000 Hz (b-d) octave bands. A 16-microphones circular array in front of two loudspeaker with the same level was used. The black dots denote the microphone positions. (color online)

#### 347 4.2. Leaf blower in front of a spiral-arm microphone array

348 The source considered now is a leaf blower which generates a stationary noise  
349 with a more complex frequency content. This hand-tool generates a tonal peak  
350 in mid frequencies (at 2631 Hz) and a broadband bump at higher frequencies  
351 (between 3400 and 4000 Hz). The array geometry is multi-arms logarithmic spiral  
352 shape with 41 microphones. First, the position of the source which generates  
353 the tonal peak at 2631 Hz is sought (9.a-b). Both techniques indicate that the  
354 position of this source is the air exhaust, the acoustic images are similar.

355 Then, the position of the source which generates the broadband bump is  
356 sought (9.c-d). Again, the acoustic images are similar and point to the engine  
357 as the main source with a secondary source at the air exhaust. Therefore in the  
358 case of a real source, both techniques provide similar results even if the noise  
359 radiated is tonal or broadband.

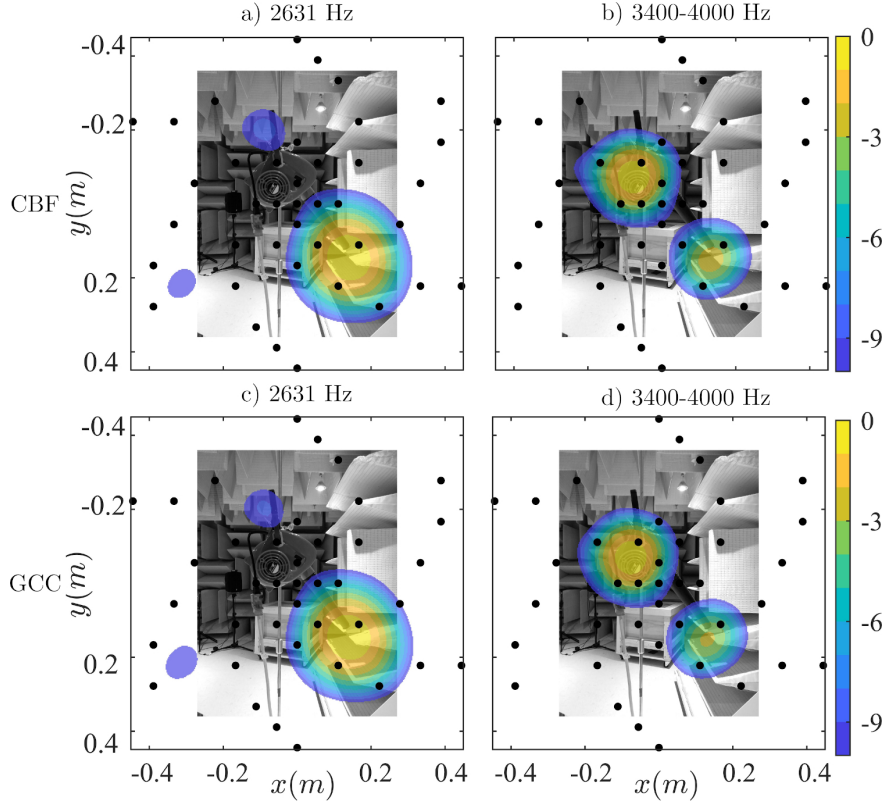


Figure 9: Acoustic images obtained with the CBF (top) and GCC (bottom). The microphone signals were filtered at 2631 Hz (a-c) and between 3400 Hz and 4000 Hz (b-d). A 41-microphones multi-arms logarithmic spiral array in front of a leaf blower was used. The black dots denote the microphone positions. (color online)

#### 4.3. Nail gun in front of a spiral-arm microphone array

The source is now a pneumatic nail gun which typically generates impulsive noise. When the trigger nail gun is pulled up, the pressurized air is rushing in the chamber which moves down the piston and impacts the nail. Then, the piston goes up, the air is released by the exhaust at the top of the nail gun, and hits the high position. The noise generated by the exhaust is one of the main noise source of this nail gun. The noise frequency content being contained

in the high frequency, the frequency band considered ranges from 2000 Hz to 5000 Hz [51]. The abilities of both techniques to localize the noise coming from the exhaust are compared in Figure 10. Both techniques provide similar results with a main lobe at the exhaust position.

