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Integrated product specifications and productivity decision making in unreliable manufacturing systems

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Abstract

This paper considers joint production control and product specifications decision making in a failure prone manufacturing system. This is with the knowledge that tight process specifications, while leading to a product of more reliable quality and higher market value, are at the same time associated with higher levels of non conforming parts, a higher rate of parts rejection and thus a lowering of overall plant productivity. The decision making is further complicated by the lack of reliability of the production process which imposes that an adequate, also to be designed, level of inventory of finished parts be maintained. The overall optimal decision policy is defined here as one that maximizes the long term average per unit time profit of a combined measure of quality and quantity dependent sales revenue, minus inventory and backlog costs, in the presence of random plant failures and random repair durations. Policy optimization is achieved via a revisited model of the Bielecki-Kumar theory for Markovian machines and a simulation and experimental design based methodology for the more general cases.

Keywords: Product specifications, Production control, Simulation, Experimental design.

1. INTRODUCTION

Although the decisions taken in response to productivity and profit making requirements have a direct impact on the quality of products, production management and quality management have traditionally been treated as independent areas of research. Indeed, when seeking to improve quality control mechanisms (positioning verification stations, frequency of sampling, etc.), current models, consider this issue separately from that of developing optimized production strategies (KANBAN, CONWIP or others). As a result, the ways in which quality and production control strategies interact, remain relatively unexplored especially in a dynamic stochastic environment.

In the literature, this avenue of research has attracted recent interest. Kim and Gershwin [3] have developed continuous quality and quantity models punctuated by random discrete jumps in the states of quality and breakdown of the system. Based on the assumption that a machine will continue to produce defective parts until its operation is corrected, the main objective of their work is to study the interaction of quality and productivity. More recently, Kim and Gershwin [4] have extended this model and proposed approximation methods for the analysis of production line performance. Although these models represent a pioneering contribution, we observe that possible modeling difficulties can occur in that parts production is modeled as fluid, yet quality remains attached to discrete parts. In this context, we conducted preliminary work to develop a model in which both production and quality are treated as continuous variables (Mhada et al. [2]). In [2], the main objective is to extend the Bielecki Kumar theory [1], where the Markovian, failure prone machine considered produces always good quality items, to a case where it can produce a mixture of good and defective items. In order to extend the theory to more complex situations where the machine is facing non exponential failure and repair time distributions, a combined simulation and experimental design based approach is also proposed.

In the same direction, Colledani and Tolio ([5], [6], [7]) have proposed a discrete time Markovian model to study the impact of quality control on the performance of production lines. In these works they considered the case where quality is controlled by statistical techniques (Statistical Process Control - SPC). This latter work led us, in the current paper, to consider the problem with a different perspective of the interdependence of production and finished products quality governed by the product specification limits after a 100% fully reliable inspection plan. Hence, an unreliable system producing a mixture of good and defective items where the design of the product specifications and the production control strategies involve an economic decision making process is considered. In the literature many works have addressed the issue of optimizing the design of product specifications in a quality decision making process. The reader is referred to Kapur and Cho [8], Phillips and Cho [9] and Jeh-Nan Pan and Jianbiao Pan [10] for more details on this issue. It is important also to note that several other researchers have studied the interaction between manufacturing activities (production & maintenance) and quality decision making, however only in a deterministic context. Integrated models were proposed and the benefits of such an approach were established. In particular, the reader is referred to Ben Daya and Rahim [11] and Ben Daya [12]. In the context of a failure prone manufacturing system, our objective here is to find an "optimal" combined product specification limits, and production control strategy within a (parameterized) class of policies combining the hedging inventory level parameter of the Bielecki-Kumar theory [1], and the engineering *Lower* and *Upper* specification limits of the product. Optimality is defined here as a long term average per unit time measure of quality and quantity dependent revenue from sales, minus inventory and backlog costs. A comparative study is carried out to assess the benefits of the proposed integrated decision making process, versus a more traditional disjoint product design specifications and production control strategy design process.

Following the lead of Gharbi and Kenne [13] in the combined use of control theoretic analysis for candidate decision rules parameterization and simulation-based experimental design approaches for parameter optimization of production in manufacturing systems, we apply a similar hybrid methodology for the study of the joint product specification and production control for non exponentially distributed machine failure and repair times. As a first step, a running net profit function is constructed relating current per unit time sales minus current finished parts stock/backlog per unit time costs to the parameters of the selected production and product design strategies acting as control factors. For each configuration of input factor values, a simulation model is used to determine the corresponding output, more specifically, the long term average system profit. An input-output data set is then generated through the simulation model. The experimental design is used to determine significant factors and/or their interactions, and the response surface methodology is applied to the input-output data obtained in order to estimate first, an approximation of the objective function and secondly, the related optimum. Details on the combination of analytical approaches and simulation-based statistical methods can be found in Gharbi and Kenne [13] and in the references they provide.

The reminder of the paper is organized as follows. Section 2 presents the statement of the problem. Section 3 presents the case study considered and an analytical solution for the basic cases governed by exponential distributions. The simulation-based experimental design methodology proposed for the more general cases is developed in Section 4. We present associated numerical results in Section 5. A comparative study between the integrated (production and product design) versus dissociated (production or product design) control strategies is detailed in Section 6. Section 7 contains concluding remarks.

2. PROBLEM STATEMENT

The manufacturing system under study consists of an unreliable machine producing one part type P. We consider a fluid model of parts production with a fraction (1-β) of the total parts produced considered as conforming, and a fraction β considered non conforming (the value of β will be discussed below). As shown in Figure 1, finished parts first accumulate in an inspection buffer where storage costs accrue. Furthermore, it is assumed that parts are inspected, according to a 100% inspection plan, at a rate consistent with an extraction rate d of good parts, and that for each single conforming part detected, a fraction β of non conforming parts is also detected and directed towards a non conforming parts buffer for either rework or elimination. A storage cost of zero is assumed for non conforming parts *once they are detected*. In general, non conforming parts are associated with two types of cost: a direct cost related to the cost of temporary storage in the inspection buffer; an indirect cost paid for in terms of *reduced productivity* of the manufacturing system.

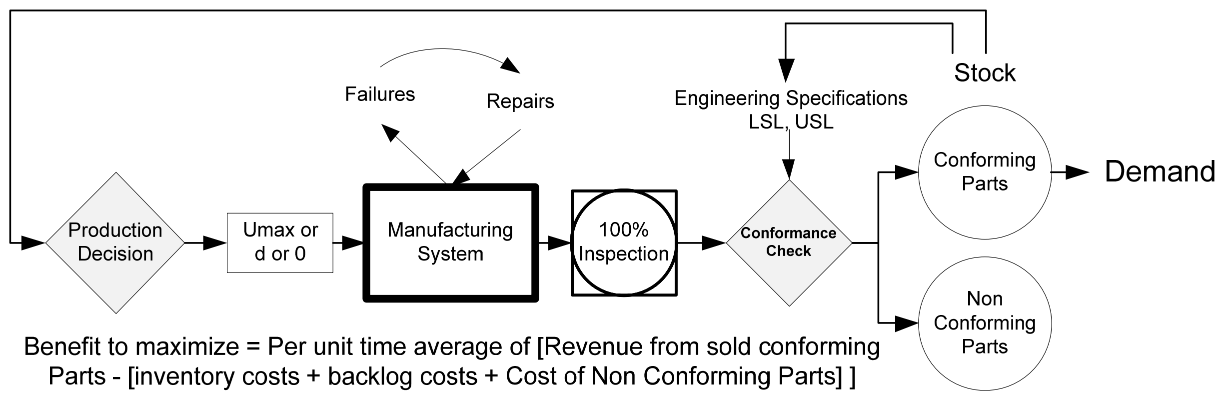

Figure 1: Manufacturing system under study

The state of the system at time *t* has three components:

- A two component continuous part, which describes the cumulative surplus of conforming and non-conforming items as measured by $x_1(t)$ and $x_2(t)$ respectively, with $x(t) \equiv x_1(t) + x_2(t)$.
- A discrete part, which describes the operational mode of the machine at time *t*.

