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TECHNICAL REPORT

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APPROCHE HIERARCHISEE APPLIQUEE AU CONTROLE DES TAUX DE PRODUCTION ET DE MAINTENANCE DES SYSTEMES MANUFACTURIERS

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SOMMAIRE

Les systèmes de production sont en général constitués de plusieurs machines interconnectées qui produisent simultanément plusieurs types de pièces. Ces machines sont sujettes à des pannes et réparations aléatoires. La dynamique des pannes de ces machines dépend de leurs âges et leur disponibilité dépend des stratégies de maintenance adoptées. Dans ce contexte, le problème de planification de la production et de la maintenance de ces types de systèmes de production peut être formulé comme un problème d'optimisation stochastique de grande dimension. Cette dimension croît de façon exponentielle avec le nombre de machines et de types de pièces considérées. Les problèmes d'optimisation stochastique associés à une telle dimension sont complexes et très difficiles à résoudre de nos jours.

L'objectif de cette recherche est de déterminer une loi de commande sous optimale, constituée des taux de production et de maintenance, qui approxime la loi de commande de ces problèmes complexes. Pour formuler le problème d'optimisation à résoudre, nous proposons une approche de commande hiérarchisée à deux niveaux. Cette approche consiste à utiliser les méthodes des perturbations singulières pour transformer le problème de commande stochastique initial en un problème de commande déterministe équivalent. L'approche de Kushner est par la suite appliquée au problème déterministe pour obtenir un problème de décision markovien. En résolvant ce problème de faible dimension à l'aide des méthodes numériques, nous obtenons une loi de commande à partir de laquelle nous construisons la loi de commande recherchée pour le problème d'optimisation stochastique initial. Une méthode heuristique est proposée pour une telle construction. Des modèles de simulation sont enfin utilisés pour valider l'approche de

commande proposée dans cette recherche. Les résultats de la recherche sont appliqués sur un exemple de système de production constitué de deux machines produisant deux types de pièces.

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ABSTRACT

This paper deals with the production and preventive maintenance planning control problem for a multi-machine flexible manufacturing system (FMS). A two-level hierarchical control model is developed according to the discrepancy between the time scale of the discounting cost event and the one of the machines states process. The proposed model extends the classical singular perturbation approach by considering age dependent machines failure rates and controlling both production and preventive maintenance rates. We replace the stochastic optimal control problem by a deterministic control problem called limiting control problem. With this approach, we compute an age dependent near-optimal control policy of the stochastic initial control problem from the optimal solution of the equivalent limiting control problem. A numerical example is used to illustrate the procedure and to show the reduction of the control problem size.

Key words : Production Control, Preventive Maintenance, Hierarchical Control, Singular Perturbations, Flexible Manufacturing Systems.

TABLE OF CONTENTS

LIST OF FIGURES

1   INTRODUCTION

Today's manufacturing systems have more sophisticated and dynamic nature than the ones in the past. Most of them are large dynamic systems characterized by several machines and a large number of different products. Moreover, such systems are subject to discrete events such as machine failures and repairs, demand fluctuations, etc. Thus, in many instances, manufacturing systems are unreliable systems. Over time, their dynamics moves from one mode to another according to various kinds of discrete events. The control policies of such systems must be able to react to these events and to minimize a given criteria index. The related control problem is and extremely complex one. This complexity is mainly due to the unreliability of the systems and the large number of machines and products involved.

The problem of controlling manufacturing systems with unreliable machines was formulated as a stochastic control problem by Older and Suri (1980). The failures and repairs processes were supposed to be described by homogeneous Markov processes. The related optimal control model fails in the category of problems studied previously by Rishel (1975). Investigation in the same direction gives rise to the analytical solution of the one-machine one-product manufacturing system obtained by Akella and Kumar (1986). In the case of non-homogeneous Matkov processes, the control problem becomes more complex. In this sphere, Boukas and Haurie (1990) considered the fact that the failure probabilities of a machine depend on its age and they added to the existing models the possibility to do preventive maintenance. The related age dependent set of dynamic programming equations were solved numerically for the two-machine one-product manufacturing system. However, with the numerical scheme presented by Boukas and Haurie (1990), it remains difficult to obtain the optimal control of a large scale FMS (more than two machines). The only way to cope with such a difficulty is to develop heuristical methods based on the reduction of the size of the considered control problem. Hence, different approaches have been proposed in the literature in order to derive simple near-optimal control policies in manufacturing.

The concept of hedging point policy, introduced by Kimemia and Gershwin (1983), is one of the simple ways of finding suboptimal control policies in manufacturing. For further details on this concept, we refer the reader to the age dependent hedging point concept presented by Boukas et al. (1995b), Kenne et al. (1997) and Kenne and Gharbi (2000). For large scale FMS, the derivation of suboptimal policies based on this concept seems to be difficult due to the computation of the threshold levels.

Another approach is to develop hierarchical control methods based on the particular structure of the system. This can be done by using the singular perturbation approach. The main idea of this approach is to simplify the complexity of a large system control problem according to the discrepancy between the time scales of different events. By replacing fast processes by their mean values, one can construct a deterministic limiting problem, which is computationally more tractable. Details on this approach can be found in Lehoczky et al. (1991), Sethi and Zhang (1994b) and Soner (1993).