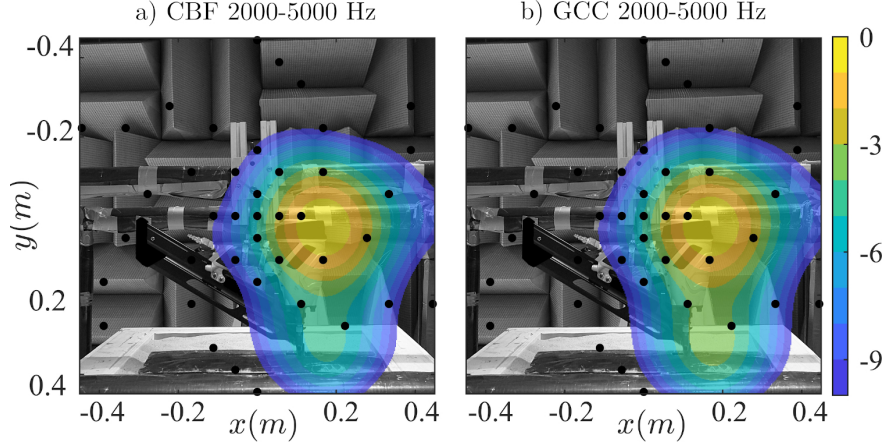


Figure 10: acoustic images obtained with the CBF (left) and GCC (right). The microphone signals were filtered between 2000 Hz and 5000 Hz. A 41-microphones spiral-arm array in front of a nail gun was considered. The black dots denote the microphone positions. (color online)

#### 4.4. CBF and GCC with advanced techniques

It has been shown previously that CBF and GCC have a large main lobe for low frequency content or strong side lobes for higher frequencies (Figure 8). Many techniques have been developed for improving the acoustic image.

With the CBF, deconvolution methods have been proposed [24, 25, 26]. Although these methods improve the acoustic image by narrowing the main lobe width or removing side lobe, the computation time and complexity are usually greatly increased. A full comparison of all the deconvolution techniques with the GCC is not possible here because the number of proposed methods is too large. The deconvolution method selected in this study is CLEAN-SC [35]

381 which is well known for its efficiency and short computation time.

382 This deconvolution technique is compared with the GCC based on the ge-  
383 ometric mean with the weighting function  $\rho$ -PHAT-C. Although, other GCC  
384 improvements exist [52, 53], the latter has been chosen due to its performance  
385 with acoustic imaging [41]. Both improvements of the GCC are almost straight-  
386 forward. Indeed, the geometric mean only modifies the average process of Eq. 10  
387 by a product and the weighting function only divides the cross spectrum of the  
388 microphone signal by its absolute value at the power  $\rho$  and coherence of the mi-  
389 crophone signals. These improvements do not greatly increase the computation  
390 time and complexity and are not based on iterative process. This technique is  
391 called GEO- $\rho$ -PHAT-C.

392 Both techniques are compared in the case of two loudspeakers in front of a  
393 circular microphone array (Figure 11). First, the frequency band considered is  
394 the 500 Hz octave band. In this case, the loudspeakers are too close and the  
395 CBF and GCC are not able to separate them (Figure 11.a-c). The result is a  
396 main lobe in between the loudspeaker created by the merging of the main lobes.  
397 Although the frequency content is low, the GEO- $\rho = 1, 3$ -PHAT-C is able to  
398 separate both loudspeakers and to identify the woofers as the main sources.  
399 The CLEAN-SC technique being based on the peak value of the CBF exhibits  
400 a source in between the loudspeakers. This technique is not able to separate  
401 both sources.