It is described by the random variable $\xi(t)$ with value in $M = \{1,0\}$, where:

$$\xi(t) = \begin{cases} 1 & \text{the machine is available (operational)} \\ 0 & \text{the machine is unavailable (under repair).} \end{cases}$$

At this stage of the analysis, $\xi(t)$ is assumed to evolve according to a Markov chain with transition rates matrix (this assumption will be later relaxed when we consider experimental design methods for non Markovian machines):

$$\mathrm{T} = \begin{bmatrix} -q_{01} & q_{01} \\ q_{10} & -q_{10} \end{bmatrix}$$

At any given time, the production rate $u(t)$ of the machine has to satisfy its capacity constraint. This constraint states that the machine cannot be utilized for more than 100% of its capacity and can be expressed as: $0 \le u(t) \le U_{Max} \times \xi(t)$, $\xi(t) \in M$, where U_{Max} is the maximum production rate.

As mentioned previously, the production system produces conforming and non conforming items. Items are classified as non-conforming according to the severity of engineering specifications [14]; let *LSL* and *USL* denote the engineering *Lower* and *Upper* specification limits respectively. These limits can be defined in terms of the engineering specified mean μ and the *half range* δ as: $LSL = \mu - \delta$; $USL = \mu + \delta$. Several indicators can measure the capability of the process as to whether it meets adequately the specifications limits. The C_{pk} indicator can be defined as follows [14]:

$$C_{pk} = \frac{Min\left\{\left(USL - \overline{X}\right);\left(\overline{X} - LSL\right)\right\}}{3\,\sigma}$$

where $\overline{X}$ is the population mean and σ is the population standard deviation.

The following assumptions are considered in this paper:

1- The process is assumed to be centered and under control
2- During the engineering phase and due to market constraints, the design is governed by degrees of freedom in the decision on the product specification limits as expressed by: $LSL_{\min} \le LSL < \overline{X}$; $\overline{X} \le USL < USL_{\max}$.

To determine the fraction of non-conforming items for a centered process, one first calculates the gap between the specifications limits and the process mean $\overline{X}$ in standard deviation units as follows:

$$K_1 = \frac{LSL - \overline{X}}{\sigma}; K_2 = \frac{USL - \overline{X}}{\sigma}$$

As shown in Figure 2, using the standardized normal distribution, non-conforming items β can be measured as follows:

$$\beta = 1 - \Pr\ (K_1 < K < K_2) \tag{1}$$

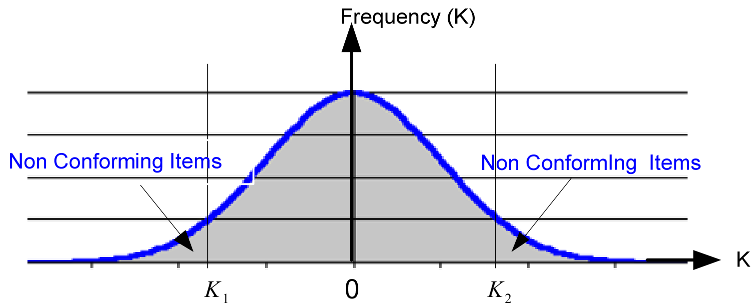

Figure 2: Rate of non - conforming items

The dynamics of the stock levels is given by the following differential equations.

$$
\begin{aligned}
&\dot{x}_1(t) = (1-\beta(LSL,USL))u(t) - d \\
&x_1(t) = (1-\beta(LSL,USL))x(t) \qquad for\; x_1(t) \geq 0 \\
&x_2(t) = \beta(LSL,USL)x(t) \qquad for\; x(t) \geq 0
\end{aligned}
\tag{2}
$$

Our decision variables are production rate $u(t)$ and the engineering specifications of the product LSL and USL which have a direct impact on the capability of the manufacturing system under study and the rate of non - conforming items. Recent work on the analysis of model (2) (Mhada et al. [2]), but not including any design of product specifications in a quality decision making process, has established that the optimal production strategy continues to belong to the class of constant threshold (or hedging) policies. We choose to restrict the production policy to such a class (parameterized as in (3) below, by a single storage parameter z), and look for the joint choice of storage parameter z and the product related specifications LSL and USL which maximizes the long term per unit time net profit defined in equation (4) below. The structure of the hedging policy with parameter z is as follows:

$$
u(t) = \begin{cases} U_{Max} & \text{if} \quad x_1(t) < z,\ \xi(t) = 1 \\ d & \text{if} \quad x_1(t) = z,\ \xi(t) = 1 \\ 0 & \text{whenever} \quad x_1(t) > z \text{ or } \xi(t) = 0 \end{cases}
\tag{3}
$$

Furthermore, the system considered involves conforming and non conforming inventory costs, and backlog (negative conforming parts storage) costs. For conforming parts, whenever the associated stock $x_1(t)$ is positive, a per unit time inventory cost $c_1^+ x_1(t)$ is charged, while if negative, a per unit time backlog cost $(-c_1^- x_1(t))$ is charged. If the stock of non conforming items is positive, a per unit time cost c_2 is charged. For maximum generality and to somehow account for productivity loss and additional inspection and handling costs caused by the presence of non conforming parts, we shall make c_2 different and in general significantly higher than c_1^+. The market price of items produced depends on the degree of conformity required for these items as defined by the sizes of *LSL* and *USL*. Let $S(LSL,USL)$ designate the corresponding (assumed known) reward function per unit part sold (all produced parts are assumed to be sold). The profit rate is the difference between the revenue generated by sales (realized demand) and the related costs. Thus summarizing, the long-run average profit is parameterized by hedging policy parameter *z* and conformity specification parameters *LSL* and *USL* as follows:

$$J\left(z,LSL,USL\right)=\lim_{t\to\infty}\frac{1}{T}E\left[\int_0^T\left(S\left(LSL,USL\right)\times d-\left(c_1^+\times x_1^+\left(t\right)+c_1^-\times x_1^-\left(t\right)+c_2\times x_2\left(t\right)\right)\right)dt\right] \quad (4)$$

where $x_1^+ = \max(0, x_1)$ and $x_1^- = \max(-x_1, 0)$.