In this paper, we will extend the production rate control model presented in Lehoczky et al. (1991), Soner (1993) and Sethi et Zhang (1994a) to the case where the capacity of the system is age dependent and some jump rates are influenced by the control policy. The production and maintenance rates of the FMS will be derived from the numerical solution of the equivalent limiting problem. The purpose of the paper is to show how some large-scale stochastic control problems involving different time scale processes can be approximated in such a way that they become numerically tractable. Our main contribution is the derivation of a FMS age dependent control policies using the singular perturbation approach. We will illustrate our approach on a numerical example, which consists of two machines producing two part types.

The paper is organized as follows. In section 2, we present the problem formulation of the production and maintenance rates control of a FMS subject to different time scale processes. Section 3 is devoted to the related optimal control problem. We develop the

limiting control problem and the construction of the suboptimal solution of the original problem in section 4. The proposed numerical algorithm is presented in section 5. In the last section, we apply the proposed method to a numerical example to illustrate the effectiveness of the method.

2   PROBLEM STATEMENT

In this section, we present an explicit singular perturbation form of the control model for a multiple identical machines system. Thus, a hierarchical approach for solving the control problem of the FMS is the main frame of the proposed formulation. The system under study consists of *m* identical machines producing *n* different part types. The operational mode of the machine *i* can be described by a stochastic process $\zeta_i(t)$ $(1 \le i \le m)$. such a machine is available when it is operational $(\zeta_i(t)=1)$ and unavailable when it is under repair $(\zeta_i(t)=2)$ or under preventive maintenance $(\zeta_i(t)=3)$. We then have $\zeta_i(t) \in B_i = \{1,2,3\}$. We can describe the operational mode of the FMS by the random vector $\zeta(t) = (\zeta_1(t), \cdots, \zeta_m(t))$ with values in $B = B_1 \times \cdots \times B_m$.

Let $a = (a_1, \cdots, a_m)'$ and $\omega = (\omega_1, \cdots, \omega_m)'$ denote the vector of machines ages and the vector of preventive maintenance rates respectively. The process $\zeta(t)$ is modelled by a continuous time Markov chain defined by machines ages and control dependent transition rates matrix $Q(a,\omega) = [\lambda_{\alpha\beta}(a,\omega)]$ with $\sum_{\beta=1}^{M} \lambda_{\alpha\beta}(a,\omega) = 0$, $\alpha = 1, \cdots, M$, with $M = \mathrm{card}(B)$. The transition rates $\lambda_{\alpha\beta}(.)$ are derived from the combination of those of the dependent processes $\zeta_i(t), i = 1, \cdots, m$. One can describe $\zeta_i(t)$ statistically by the following state probabilities:

$$P[\zeta_i(t+\delta t) = \beta | \zeta_i(t) = \alpha] = \begin{cases} \lambda_{\alpha\beta}^i(.)\delta t + o(\delta t) & \text{if } \alpha \ne \beta \\ 1 + \lambda_{\alpha\beta}^i(.)\delta t + o(\delta t) & \text{if } \alpha = \beta \end{cases} \tag{1}$$

where

$$\lambda_{\alpha\beta}^i(.) \ge 0 \quad \lambda_{\alpha\alpha}^i(.) = -\sum_{\beta \ne \alpha} \lambda_{\beta\alpha}^i(.) \quad (\alpha, \beta \in B_i) \text{ and } \lim_{\delta t \to 0} \frac{o(\delta t)}{\delta t} = 0$$

For the example of two machines, the set of possible values of $\zeta(t)$ can be determined from the values of $\zeta_1(t)$ and $\zeta_2(t)$ as illustrated in table I.

Table I. Modes of a two-machine manufacturing system

$\zeta_1(t)$	1	1	1	2	2	2	3	3	3
$\zeta_2(t)$	1	2	3	1	2	3	1	2	3
$\zeta(t)$	1	2	3	4	5	6	7	8	9

Our approach is used when the rate of change in the machine states is much larger than the rate at which the cost is discounted. In this paper, we assume a constant demand rate without any loose of generality. Two time scales are then considered: the discounting cost event and the machines states process time scales. When the difference between the two time scales is very large, one can split the time. Thus, the transition rates for the system, $\lambda_{\alpha\beta}(.)$, can be expressed as $\varepsilon^{-1}q_{\alpha\beta}(.)$ where $q_{\alpha\beta}(.)$ and the discount rate have the same magnitude. We can describe the behaviour of the system by a process $\zeta^{\varepsilon}(t)$ associated to the infinitesimal generator $Q^{\varepsilon}(.)$ defined as follows:

$$Q^{\varepsilon}(.) = \frac{1}{\varepsilon}\left[q_{\alpha\beta}(.)\right] \qquad \alpha,\beta \in B \tag{2}$$