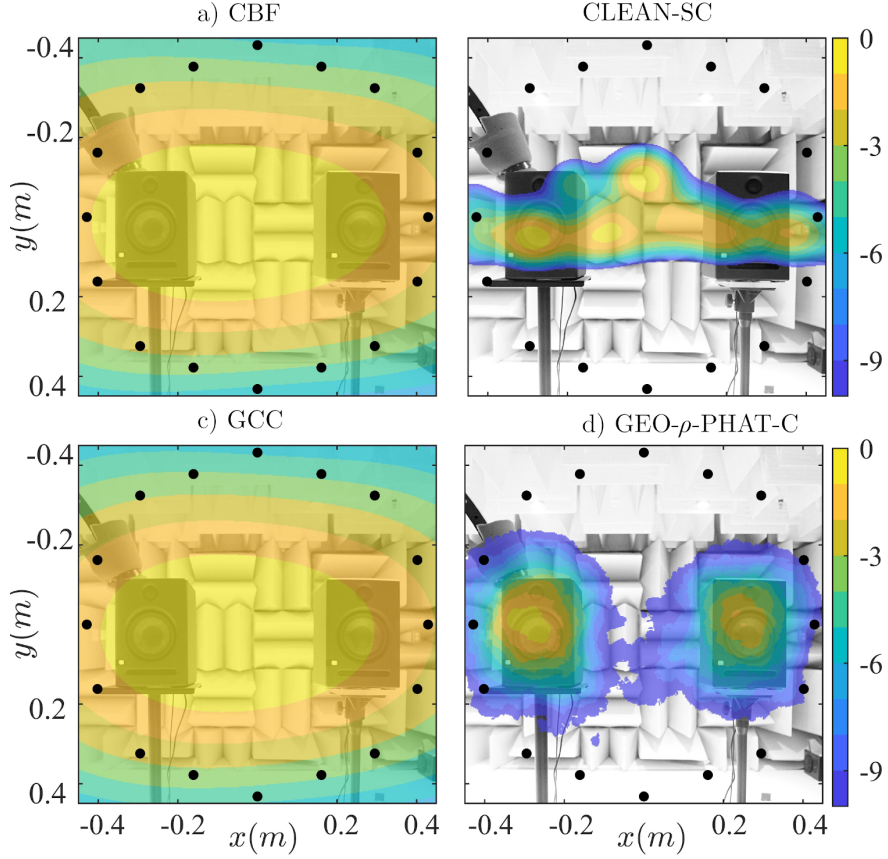


Figure 11: Acoustic images obtained a) CBF, b) CLEAN-SC, c) GCC and d) GEO- $\rho$ -PHAT-C. A 16-microphones circular array in front of two loudspeaker was used. The microphone signals were filtered in the 500 Hz octave band. The black dots denote the microphone positions. (color online)

Then, the third octave band 3150 Hz is investigated. This frequency band is slightly larger than the cross over frequency between the woofer and tweeter. The CBF and GCC provide similar acoustic images with main lobes a little bit below the tweeter positions and strong side lobes (Figure 12). The GEO- $\rho = 1,3$ -PHAT-C perfectly detects both tweeters and removes all side lobes. The CLEAN-SC also removes the side lobes but identify the main sources such

408 as the CBF i.e. a little bit below the tweeter positions.

409 To conclude, the GEO- $\rho$ -PHAT-C provides similar results than CLEAN-SC  
 410 but without using iterative process and without increasing the computation time  
 411 and complexity.