Let *A* denote the set of admissible parameters (z, *LSL* and *USL*). The production and product design control problem considered herein is to determine the admissible parameter vector (z, *LSL* and *USL*) that maximizes *J*(.) given by (4) considering equations (1) to (3). This is a feedback control that specifies the control actions when the system is in a given state $(x_1(t), x_2(t), \xi(t))$. If the controlled manufacturing system is considered ergodic, the corresponding optimal objective function, then independent of the initial conditions, $v(.)$ is given by:

$$v=\underset{(z,LSL,USL)\in A}{Sup}\; J\left(z,LSL,USL\right) \quad (5)$$

3. CASE STUDY

The case study considered consists of a continuous industrial process producing metal discs. It is assumed that on the basis of the client's requirements, the range of acceptable diameter values has initially been set equal to 5 ± .15 cm. Practical production and quality control measurements indicate that the mean value of the diameter of the controlled items is 5 cm with a standard deviation of 0.1 cm. Based on these practical results, the capability of the process is 0.5 with a rate of non conforming items $\beta = 13.36\%$. Such performance is deemed inadequate in view of the high cost incurred for non conformity (unless otherwise agreed, non conforming parts are to be scrapped). To deal with the problem, an agreement is reached with the customer. The latter accepts to buy the items that lie outside the specifications (5 ± .15 centimetres) at a lower price. The customer has indicated that this difference in price will compensate for the cost of filling the gaskets associated with discs the diameters of which are below LSL. However any discs with diameter exceeding the USL will be deemed unacceptable. In this case, a rework process has been settled on. Thus, a shortfall equal to the unit cost of a reworked unit is considered.

Results for the capability of the process as estimated from previous observations are summarized in Table 1 below.

Table 1: Capability Vs. Reward

LSL	USL	Client Specifications	Real Production	C_{pk}	β	Remarks $S(LSL,USL)$	Remarks C_{pk}
4.85	5.15	5 ± .15	$\overline{X} = 5\,; \sigma = 0.1$	0.5	13.36 %	Excellent	Very Poor
4.8	5.2	5 ± .15	$\overline{X} = 5\,; \sigma = 0.1$	0.6667	4.55 %	Good	Poor
4.7	5.3	5 ± .15	$\overline{X} = 5\,; \sigma = 0.1$	1	0.27 %	Average	Acceptable
4.5	5.5	5 ± .15	$\overline{X} = 5\,; \sigma = 0.1$	1.6667	5.7e-4 %	Poor	Good

These results clearly show that a more detailed analysis should be carried out to determine the choice of LSL and USL and thus the capability of our process and the reward per unit for conforming items. Moreover, since shortage costs will be incurred each time a demand is not filled in addition to the holding costs, a production strategy should be adopted to guarantee the best overall profit.

3.1. Analytical solution for a Markovian machine

Based on our aforementioned research [2], where an extended model of the Bielecki-Kumar theory for a failure prone machine producing *a mixture* of good and defective items is proposed, this section aims at subsuming the above optimization problem within that extended framework. Assuming that the mean product specification μ is always met, the sale price becomes a function of the half-range δ. In order to maximize the long-run average profit (4), we then consider the following steps:

1- For a given value of δ, and the corresponding $\beta(\delta)$ based on equation (1), we determine the resulting items price $S(\delta) \equiv S(USL, LSL)$ based on the assumed mathematical expression below, where *g,h,* are respectively some given positive and negative constants:

$$S(\delta) = g \times e^{h\delta} \tag{6}$$

2- For a given value of $\beta(\delta)$, we determine the hedging point level $z^*(\delta)$ which minimizes $J_1(z,\delta)$, the long term average combined storage and backlog cost under the hedging policy (3), with hedging parameter z, mid range parameter δ, and defined by:

$$J_1(z,\delta) = \lim_{t\to\infty} \frac{1}{T} E\left[\int_0^T \left(\left(c_1^+ \times x_1^+(t) + c_1^- \times x_1^-(t) + c_2 \times x_2(t)\right)\right) dt\right] \tag{7}$$

3- Compute the long-run average profit (8)

$$J(z,\delta) = \lim_{t\to\infty} \frac{1}{T} E\left[\int_0^T \left(S(\delta) \times d - \left(c_1^+ \times x_1^+(t) + c_1^- \times x_1^-(t) + c_2 \times x_2(t)\right)\right) dt\right] \tag{8}$$

Note that the cost defined by equation (7) could be represented as a function of the inventory variable $x_1(t)$ only, as follows:

$$\begin{aligned} J_1(z,\delta) &= \lim_{t\to\infty} \frac{1}{T} E\left[\int_0^T \left(\left(c_1^+ \times x_1^+(t) + c_1^- \times x_1^-(t) + c_2 \times \frac{\beta(\delta)}{1-\beta(\delta)} x_1^+(t)\right)\right) dt\right] \\ &= \lim_{t\to\infty} \frac{1}{T} E\left[\int_0^T \left(\left((c_1^+ + c_2 \times \frac{\beta(\delta)}{1-\beta(\delta)}) \times x_1^+(t) + c_1^- \times x_1^-(t)\right)\right) dt\right] \end{aligned}$$

With the dynamic behaviour of $x_1(t)$ when $0 < x_1(t) < z$ defined as:

$$\frac{dx_1^+(t)}{dt} = U_{\max} \times (1-\beta(\delta)) \times I\left[\xi(t) = 1\right] - d$$

The problem formulated in this way can be reduced to the Bielecki-Kumar framework [1] with both a higher holding cost ($c_1^{+*} = (c_1^+ + c_2 \times \frac{\beta(\delta)}{1-\beta(\delta)})$) and a smaller maximal production rate ($U_{\max}^* = U_{\max} \times (1-\beta(\delta))$). Accordingly, we can conclude that an optimal hedging point $z^*(\delta)$ and an optimal combined storage and backlog long term average per unit time cost $J_1(z^*(\delta))$ exist and can be calculated as follows:

Optimal hedging point (conforming items) $z^*(\delta)$

Following the development given in [1], one should investigate the system parameters and some boundary conditions to determine the value of $z^*(\delta)$. In fact, given the stochastic process governing the machine we can calculate the expected capacity of the system available to respond to the given demand rate. This capacity is given by:

$$U_{Max}(1-\beta(\delta))\frac{q_{01}}{q_{01}+q_{10}}$$

If the capacity of the system is insufficient or just exactly enough to meet the demand, it is clear that the value of $z^*(\delta)$ will be equal ∞. Thus $z^*(\delta)$ will be infinite if:

$$\left[U_{Max}(1-\beta(\delta))\times\frac{q_{01}}{q_{01}+q_{10}}\leq d\right]\Rightarrow\left[\beta(\delta)\geq 1-\frac{d}{U_{Max}}\times\frac{q_{01}+q_{10}}{q_{01}}\right]$$

Now, if the system has enough capacity to respond to the demand rate i.e. if:

$$\beta(\delta)<1-\frac{d}{U_{Max}}\times\frac{q_{01}+q_{10}}{q_{01}}$$

then $z^*(\delta)$ will be a finite stock (zero or positive). Furthermore, given that z is the safety stock to hedge against future production shortages, it is of interest to identify the conditions under which its optimal value $z^*(\delta)$ will be positive; more particularly we wish to study the influence of the quality parameter $\beta(\delta)$ on the size of that safety stock.