The singular perturbation parameter ε is used here to express the hierarchical structure of the proposed approach. With $\varepsilon \ll 1$, one can derive, from the formulation of the initial stochastic problem, an equivalent deterministic problem. The related control approach is illustrated in figure 1. Both control problems (stochastic and deterministic) are described by dynamic programming equations (DPE) presented in sections 3 and 4. The significance of the structure depicted in figure 1, for the decision-making hierarchy, is that the planning level management can ignore the day-to-day fluctuations in machines capacities, or generally, the details of shop floor events. This is given by the

limiting problem formulated at l. The operational level management can then derive approximate optimal policies for running the actual (stochastic) manufacturing system. We will provide the control for this level, namely level 2 of the proposed hierarchy, by construction a near-optimal control policy from the one of the limiting problem (level 1).
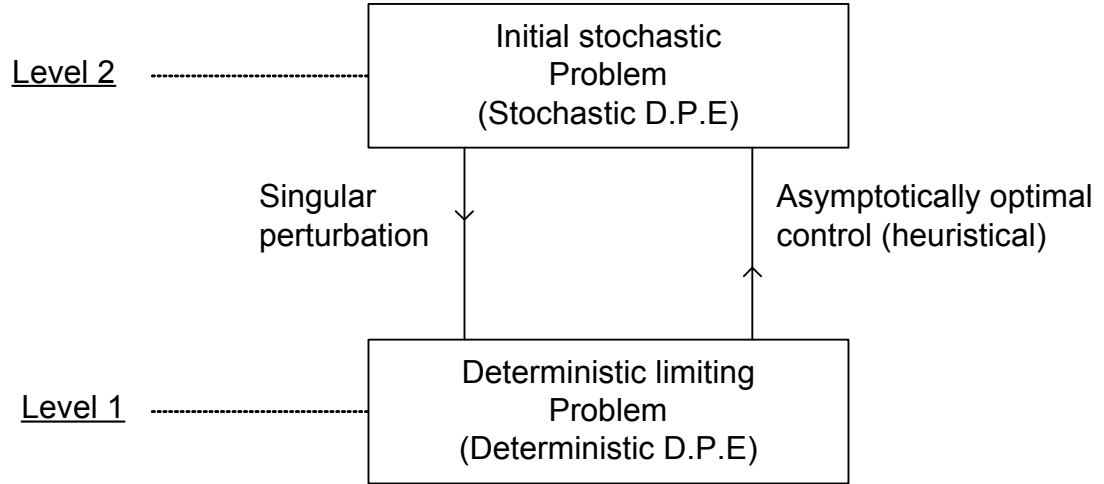
Figure 1. Hierarchical control approach

In order to increase the system capacity or the availability of the machines, we assume that the transition rate from the operational mode to the preventive maintenance mode for each machine is a control variable called $\omega_i(t), i = 1, \cdots, m$. The inverse of the control variable $\omega_i(t)$ represents the expected delay between the call for the technician and his arrival as defined by Boukas and Haurie (1990).

Let $\{Z^\alpha(t), t \geq 0\}, \alpha \in B$, be a finite-state stochastic process corresponding to the number of operational machines at time t with values in $\varepsilon = \{0, 1, \cdots, m\}$. Let $u = (u_1, \cdots, u_n)'$ denotes the vector of production rates. The set of the feasible control policies $K(z)$, with $z = Z^\alpha(t)$, depend on the process $\zeta^\epsilon(t)$, and is given by:

$$K(z) = \begin{cases} (u(t), \omega(t)) \in \mathfrak{R}^{n+m}, u_i(t) \geq 0, \sum_{\rho=1}^{n} u_p(t)\gamma_p \leq z, \quad u_p = \sum_{i=1}^{m} u_{ip} \\ \text{and } 0 \leq \omega_j(t) \leq \omega_{max} \end{cases} \quad 0 \leq z \leq m \quad (3)$$

where γ_p is the processing time of the part-type p and ω_{max} is the maximal preventive maintenance rate of each machine.

The system behaviour is described by a hybrid state comprising both a discrete and a continuous component. The discrete component consists of the discrete event stochastic process $\zeta^\epsilon(t)$. The continuous component consists of continuous variables $x = (x_1, \cdots, x_n)'$ and $a = (a_1, \cdots, a_m)'$ corresponding to the inventory/backlog of products and the cumulative ages of machines. These state variables are described by the following differential equations:

$$\dot{x}(t) = u(t) - d \qquad x(0) = 0 \tag{4}$$

$$\dot{a}(t) = f(u(t)) \qquad a(0) = a \tag{5}$$

where x, a and d are given initial surplus or backlog, initial machines ages and demand rates vectors respectively. Let $\tilde{x} = (x, a)$ and $\tilde{u} = (u, \omega)$.

Let $G(\alpha, \tilde{x}, \tilde{u})$ be the cost rate defined as follow:

$$G(\alpha, \tilde{x}, \tilde{u}) = c^+ x^+ + c^- x^- + c^\alpha, \qquad \forall \alpha \in \varepsilon \tag{6}$$

where c^+ and c^- are costs incurred per unit produced parts for positive inventory and backlog respectively, $x^+ = (\max(0, x_1), \cdots, \max(0, x_n))'$, $x^- = (\max(-x_1, 0), \cdots, \max(-x_n, 0))'$ and c^α are given constants.