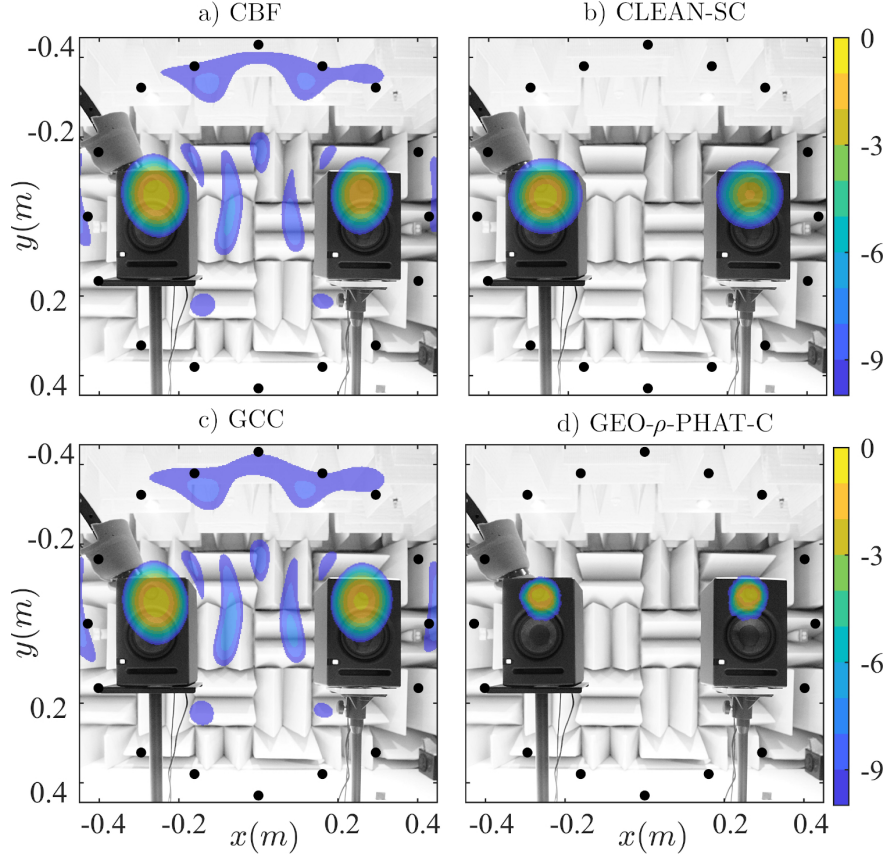


Figure 12: Acoustic images obtained a) CBF, b) CLEAN-SC, c) GCC and d) GEO- $\rho$ -PHAT-C. A 16-microphones circular array in front of two loudspeaker was used. The microphone signals were filtered in the 3150 Hz third octave band. The black dots are the microphone positions. (color online)

## 412 5. Conclusion

413 Microphone arrays have become a standard tool to perform source localiza-  
414 tion or acoustic imaging. The post-processing of the microphone signals can  
415 be done in the time or frequency domain. In acoustic imaging, the standard  
416 technique is the conventional beamforming (CBF) based on the cross spectral  
417 matrix (frequency domain). However, acoustic images can be also obtained with  
418 the time domain technique such as the generalized cross-correlation (GCC). The  
419 objective of this work was to compare both techniques.

420 First, the computation of the CBF and GCC has been presented in details  
421 in the first section. Then, a criterion based on a covariance ellipse, denoted  
422 EAR, has been introduced to compare the acoustic images obtained numeri-  
423 cally with each technique. This criterion allows for characterizing either the  
424 main lobe surface or side lobe influence. The CBF and GCC provided similar  
425 EAR criterion in the case of a single source. The abilities of both techniques  
426 for estimating a source amplitude difference or for localizing a single source in  
427 presence of an additive noise have also shown similar results. The main differ-  
428 ence was the computation time which was faster with the GCC than the CBF  
429 (at the exception of very large grids).

430 Then, experimental data were used in order to compare both techniques, the  
431 sources were two loudspeakers, a leaf blower (stationary tonal and broadband  
432 noise) or a nail gun (impulsive noise). Again, the acoustic images provided by  
433 both techniques were similar. The main difference was when improved tech-  
434 niques were used. The deconvolution technique CLEAN-SC (usually used with  
435 the CBF) was compared with the GCC with the geometric mean associated to  
436 an improved weighting function. Both techniques were selected for their fast  
437 computation time. In the case of low frequency sources, the improved GCC  
438 outperformed the CBF, GCC and CLEAN-SC and was able to separate them.



439 For higher frequency source, the improved GCC enhanced the noise source map  
440 (as compared to GCC and CBF) in similar way than CLEAN-SC.

441 To conclude, the CBF and GCC provide similar performance, the main  
442 differences lies in the computation time and the use of improved techniques.  
443 A next step of this work would be a larger comparison of time and frequency  
444 domain techniques based on sparse representation.

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