Following [1], we can define the equation of a boundary separating the region where $z^*(\delta)$ is equal to zero from that where it is strictly positive. This equation is given by:

$$\frac{U_{Max}(1-\beta(\delta))(c_1^-+(c_1^++\frac{c_2\beta(\delta)}{(1-\beta(\delta))}))q_{10}}{(c_1^++\frac{c_2\beta(\delta)}{(1-\beta(\delta))})(U_{Max}(1-\beta(\delta))-d)(q_{10}+q_{01})}=1 \tag{9}$$

If the left hand-side expression is greater than 1, then $z^*(\delta)$ is strictly positive; it is zero otherwise.

Equation (9) defines the following quadratic expression in $(1-\beta(\delta))$, and whose sign we wish to investigate as a function of $\beta(\delta)$:

$f(\beta) = a(1-\beta)^2 + b(1-\beta) + c \equiv a\ X^2 + b X + c$ where:

$a \equiv U_{Max}\, q_{10}\, c_1^- - U_{Max}\, q_{01}\, (c_1^+ - c_2)$

$b \equiv d\,(q_{01} + q_{10})(c_1^+ - c_2) - U_{Max}\, q_{01}\, c_2$

$c \equiv d\,(q_{01} + q_{10})\, c_2$

and $0 < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$

Here, *a* and *c,* are positive, while *b* is negative. The discriminant of this quadratic equation is:

$$\Delta = b^2 - 4ac = [U_{Max} q_{01} c_2 - d\,(q_{01}+q_{10})(c_2 - c_1^+)]^2 - 4\ U_{Max}\, q_{10}\, d\,(q_{01}+q_{10}) c_2\, c_1^-$$

We distinguish the following cases:

(1) $\Delta = 0$: i.e.: $c_1^- = \dfrac{[U_{Max}\, q_{01}\, c_2 - d\,(q_{01}+q_{10})(c_2 - c_1^+)]^2}{4\ U_{Max}\, q_{10}\, d\,(q_{01}+q_{10})\ c_2}$

In this case $f(\beta) > 0\ ; \forall\ 0 < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$.

(2) $\Delta < 0$: i.e. $c_1^- > \dfrac{[U_{Max}\, q_{01}\, c_2 - d\,(q_{01}+q_{10})(c_2 - c_1^+)]^2}{4\ U_{Max}\, q_{10}\, d\,(q_{01}+q_{10})\ c_2}$

In this case $f(\beta) = a\left[(X(\beta) + \dfrac{b}{2a})^2 - \dfrac{b^2-4ac}{4a^2}\right]$; here the sign of $f(\beta)$ is dictated by the sign of a. Since $a > 0$ then $\forall\ 0 < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}, f(\beta) > 0$.

(3) $\Delta > 0$: i.e. $c_1^- < \dfrac{[U_{Max}\, q_{01}\, c_2 - d\,(q_{01}+q_{10})(c_2 - c_1^+)]^2}{4\ U_{Max}\, q_{10}\, d\,(q_{01}+q_{10})\ c_2}$

In this case $f(\beta) = a(X(\beta) - r_1)(X(\beta) - r_2)$ where $r_1 = \dfrac{-b-\sqrt{\Delta}}{2a}$, $r_2 = \dfrac{-b+\sqrt{\Delta}}{2a}$ and $0 \le r_1 \le r_2$. We recall that $X(\beta) = 1 - \beta(\delta)$ and $0 < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$.

So the sign of $f(\beta)$ according to the different scenarios when $\Delta > 0$ is as follows:

- If $0 < \beta(\delta) < \max\left[0, 1-r_2\right]$ or $\min\left[1-r_1, 1-\dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}\right] < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$ then $f(\beta) > 0\ ; \forall\ 0 < \beta(\delta) < 1 - \dfrac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$

- If $\max(1-r_1, 0) < \beta(\delta) < \min\left[1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}, (1-r_2)\right]$ then

$$f(\beta) < 0 \; ; \forall \; 0 < \beta(\delta) \; < 1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$$

In the case studied before (i.e. $\Delta > 0$), we distinguish an important region: it is when $\max(1-r_2, 0) = 1-r_2$. This region is defined by $0 < \beta(\delta) \; < 1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$ and $C_n^2 < c_1^- < C_n^1$ with $C_n^2 \equiv \left(\frac{q_{01}}{q_{10}} - \frac{d(q_{01}+q_{10})}{U_{Max}\, q_{10}}\right) c_1^+$ and $C_n^1 \equiv \frac{[U_{Max}\, q_{01}\, c_2 - d\,(q_{01}+q_{10})\,(c_2 - c_1^+)]^2}{4\, U_{Max}\, q_{10}\, d\,(q_{01}+q_{10})\; c_2}$. For all c_1^- satisfying the above inequality the optimal hedging point z^* displays the unusual behavior that it is

$$\begin{cases} \text{Positive} & \text{for} \quad 0 < \beta(\delta) < 1-r_2 \\ \text{Null} & \text{for} \quad 1-r_2 < \beta(\delta) < 1-r_1 \\ \text{Positive} & \text{for} \quad 1-r_1 < \beta(\delta) < 1-\frac{d\,(q_{01}+q_{10})}{q_{01}U_{Max}}. \end{cases}$$

This is confirmed in Figure 5 below where, for a fixed c_1^- lying between C_n^2 and C_n^1, for small values of β, the optimal hedging point z^* is at first positive and as β increases, becomes zero; ultimately, as β increases further, z^* becomes positive again. This can possibly be interpreted intuitively as follows: as β increases, two conflicting effects result; on the one hand, it gets more and more expensive to store a given critical level of good parts (because many bad parts have to be stored at the same time as well), and this tends to drive the optimal hedging level towards zero. On the other hand, as β increases even further, the machine productivity gets lower and lower and this can produce expensive deep incursions in the negative inventory regions. At some point it pays more to start having a positive inventory of good parts again.

The figure 3 below shows the possible scenarios and is a summary of the above.

$$c_1^- = \left(\frac{q_{01}}{q_{10}} - \frac{d(q_{01}+q_{10})}{U_{Max}\, q_{10}}\right) c_1^+ \qquad\qquad c_1^- = \frac{[U_{Max}\, q_{01}\, c_2 - d\,(q_{01}+q_{10})\ (c_2 - c_1^+)]^2}{4\, U_{Max}\, q_{10}\, d\,(q_{01}+q_{10})\ c_2}$$

c_1^-	$a>0$ and $\Delta>0$	$a>0$ and $\Delta>0$	$a>0$ and $\Delta<0$
$f(\beta)>0$ i.e. $z^*(\delta)>0$	$\min\left[1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}},\ 1-r_1\right]<\beta(\delta) <1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$	$0<\beta(\delta)<1-r_2$; OR $\min\left[1-r_1, 1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}\right]<\beta(\delta) <1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$	$0<\beta(\delta)<1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}}$
$f(\beta)<0$ i.e. $z^*(\delta)=0$	$0<\beta(\delta)<\min\left[1-r_1,\ (1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}})\right]$	$1-r_2<\beta(\delta)<\min\left[1-r_1,\ (1-\frac{d(q_{01}+q_{10})}{q_{01}U_{Max}})\right]$	Impossible

Figure 3: A summary of the possible scenarios

Equation (10) below summarizes the value of $z^*(\delta)$ with respect to the aforementioned conditions:

$$z^*(\delta)=\infty \quad \text{if} \quad \frac{U_{Max}(1-\beta(\delta))-d}{q_{10}} \le \frac{d}{q_{01}}$$