Our objective is to control the production rate u(t) and the preventive maintenance rate $\omega(t)$ (or the control policy $\tilde{u}(t)$) so as minimize the expected discounted cost given by :

$$J^{\varepsilon}(\alpha, \tilde{x}, u) = E\left\{\int_0^{\infty} e^{-pt} G(\alpha, \tilde{x}, u) dt \middle| x(0) = x, a(0) = a, Z^{\varepsilon}(0) = \alpha\right\} \tag{7}$$

subject to constraints given by equations (1) to (6). The value function of such a problem is :

$$\upsilon^{\varepsilon}(\alpha, \tilde{x}) = \inf_{\tilde{u} \in K(\alpha)} J^{\varepsilon}(\alpha, \tilde{x}, \tilde{u}) \qquad \forall \alpha \in \varepsilon \tag{8}$$

In the next sections, we present the elementary properties of the value function $\upsilon^{\varepsilon}(.)$ and the derivation of the limiting control problem as ε goes to 0.

15

3   PROPERTIES OF THE VALUE FUNCTION

In this section, we present some properties of the value function $\upsilon^{\epsilon}(.)$ by using the method of viscosity solution of HJB equations. The dynamic programming equations, known as HJB equations, associated with the optimal control problem under study can be written formally as follows :

$$\rho\upsilon^{\epsilon}(\alpha,\widetilde{x}) = \min_{\widetilde{u}\in K(\alpha)}\left\{(u-d)\upsilon_{x}^{\epsilon}(\alpha,\widetilde{x}) + f(u)\upsilon_{a}^{\epsilon}(\alpha,\widetilde{x}) + G(\alpha,\widetilde{x},\widetilde{u}) + \frac{1}{\epsilon}\sum_{\beta=0}^{m}q_{\alpha\beta}\upsilon^{\epsilon}(\beta,\widetilde{x})\right\} \quad (9)$$

for all $\alpha\in\epsilon$. In what follows, we show that the value function $\upsilon^{\epsilon}(\alpha,.)$ is locally Lipschitz and is the viscosity solution of the HJB equations.

The reader is referred to Sethi et al. (1998) for some results concerning convex functions. Before we move on, we give some definitions. For any function $f(\cdot) : R \to R$, the super differential $D^{+}f(\widetilde{x})$ and the sub differential $D^{-}f(\widetilde{x})$ are defined respectively as follows :

$$D^{+}f(\widetilde{x}) = \left\{r\in R^{n+m} : \limsup_{h\to 0}\frac{f(\widetilde{x}+h)-f(\widetilde{x})-h.r}{|h|} \leq 0\right\}$$

$$D^{-}f(\widetilde{x}) = \left\{r\in R^{n+m} : \liminf_{h\to 0}\frac{f(\widetilde{x}+h)-f(\widetilde{x})-h.r}{|h|} \geq 0\right\}$$

Definition 3.1.   The value function $\upsilon^{\epsilon}(.)$ is a viscosity solution of (9) if the following conditions hold : $\upsilon^{\epsilon}(.)$ is continuous and $0 \leq \upsilon^{\epsilon}(\alpha,\widetilde{x}) \leq C\left(1+|\widetilde{x}|^{C_{k}}\right)$;

$$\forall r\in D^{+}\upsilon^{\epsilon}(\alpha,\widetilde{x}) = \left\{r\in R^{m+n} : \limsup_{h\to 0}\left[\upsilon^{\epsilon}(\alpha,\widetilde{x}+h)-\upsilon^{\epsilon}(\alpha,\widetilde{x})-\widetilde{h}.r\right]|h|^{-1} \leq 0\right\}$$

$$\rho\upsilon^{\epsilon}(\alpha,\widetilde{x}) \leq \min_{\widetilde{u}\in K(\alpha)}\left\{f(.)r + G(\alpha,\widetilde{x},\widetilde{u}) + \frac{1}{\epsilon}\sum_{\beta=0}^{m}q_{\alpha\beta}\upsilon^{\epsilon}(\beta,\widetilde{x})\right\}$$

$$\forall r \in D^- \upsilon^\epsilon(\alpha, \tilde{x}) = \left\{ r \in R^{m+n} : \liminf_{h \to 0} \left[\upsilon^\epsilon(\alpha, \tilde{x} + h) - \upsilon^\epsilon(\alpha, \tilde{x}) - \tilde{h}.r \right] |h|^{-1} \leq 0 \right\}$$

$$\rho \upsilon^\epsilon(\alpha, \tilde{x}) \geq \min_{\tilde{u} \in K(\alpha)} \left\{ f(.).r + G(\alpha, \tilde{x}, \tilde{u}) + \frac{1}{\epsilon} \sum_{\beta=0}^{m} q_{\alpha\beta} \upsilon^\epsilon(\beta, \tilde{x}) \right\}$$

where $f(.) = (u - d, f(u))'$.

A similar definition can be found in Clarke (1983). For more information and discussions on viscosity solutions, we refer the reader to Sethi et Zhand (1994a) and references therein.

Theorem 3.1. The value function $\upsilon^\epsilon(\alpha, \tilde{x}), \alpha \in \epsilon$, defined in (8), is the only viscosity solution of the dynamic programming equations (9).

Proof. The proof of this theorem can be found in Kenne (1987) and Sethi and Zhang (1994b).

Let us now present a verification theorem that provides a sufficient condition for optimal control.

Theorem 3.2. (Verification Theorem) Let

$$H(\alpha, x, \upsilon(x, \cdot), \upsilon_x(x, \alpha), u, \omega) = (u - d)\upsilon_x^\epsilon(\alpha, \tilde{x}) + G(\alpha, \tilde{x}, \tilde{u}) + \frac{1}{\epsilon} \sum_{\beta=0}^{m} q_{\alpha\beta} \upsilon^\epsilon(\beta, \tilde{x})$$

with $\upsilon(\alpha, x)$ denoting the differentiable solution of the HJB equations (9) such that :

$$0 \leq \upsilon(x, \alpha) \leq C\left(1 + |x|^{C_k}\right)$$

Then

i) $\upsilon(x, \alpha) \leq J(x, \alpha, u(\cdot), \omega(\cdot))$ for all $(u(\cdot), \omega(\cdot)) \in K(\alpha))$

ii)   If $\left(u^*(x,\alpha), \omega^*(x,\alpha)\right)$ is an admissible feedback control such that :

$$\min_{(u,\omega)\in K(\alpha)} H(\alpha,x,\upsilon(x,\cdot),\upsilon_x(x,\alpha),u,\omega)=H(\alpha,x,\upsilon(x,\cdot),\upsilon_x(x,\alpha),u_*(x,\alpha),\omega_*(x,\alpha))$$

then :

$$J\left(x,\alpha,u^*(\cdot),\omega^*(\cdot)\right)=\upsilon(x,\alpha)$$

with $\left(u^*(\cdot),\omega^*(\cdot)\right)=\left(u^*(x(\cdot),\alpha(\cdot)),\omega^*(x(\cdot),\alpha(\cdot))\right)$

Thus $\left(u^*(\cdot),\omega^*(\cdot)\right)$ is optimal

Proof. The proof of this theorem can be found in Boukas et al. (1995a).

The optimal control policy $\left(u^*(\cdot),\omega^*(\cdot)\right)$ is the solution of HJB equations described by (9). Given the large size of (9) for multiple-machine, multiple-part-type manufacturing system, one need here to proceed to the reduction of the system size through the limiting control problem.

4 LIMITING CONTROL PROBLEM

We derive here two equivalent formulations of the limiting problem in order to show that the value function of the initial stochastic problem converges to one of the related deterministic control problem. Let $\upsilon(\widetilde{x}) = \lim_{\epsilon \to 0} \upsilon^\epsilon(\alpha, \widetilde{x})$. The proposed approach is based on the stationary distribution of the stochastic process, computed here in terms of the mean values of the machines ages and preventive maintenance rates.

Let a_M and $\overline{\omega}$ denote the mean values of the machines ages and preventive maintenance rates respectively. If a_i and $\omega_i, i = 1, \cdots, m$, are replaced in the model by a_M and $\overline{\omega}$, one obtain a constant transition rates matrix $Q(a_M, \overline{\omega})$. The finite state Markov chain associated to such a matrix is then homogeneous with stationary or limiting probabilities $v = (v_1, v_2, \cdots, v_M)'$ given by :

$$vQ(.) = 0 \quad \text{and} \quad \sum_{i=1}^{M} v_i = 1 \tag{10}$$

With these probabilities, the convergence property of the initial stochastic control problem to the limiting problem when $\varepsilon \to 0$ is established in Soner (1993). Let us now discuss on the meaning and determination of the mean value of a machine.

In early works we have proposed models for the planning of the production and maintenance problem (see Kenne et al. (1997), Kenne and Gharbi (1999) and Gharbi and Kenne (2000)) related to small size manufacturing systems. In all the models proposed, we used the fact that the stock level, $x(\cdot)$, and the aging, $a(\cdot)$, are described by differential equations of the types given by (4) and (5) respectively. Based on those models, we obtained that there exist two critical age value A_i and B_i, for a machine i, $i = 1, \cdots, m$, defined as follow :

A_i The age of the machine i at which it is necessary to stock parts. Before this age, the machine is assumed to be new, and a production at the demand rate is suggested if the other machines are new.

B_i The age at which the machine i is sent to preventive maintenance when a comfortable level of inventory is achieved. In such a situation, the machine will be sent to preventive maintenance randomly (i.e., there is a random delay from the machine age B_i to the maintenance time).

By assuming that the jump rates are functions of the age $a(\cdot)$, we model correctly the fact that the more we use a machine, the more its probability to failure will increase with age. Hence, we assume herein that the mean value of a machine age a_M is between A_i and B_i.

This assumption has been verified through off-line simulation experiments. Note that negative inventories are minimised by setting a_M to A_i. However, this is not necessary the best way to select such a value. We usually resort to simulation modelling as in Pritsker et al. (1997) to determine the mean age value for a given machine.