If $\frac{U_{Max}(1-\beta(\delta))-d}{q_{10}} > \frac{d}{q_{01}}$:

$$z^*(\delta)=0 \quad \text{if} \quad \frac{U_{Max}(1-\beta(\delta))(c_1^- + (c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))}))q_{10}}{(c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))})(U_{Max}(1-\beta(\delta))-d)(q_{10}+q_{01})} \le 1$$

$$z^*(\delta)=\frac{1}{\frac{q_{01}}{d}-\frac{q_{10}}{(U_{Max}(1-\beta(\delta))-d)}}\log\left(\frac{U_{Max}(1-\beta(\delta))q_{10}(c_1^- + (c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))}))}{(c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))})(U_{Max}(1-\beta(\delta))-d)(q_{10}+q_{01})}\right) \quad \text{otherwise} \tag{10}$$

Optimal storage and backlog cost:

$$J_1(z^*(\delta))=\frac{c_1^- q_{10} U_{Max}(1-\beta(\delta))d}{(q_{10}+q_{01})(rU_{Max}(1-\beta(\delta))-q_{01}d-q_{10}d)} \quad \text{if} \quad z^*(\delta)=0$$

$$J_1(z^*(\delta))=\frac{(c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))})d}{q_{01}+q_{10}} + \frac{(c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))})}{\frac{q_{01}}{d}-\frac{q_{10}}{U_{Max}(1-\beta(\delta))-d}}\log\left(\frac{U_{Max}(1-\beta(\delta))q_{10}(c_1^- + (c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))}))}{(c_1^+ + \frac{c_2\beta(\delta)}{(1-\beta(\delta))})(U_{Max}(1-\beta(\delta))-d)(q_{10}+q_{01})}\right)$$

$$\text{if} \quad z^*(\delta)>0$$

Now given $z^*(\delta)$ as defined in (10) above, it becomes possible to optimize with respect to δ an expression of the net profit, $J(z^*(\delta),\delta)$, itself already optimized with respect to the hedging point level z in the hedging control policy.

3.2. Numerical results

The production and operating state of the parameters of the system considered, as well as various cost coefficients and the reward per unit for conforming items are presented in Table 2 below.

Table 2: system parameters

c_1^+	c_1^-	c_2	U_{Max}	d
1	18	5	1.5	1
q_{10}	q_{01}	g	h	
0.01	0.1	195.9265	-4.5151	

Figure 4 is a plot of the average total profit $J(z^*(\delta),\delta)$, pre optimized with respect to the hedging level, as a function of δ.

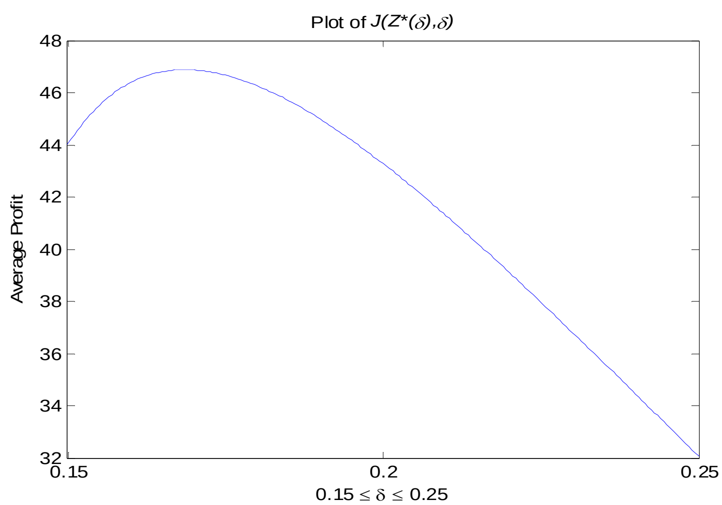

Figure 4: Hedging level pre optimized average profit $J(z^*(\delta),\delta)$

Figure 5 illustrates the regions where $z^*(\delta)$ is equal to zero or strictly positive for all parameters except c_1^- and U_{Max} as in Table 2. We notice that for some values of c_1^-, the optimal hedging point $z^*(\delta)$ can successively go through $z^*(\delta) > 0$, $z^*(\delta) = 0$, $z^*(\delta) > 0$, and $z^*(\delta) = +\infty$ as $\beta(\delta)$ increases .

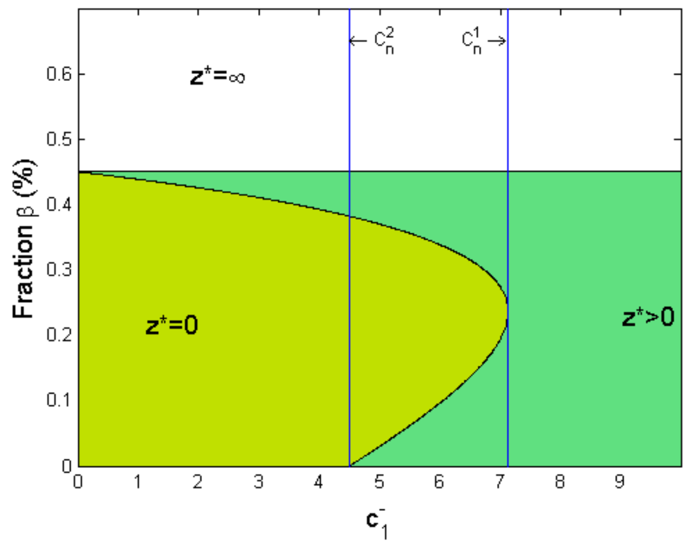

Figure 5: Regions of $z^*(\delta) > 0$, $z^*(\delta) = 0$, and $z^*(\delta) = +\infty$ in the $\beta - c_1^-$ plane for $U_{Max} = 2$, and remaining parameters as in Table2.

To illustrate the effect of the cost variation on the design parameters, a sensitivity analysis was conducted. Table 3 details the cost variations, and presents the optimal parameters and the optimal profits for the sensitivity analysis cases. The results confirm that when backlog cost increases (cases 2 and 3) both the hedging level z and δ increase. When δ increases, the rate of non-conforming items β decreases. This result confirms the fact that, when facing higher backlog costs, the system must guarantee a certain level of conforming items (thus safety stock z increases and conformity specifications δ become more relaxed). Clearly, in practice, such relaxation of quality cannot be carried out indefinitely. One would then need to impose an absolute upper bound on δ and consider the corresponding *constrained optimization* problem. When the unit cost of non - conforming items increases (cases 4 and 5), the hedging level z decreases and δ increases. This result makes sense since with a higher unit cost of non - conforming items, the system must react to decrease the rate of production of such items. Thus the system must keep a lower level of z as well relax conformity requirements.

Table 3: Optimal design factors and profits

Cases	c_1^+	c_1^-	c_2	δ^*	$S(\delta^*)$	z^*	β^*	Profit
Basic	1	18	4	0.1685	91.5555	20.5600	0.0920	46.8853
2	1	19	4	0.1695	91.1430	21.2059	0.0901	45.8506
3	1	20	4	0.1705	90.7324	21.8188	0.0882	44.8740
4	1	18	5	0.1720	90.1200	19.6429	0.0854	45.2815
5	1	18	6	0.1750	88.9076	18.8960	0.0801	43.8578

In real manufacturing systems, non exponential machine failure and repair times is frequently encountered situation. We refer the reader to Law and Kelton [15], chapter 6, for details on commonly used failure and repair time probability distributions.