By using equation (10) and multiplying both side of (9), at $\alpha = i$, by $v_i(.)$ and sum over $i \in \varepsilon$, we have :

$$\rho \sum_{i=0}^{m} v_i \upsilon^{\varepsilon}(i,x) = \sum_{i=0}^{m} v_i \min_{\widetilde{u} \in K(i)} \left\{ (u^i - d) \upsilon_x^{\in}(i,\widetilde{x}) + f(u^i) \upsilon_{a_M}^{\in}(i,\widetilde{x}) + G(i,\widetilde{x},\widetilde{u}^i) \right\} \qquad (11)$$

Let $\in \to 0$, one have

$$\rho \upsilon(i,\widetilde{x}) = \sum_{i=0}^{m} v_i \min_{\widetilde{u} \in K(i)} \left\{ (u^i - d) \upsilon_x^{\in}(\widetilde{x}) + f(u^i) \upsilon_{a_M}^{\in}(\widetilde{x}) + G(i,\widetilde{x},\widetilde{u}^i) \right\} \qquad (12)$$

Let us define a deterministic control process U(t) in a control space A^0 given by :

20

$$A^0 = \left\{ \begin{array}{l} U(t) = \left(\left(u^0(t), \overline{\omega}^0(t) \right), \left(u^1(t), \overline{\omega}^1(t) \right), \ldots, \left(u^m(t), \overline{\omega}^m(t) \right) \right) : 0 \le p = \sum_{p=1}^{n} \gamma_p u_p^i(t) \le i, \\ 0 \le \overline{\omega}^i(t) \le \omega_{max} \end{array} \right\} \quad (13)$$

We define the control problem P^0 as follows :

$$\min J^0\left(\tilde{x}, U(.)\right) = E \int_0^\infty e^{-\rho t} \sum_{i=0}^{m} v_i G\left(i, \tilde{x}(t), \tilde{u}^i(t)\right) dt \quad (14)$$

subject to

$$\dot{x} = \sum_{i=0}^{m} v_i u^i(t) - d \qquad x(0) = x \quad (15)$$

$$\dot{a}_M = \sum_{i=0}^{m} v_i f\left(u^i(t)\right) \qquad a_M(0) = a_M \quad (16)$$

with the value function given by

$$\upsilon(\tilde{x}) = \inf_{U(.) \in A^0} J^0\left(\tilde{x}, U(.)\right)$$

The structure of (12) suggests us another formulation of the limiting problem. This is done by replacing $Z^\epsilon(t)$, or $i \in B$, by $\overline{k}(\overline{a}_M, \overline{\omega}) = E\left[Z^\epsilon(t)\right] = \sum_{i=0}^{m} i v_i(.)$. For any $r \in R^{m+1}$, we then have

$$\sum_{i=0}^{m} v_i \min_{\tilde{u} \in K(i)} \left\{ f\left(\tilde{u}^i\right) \cdot r + G\left(i, \tilde{x}, \tilde{u}^i\right) \right\} = \min_{\tilde{u} \in K(\overline{k}(.))} \left\{ f\left(\overline{\overline{u}}\right) \cdot r + \overline{G}\left(\tilde{x}, \overline{\overline{u}} \right) \right\} \quad (17)$$

with $\overline{G}\left(x, \overline{\overline{u}}\right) = \sum_{i=0}^{m} v_i G\left(i, x, \overline{\overline{u}}\right)$ and $\overline{\overline{u}} = (\overline{u}, \omega)$ where $\overline{u} = \sum_{i=0}^{m} v_i u^i(t)$. The equation (17) is derived with the conjecture of the linearity of $f(.)$ on u. The equation (12) can then be written as

$$\rho\upsilon(\widetilde{x}) = \min_{\overline{\overline{u}} \in K(\overline{k}(.))} \left\{ (\overline{u} - d)\upsilon_x(\widetilde{x}) + k\overline{u}\upsilon_{a_M}(\widetilde{x}) + \overline{G}(\widetilde{x}, \overline{\overline{u}}) \right\} \qquad (18)$$

The alternative formulation of the limiting problem, $\widetilde{P}^0$, can then be defined by :

$$\min \widetilde{J}^0(\widetilde{x}, \overline{\overline{u}}(.)) = \int_0^\infty e^{-\rho t}\overline{G}(\widetilde{x}, \overline{\overline{u}})dt \qquad (19)$$

subject to

$$\dot{x} = \overline{u}(t) - d \qquad x(0) = x \qquad (20)$$

$$\dot{a}_M = k\overline{u} \qquad a_M(0) = a_M \qquad (21)$$

with the value function given by

$$\widetilde{\upsilon}(\widetilde{x}) = \inf_{\overline{\overline{u}} \in \widetilde{A}^0} \widetilde{J}^0(\widetilde{x}, \overline{\overline{u}}(.)) \qquad (22)$$

The value function $\widetilde{\upsilon}(\widetilde{x})$ is equivalent to $\upsilon(\widetilde{x})$ and can be shown to be the only viscosity solution of (18) (cf. theorem 3.1). It is interesting to note that the obtained limiting problem does not depend on the explicit form of $Q(.)$, only on $\overline{k}(a, \omega)$, which is the mean machine availability. This makes the limiting problem very simpler to solve.

Given that the previous equations was obtained for given $\overline{\omega}$, the overall optimization problem can be described by the following deterministic HJB equations :

$$\rho\upsilon(\widetilde{x}) = \min_{0 \le \overline{\omega} \le k\omega_{max}} \left\{ \min_{\overline{u} \in K(\overline{k}(.))} [(\overline{u} - d)\upsilon_x(\widetilde{x})] + \frac{1}{m}\left(\sum_{j=1}^n k_j d_j\right)\upsilon_{a_M}(\widetilde{x}) + \overline{G}(\widetilde{x}, \overline{\omega}) \right\} \qquad (23)$$

Based on the structure of the stochastic control presented in Boukas and Haurie (1990) and Kenne (1997), we provide a machine age dependent control policy through the solution of (23). The following method is applied to build such a policy.

22

- Consider an equivalent machine and let $\bar{a}_m$ be its age mean value, called here equivalent machine mean age.
- Select δ and a_M to define the following set of equivalent machine mean age values.

$$\Omega = \{\bar{a}_m : a_M - \delta \le \bar{a}_m \le a_M + \delta\} \tag{24}$$

- How to determine machine age dependent preventive maintenance and production rates

(i) For any $a_i(t) \in \Omega, i = 1, \cdots, m$, set the maintenance rate to the one of the limiting problem if there is a significant stock level. The production rate is then set to its maximal value.

(ii) For any $a_i(t) < a_M - \delta, i = 1, \cdots, m$, the machine i is supposed to be new. Hence, the preventive maintenance rate is set to zero and the production rate is set to the demand rate.

(iii) For any $a_i(t) > a_M + \delta, i = 1, \cdots, m$, the machine i is supposed to be old. The preventive maintenance and production rates are set to their maximal values.

Such a method is used in the next section to determine the age dependent control policy for the limiting problem.

5   HEURISTICAL OPTIMAL FEEDBACK CONTROL

From the solution of the limiting problem, it has been shown in Lehoczky et al. (1991) how to construct a near-optimal control policy of the original stochastic control problem in the case of homogeneous Markov processes. With such a process, the control policy consists of the production rate and is approximated in Lehoczky et al. (1991) by the following theorem :

Theorem 5.1. If $\overline{u}^*(.)$ is the optimal production rate of the limiting problem related to the homogeneous Markov process, one can approximate the optimal production rate related to the stochastic original problem by the following equation :

$$u(\widetilde{x},l) = l\frac{\overline{u}^*(\widetilde{x})}{\overline{v}(.)} \qquad 0 \leq l \leq m \tag{25}$$

where l is the number of operational machines. The production rate of the operational machine i at mode α is given by :

$$u_i(\widetilde{x},\alpha) = \frac{u(\widetilde{x},l)}{\alpha} \qquad 0 \leq \alpha \leq m \tag{26}$$

Proof. The proof of this theorem is presented in Lehoczky et al. (1991).

This theorem states that the production rate of a FMS is equally distributed to operational machines at a given time. The proposal of this paper is mainly based on the machine age and control dependent dynamics of the machines modes (i.e., non homogeneous Markov process). Thus initial identical machines become different while producing due to the fact that their age's trajectories are obviously different. With such an observation, theorem 5.1 cannot be generally valid for the construction of a stochastic near-optimal control when the age is introduced in the model.

Given that the control policy is age dependent in the case considered herein, we first proceed with the numerical solution of (23) for a two-machine, one-product system $(m = 2$ and $n = 1)$, to illustrate the structure of the optimal stochastic control policy. From such a structure, we will next derive and heuristical method for the construction of a near-optimal control policy for a large class of FMS. The solution of (23) for large systems can be easily determined, as in section 6, using numerical techniques. From the control policy of the two-machine, one-product FMS, derived from a numerical solution of equation (23) and the related age dependent control policy, it is interesting to observe the following :

- When there is a backlog (i.e., the surplus $x(t) < 0$), the production rates are set to their maximal values and the preventive maintenance rates are set to zero for both machines;

- When $x(t) = 0$, the production rate of the FMS is either equal to the demand rate for small machines age's values or greater than the demand rate for large machines age's values.

- When there is an inventory (i.e., $x(t) > 0$), the machine with a higher age is frequently used compared with the new one. We note that the preventive maintenance domain decreases when the machines ages increase.

Based on these results, we propose the following heuristical method for the construction of near-optimal control policy for the original stochastic control problem.

Step 1. Consider a *q*-finite partition C_0, C_1, …, C_q which consists of q + 1 sets of machines ages values such that the dynamics of a machine age moves from C_i to C_{i+1} and from C_k, $(k \neq 0)$ to C_0 after each intervention on the machine (repair or maintenance). The partition is such that :

$$\bigcup_{i=0}^{q+1} C_i = \wedge \quad \text{and} \quad \bigcap_{i=0}^{q+1} C_i = \emptyset \tag{27}$$