With non-exponential failure and repair time distributions, analytical approaches such as the one detailed in this section become more difficult to develop. Indeed, while it is often possible, although not guaranteed, to embed the model back into the Markovian framework by approximating failure and repair time non exponential distributions via phase-type distributions ([16]), this comes at the cost of a generally significant increase in the dimension of the Markovian systems studied. More importantly, one has to relinquish reliance on analytical expressions for both cost and optimal hedging point as presented in Section 3.1. In order to deal with such situations, in the next Section, we choose instead to investigate an alternative approach which has been successfully used to control production and preventive maintenance activities in a manufacturing system configuration (see Gharbi and Kenne [13] and the references therein). This approach combines the descriptive capacities of continuous/discrete event simulation models with analytical models, experimental design (DOE) and response surface methodology (RSM) approach.

4. SIMULATION BASED APPROACH

In this section, we present the procedure for varying the control factors simultaneously so as to obtain the appropriate relationship between the long-run average profit (4) and significant main factors (z, δ) and interactions.

The structure of the proposed control approach as summarized in Figure 6 consists of the following sequence of steps:

1. The *Control problem statement*, as exemplified in Section 2, consists in the formulation of an optimal control problem for the joint production control and quality specification of an unreliable manufacturing system with the aim of optimizing long term average profit. This control formulation entails the development of a dynamic model where the class of quality controls to be designed and their impact on production and profits are specified, and where the impact of production inputs on stocks dynamics is represented. Also, the objective function (profit/cost) has to be specified. At this stage, while one would ideally try to compute the (unrestricted) optimal feedback policy for the problem at hand, response surface methods focus instead on the more realistic goal of constructing optimal feedback policies over a restricted class of *parameterized* control policies, defined as the admissible set of policies.
2. The *control factors* z, for production rates control and *LSL* and *USL* for product design specifications, parameterize the class of proposed joint control policies. Indeed, in this particular case, previous work indicates that the class of single parameter hedging control policies are a reasonable choice for admissible production control policies. Let z be the associated generic inventory level. Furthermore, let *LSL* and *USL* be the generic parameters associated with a member of the selected class of admissible quality decision making policies.
3. A discrete/continuous event *simulation model* which describes the continuous dynamics of the system (2) and its discrete stochastic behaviour is developed using the Visual SLAM language [17]. This model consists of several networks, each of which describes a specific task in the system (i.e., demand generation, control policy, states of the machines, inventory control..., etc.). The *simulation model* uses the admissible (parameterized) control policies defined in the previous step as inputs for conducting a preliminary series of numerical experiments aimed at evaluating the degree of dependence of manufacturing system performance on the various control factors (policy parameters). Hence, for a set of values of the control factors, the long-run average profit is obtained from the simulation model.

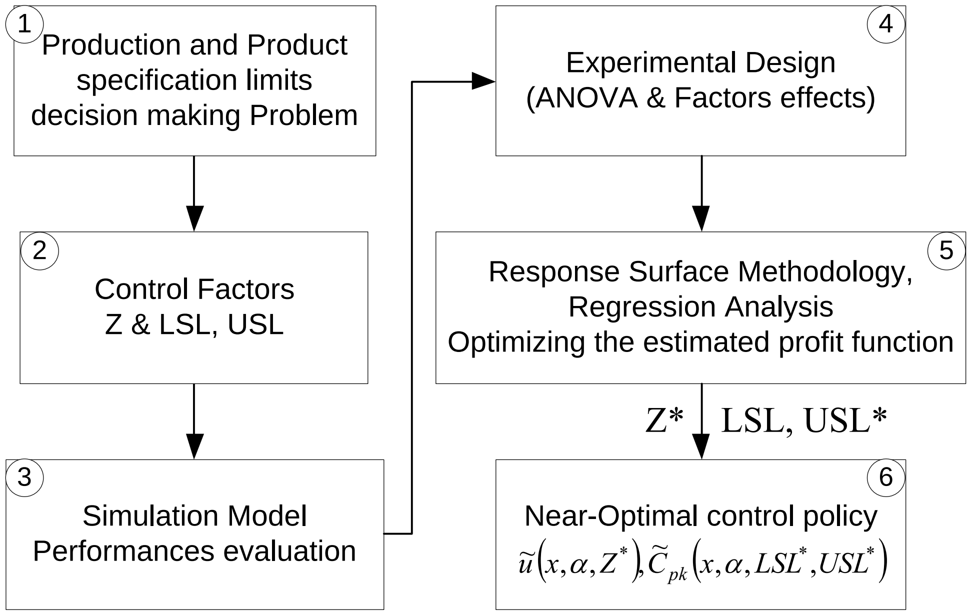

Figure 6: Simulation Based Design Methodology

4. Given the preliminary results in step 3, the *experimental design* approach defines how the control factors must be varied in order to best determine the effects of the main factors and their interactions (i.e., analysis of variance or ANOVA) on the objective function. Its aim is to use statistical know how to minimize the set of required (computationally costly) simulation experiments (see [18] for more details).
5. The *response surface methodology* is then used to obtain through regression analysis over a set of basis functions, the mathematical relationship between the long-run average profit and significant main factors and interactions given in step 4. The obtained model is then optimized in order to determine the best values of factors called here z^* for production, and $LSL*$ and USL^* for product specification limits.
6. The resulting *near-optimal control policy* ($\tilde{u}\left(x_1, x_2, \xi, z^*\right), \tilde{C}_{pk}\left(z^*, LSL^*, USL^*\right)$) is thus a joint production/product specification policy to be applied to the manufacturing system.

5. EXPERIMENTAL RESULTS

The objectives of this section are to: (i) determine whether the input parameters really affect the response, (ii) estimate the relationship between the long-run average profit and significant factors, and finally, (iii) compute the optimal values of estimated factors.

5.1. Experimental design

In this study, we collect and analyze data for a steady state profit which as much as possible approximates that defined by the objective function given by equation (4).

Given that the engineering Lower and Upper specification limits (LSL and USL) can be defined in terms of the engineering specified mean μ and the half range δ as: $LSL = \mu - \delta\,; USL = \mu + \delta$, and assuming (as earlier in the paper) a centered process, we will use the half range δ as input variable to replace LSL and USL.

The experimental design is concerned with (i) selecting a set of input variables (i.e., factors δ and z) for the simulation model; (ii) setting the levels of selected factors of the model and making decisions on the conditions, such as the length of runs and number of replications, under which the model will be run.

Two independent variables and one dependent variable (the profit) are considered. The levels of independent variables or design factors must be carefully selected to ensure they properly represent the domain of interest. In order to approximate the objective function by a second-order response surface model, we selected a 3^2-response surface design since we have 2 independent variables, each at three levels. The levels of the independent variables were selected as in Table 4.

Table 4: Levels of the independent variables

	Low level	Center	High level
δ	0.15	0.2	0.25
z	6	15	24

Five replications were conducted for each combination of the factors, and therefore, 45 (3^2 x 5) simulation runs were made. To reduce the number of replications, we used a variance reduction technique called *common random numbers* ([15]). The technique guarantees the generation of the same sequence of random numbers, thus the same failure and repair times, within the different runs of one *block* (one replication). However, a different sequence of random numbers is generated from one *block* to another. We conducted some preliminary simulation experiments using 5 replications, and noticed that the variability allows the different parameter effects to be distinguished. It is interesting to note that all possible combinations of different levels of factors are provided by the response surface design considered. The experimental design is used to study the effects that some parameters, namely δ and z, and their interactions have on the performance measure (i.e., the profit).

5.2. Statistical analysis

The statistical analysis of the simulation data consists of the multi-factor analysis of the variance (ANOVA). This is done using a statistical software application (e.g., STATGRAPHICS or SATISTICA), to provide the effects of the two independent variables on the dependent variable. Table 5 illustrates the ANOVA for the third case under study (Table 3). From Table 5, we can see that the main factors δ and z, their quadratic effects, as well as their interactions are significant at the 0.05 level (i.e., P-value < 0.05; symbol S in the last column).

The residual analysis was used to verify the adequacy of the model. A *residual* versus *predicted* value plot and *normal probability* plot were used to test the homogeneity of the variances and the residual normality, respectively. We concluded that the normality and equality of variance led to satisfactory plots. Moreover, the R-squared value equal to 0.98 is very satisfactory. This indicates that more than 98% of the total variability is explained by the model ([18]). The model obtained includes two main factors (δ and z), two quadratic effects (δ^2 and z^2), and cross term effects (δ x z).

Table 5: ANOVA table

Analysis of Variance for Profit3 (Case 3)

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*	
z :Factor_A	3153.3	1	3153.3	1117.14	0.0000	**S**
δ:Factor_B	230.33	1	230.33	81.60	0.0000	**S**
AA	502.595	1	502.595	178.06	0.0000	**S**
AB	195.843	1	195.843	69.38	0.0000	**S**
BB	599.174	1	599.174	212.27	0.0000	**S**
blocks	173.561	4	43.3902	15.37	0.0000	
Total error	98.7929	35	2.82265			
Total (corr.)	4953.6	44				

R-squared = **98.0056** percent

5.3. Response surface methodology

The Response surface methodology is a collection of mathematical and statistical techniques that are useful for modeling and analyzing problems in which a response of interest is influenced by several variables, and the objective is to optimize this response

([18]). We assume here that there exists a function Φ of δ and z that provides the value of the profit corresponding to any given combination of input factors, i.e., $\Pr ofit = \Phi(\delta, Z)$.

The function Φ(.) is called the response surface, and is assumed to be a continuous function of δ and z. The second order model is thus given by:

$$\Phi = \beta_0 + \beta_{11} \times \delta + \beta_{12} \times z + \beta_{21} \times \delta^2 + \beta_{22} \times z^2 + \beta_3 \times \delta \times z + \varepsilon \qquad (11)$$

Where δ and z are the input variables; β_0, β_{11}, β_{12}, β_{21}, β_{22} *and* β_3 are unknown parameters, and ε is a random error. From STATGRAPHICS, the estimation of unknown parameters is performed, and the following six coefficients achieved. The values of these coefficients are:

$\beta_0 = -130.891$, $\beta_{11} = 1287.39$, $\beta_{12} = 5.15562$, $\beta_{21} = -3096.25$, $\beta_{22} = -0.0875234$, and $\beta_3 = -6.95386$.

The corresponding response surface is presented in Figure 7. The optimum is obtained for $\delta^* = 0.182978$ and $z^* = 22.183$, and the optimal profit Φ* is 44.0807.

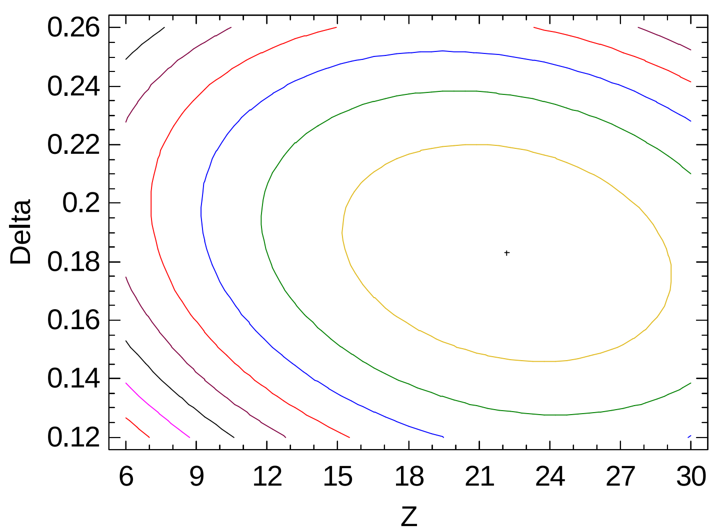

Figure 7: Profit response surface

5.4. Validation of the analytical solution with the simulation based approach

To crosscheck the robustness of the proposed approach and the validity of the simulation results we carried on several experimentations and analysis detailed in the following steps. More details on the validation issue of simulation results can be found in [19].

1. Consider the analytical optimal z^* and δ^* as input and estimate the profit J^* with the simulation model.
2. Apply the simulation based approach to develop a regression model and to find the near optimal z^*_{sim} and δ^*_{sim} and the corresponding profit.
3. Compare the estimated control policy to the optimal one through the Student's *t*-test of the following H_0 hypothesis.

H_0: The incurred cost for the estimated control policy (Y_1) is different from that given by the analytically obtained optimal hedging control policy (Y_2).

The simulation model is used to generate two data samples related to the estimated and optimal control policies. For each control policy, *N* replications are performed to obtain *N* incurred cost observations. For a $1-\alpha$ confidence level, H_0 is rejected if the value of the *t* distribution with υ degrees of freedom and a confidence level $1-\alpha$ (i.e., $t_{\upsilon,1-\alpha/2}$) is greater than the *t* given by:

$$t = \frac{\overline{Y}_1 - \overline{Y}_2}{\sqrt{\frac{\left(S_1^2 + S_2^2\right)\left(N-1\right)}{2N-2}\frac{\sqrt{2}}{N}}} \tag{12}$$

where $\upsilon = 2N-2$, $\overline{Y}_i = \left(1\big/N\right)\times\sum_{j=1}^{N} Y_{ij}, S_i^2 = \left(1\big/N-1\right)\times\sum_{j=1}^{N}\left(Y_{ij}-\overline{Y}_i\right)^2, i=1,2.$ For *N*=26 and $\alpha = 5\%$, the Student's *t*-test table gives $t_{50,0.975} = 2.145$.

Table 6: Validation

	Analytical Results		Simulation Results of the Analytical (z^*, δ^*)		Simulation Results Of the near-optimal $(z^*_{sim}, \delta^*_{sim})$		Student's *t*-test
Cases	(z^*, δ^*)	J^*	(z^*, δ^*)	$\overline{Y}_1$	$(z^*_{sim}, \delta^*_{sim})$	$\overline{Y}_2$	t
Basic	(20.56,0.1685)	46.8853	--	47.5583	(21.47,0.1789)	47.0016	0.8146246
2	(21.21,0.1695)	45.8506	--	46.7092	(21.85,0.1811)	46.2549	0.44962212
3	(21.82,0.1705)	44.8740	--	45.4521	(22.18,0.1829)	44.9603	0.7831087
4	(19.64,0.1720)	45.2815	--	45.1307	(20.73,0.1847)	45.0726	0.72389922
5	(18.89,0.1750)	43.8578	--	43.6501	(20.11,0.1895)	43.4731	0.64854362

The results presented in Table 6 clearly show that the Student's *t*-test table value is always greater than the t given by equation (12). Hence, the estimated control policy performs as well the optimal hedging control policy in the Markovian case.

5.5. Non markovian cases results

The hypothesis test validates the proposed approach and states that simulation experiments could be combined with statistical analysis to obtain very good near-optimal policies. For non-Markovian processes (i.e., non-exponential failure and repair probability distributions), the same policy is used as a near-optimal control policy with estimated stock threshold values and product specifications. As illustrative examples, we consider the following numerical values. To compare the Markovian versus the non Markovian experimental results we have fixed the standard deviation equal to the mean for the Lognormal and the Gamma distributions. More details on the use of simulation to address extension issues in the absence of an analytic solution can be found in [20].

- Failure times: Gamma, Lognormal, with mean values equal to 100 and the same standard deviation $\sigma = 100$.
- Repair times: Gamma, Lognormal, with mean values equal to 10 and the same standard deviation $\sigma = 10$.

The obtained results for the considered distribution and the different values of β are given in Table 7, where R^2 gives the proportion of the total variation in the estimated cost attributable to the variability of the process. Hence, well-fit regression models are characterized by large R^2 values (i.e., close to 1).

Given that the values of R^2 for the considered distributions are greater than that for the exponential one, we can conclude that the obtained hedging point policy is consistent with the behaviour of the considered manufacturing system.

Table 7: Results for non Markovian cases

	Exponential			Gamma			Lognormal		
Cases	$(z^*_{sim}, \delta^*_{sim})$	J^*	R^2	$(z^*_{sim}, \delta^*_{sim})$	J^*	R^2	$(z^*_{sim}, \delta^*_{sim})$	J^*	R^2
Basic	(21.47,0.1789)	47	0.98	(21.67,0.1801)	47.8	0.99	(20.00,0.179)	42.69	0.99
2	(21.85,0.1811)	46.3	0.98	(22.05,0.1821)	47.3	0.99	(20.46,0.1816)	41.58	0.99
3	(22.18,0.1829)	45	0.98	(22.58,0.1849)	46	0.99	(20.87,0.1837)	40.54	0.99
4	(20.73,0.1847)	45	0.98	(21.03,0.1877)	46.1	0.99	(19.22,0.1849)	41.36	0.99
5	(20.11,0.1895)	43	0.98	(20.81,0.1905)	44	0.99	(18.56,0.189)	40.22	0.99

It is interesting to remark that the hedging levels values, under the exponential and Gamma distributions, are very close to each other. This observation makes sense since the exponential distribution can be generated from a particular gamma distribution. Moreover, the hedging levels values, under the exponential and Gamma distributions, are greater than those under the Lognormal distribution. This observation also makes sense since the exponential and gamma distributions are characterized by a higher variability in comparison with the Lognormal distribution. These observations confirm the necessity to consider the appropriate probability distribution to characterize and analyse a given system.

6. COMPARATIVE STUDY

A comparative study for joint versus dissociated production and quality control strategies was also conducted. Consider the case where the engineering specification are fixed near the client requirements (i.e., LSL=4.81 and USL=5.19) and a hedging point policy is adopted to control the production rates. In this case the rate of non - conforming items is equal to 5.74 %. In such a situation the manager has to find the best hedging level that maximizes the profit.

We present in Table 8 the optimal profit for the same sensitivity analysis input (Table 3), conducted with the case detailed previously. It is important to note that the results presented in Table 8 were obtained under the same conditions (simulation, experimental design and RSM), and following the same approach under which the sensitivity analysis was conducted for the joint control strategies (Table 6).

Table 8: Optimal design factors and profits for the sensitivity analysis case 1

Cases	c_1^+	c_1^-	c_2	δ	z^*	β	Profit	**Profit (joint strategies)**	Gain
1	1	18	5	0.19	20.11	5.74 %	44.98	47.0016	4.49%
2	1	19	5	0.19	20.77	5.74 %	44.11	46.2549	4.86%
3	1	20	5	0.19	21.41	5.74 %	43.29	44.9603	3.86%
4	1	18	6	0.19	19.55	5.74 %	43.97	45.0726	2.51%
5	1	18	7	0.19	19.01	5.74 %	42.99	43.4731	1.12%

The results obtained show that the variation of the design parameters is in the expected direction. However, the profits for all the cases (Table 8) are lower than those under the joint strategies. To confirm these results and hence the advantage of the joint strategy, a Student test was performed in order to compare the performance of the two strategies.

The confidence interval of $P^*_{Joint} - P^*_{Disso}$ is given by (13).

$$\begin{aligned} &\overline{P}^*_{Joint} - \overline{P}^*_{Disso} - t_{\alpha/2,n-1} s.e(\overline{P}^*_{Joint} - \overline{P}^*_{Disso}) \\ &\qquad \le P^*_{Joint} - P^*_{Disso} \le \\ &\overline{P}^*_{Joint} - \overline{P}^*_{Disso} + t_{\alpha/2,n-1} s.e(\overline{P}^*_{Joint} - \overline{P}^*_{Disso}) \end{aligned} \tag{13}$$

where:

$t_{\alpha/2,n-1}$ is the student coefficient function of *n* and α, with *n* the number of replications (set at 10) and (1-α), the confidence level (set at 95%).

$s.e(\overline{P}^*_{Joint} - \overline{P}^*_{Disso}) = \frac{S_D}{\sqrt{n}}$ Standard error, $S_D^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(P^*_{Joint\,i} - P^*_{Disso\,i}\right)^2 - n\left(\overline{P}^*_{Joint} - \overline{P}^*_{Disso}\right)^2\right)$

$\overline{P}^{*}_{Joint}$ the average profit under joint strategies.

$\overline{P}^{*}_{Disso}$ the average profit under dissociate strategies.

The two configurations under study (Joint and Dissociated) were simulated with their optimal design parameters. It has been shown that in all cases, it can be concluded that $P^{*}_{Joint} - P^{*}_{Disso} > 0$ at the 95% confidence level. Consequently, it is a statistically significant statement that the joint strategies lead to higher profits.

7. CONCLUSION

In this paper we studied the joint production control and economic product design in a failure prone manufacturing system. More specifically, we have jointly considered, in a dynamic stochastic context, the engineering vision in terms of product specification and the production vision in terms of quality and production control. To determine the parameters of the best control policy within the class of hedging control policies, an analytical approach valid for exponential machines was first developed; it produces new insights concerning the dependence between of the optimal hedging point on the rate β of non conforming parts (se Figure 5). The analytical approach was then contrasted with an experimental approach based on simulation modeling, design of experiment and response surface methodology. For a particular illustrative case study, we were able to show that the profit under a joint production-quality and product design strategy could increase up to 5 % relative to that resulting from completely dissociated decision making strategies. It is interesting to note that the simulation based approach offers a versatile procedure to control manufacturing systems at the operational level as it is capable of handling more general non exponential machines than allowed by the analytical method. Furthermore, its application can be significantly enhanced thanks to an adequate initialization in the design parameter space as obtained from the analysis based on exponential machine failure and repair assumptions. More complex production and inspection architectures will be considered in the future.

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