25

where $\wedge = \{a_i : a_i = (i-1)h_a\}$, i = 1, ..., N_a for given N_a and h_a. Note that $\wedge$ is a one-dimension grid of N_a points generated by h_a. An example of partition is illustrated in figure 2 for q = 1 in the case of two-machine FMS.
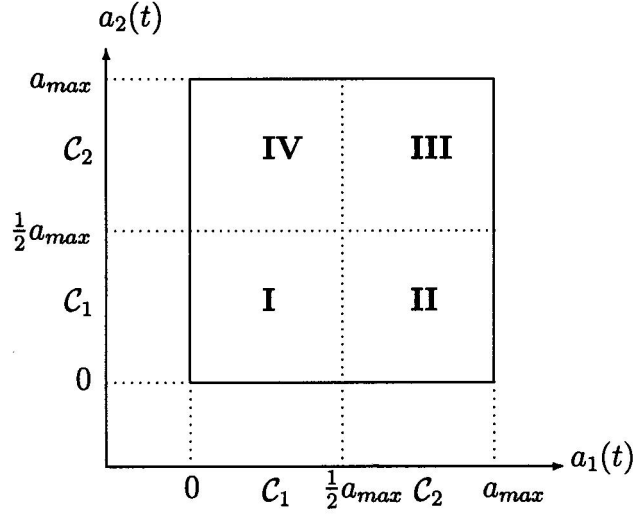

Figure 2. Classes of operational machines

Step 2. At a given time, group operational machines in classes and apply theorem 5.1 (equations (25) and (26)) for machines in the same class. A class here correspond to a set C_k, $0 \le k \le q$, of the partition mentioned in step 1. It is assumed here that machines in the same class are identical.

Step 3. Use frequently old machines such that the sum of production rates remains close to the solution of the limiting problem when the involved machines are in different classes. The production rates $u(\alpha, \tilde{x})$ of the FMS still given by the equation (25). When $x(t) > 0$ and $\overline{\overline{u}}(.) \ne 0$, one can distribute $u(\alpha, \tilde{x})$ to operational machines using the following steps :

26

(i)   For a given mode α, classify the operational machines through a decreasing order (beginning with the oldest machine, followed with the closely second, and so on). Let O denote the set of ordered machines.

(ii)   Set a maximal value to the production rate of the oldest machine and assign sequentially the maximal production rate or the demand rate to other machines until the sum of assigned production rates reaches the production rate $\tilde{u}(\alpha, \tilde{x})$. Assign zero to the production rates of the remaining machines.

(iii)   Repeat steps (i) and (ii) for all $\alpha \in \{1, \cdots, m\}$.

Step 4.   At the mode $\alpha = m$ (i.e., all machines are operational), the preventive maintenance policy of the stochastic problem can be approximated by assigning maximal preventive maintenance rate to the oldest machine and compare the sum of assigned preventive maintenance rates to $\omega(\alpha, \tilde{x})$. Continue with the assignment process until the sum of assigned rates reaches $\omega(\alpha, \tilde{x})$ and then assign 0 to the preventive maintenance rates of the remaining machines.

We will use this method in the next section in order to construct a near-optimal control policy for the numerical example developed in that section.

6   EXPERIMENTATION OF THE PROPOSED APPROACH

We give a two-machine two-part-type model as an example for the numerical experiment (m = 2 and n = 2). With such a model, we have :

$$\dot{x}_j = u_{1j}(t) + u_{2j}(t) - d_j \quad x_j(0) = x_j, \quad j = 1,2 \tag{28}$$

$$\dot{a}_i = k_a \left(u_{i1}(t) + u_{i2}(t) \right)_j \quad a_i(0) = a_i, \quad i = 1,2 \tag{29}$$

In this case we have $\zeta^\in(t) \in B = \{1,2,\cdots,9\}$ and $Z(t) \in \varepsilon = \{0,1,2\}$. The transition rates q_{12}^i depend on the machine age and are approximated by the equation :

$$q_{12}^i(.) = q_o + q_a \times a_i(t), \quad i = 1, 2 \tag{30}$$

for given q_o and q_a. The average number of operational machines is given by :

$$\bar{v}(.) = E[Z(t)] = 0 \times p_0 + 1 \times p_1 + 2 \times p_2 \tag{31}$$

with $p_0 = v_5 + v_6 + v_8 + v_9, p_1 = v_2 + v_3 + v_4 + v_7$ and $p_1 = v_1$. The considered control problem is feasible if the following condition holds :

$$\bar{v}(.) - d \geq 0 \tag{32}$$

The average cost function derived from equation (6) is given by :

$$\overline{G}(s,u,\omega) = c^+ x^+ + c^- x^- + c^2 (v_2 + v_4 + 2v_5 + v_6 + v_8) + c^3 (v_3 + v_6 + v_7 + v_8 + 2v_9) \tag{33}$$

where c^2 and c^3 represent repair and maintenance cost activities respectively.

Equation (19) is subject here to the following state constraints :

$$\dot{x}_1(t) = \overline{u}_1(t) - d_1 \qquad x_1(0) = x_1 \tag{34}$$

$$\dot{x}_2(t) = \overline{u}_2(t) - d_2 \qquad x_2(0) = x_2 \tag{35}$$

$$\dot{a}_M = k\frac{1}{2}(k_1 a_1 + k_2 a_2) \quad a_M(0) = a_M \tag{36}$$

where a_M is the mean value of the machines ages and k_1 and k_2 are given constants. The HJB equations (23) can be rewritten as follows :

$$\rho\upsilon(\tilde{x}) = \min_{0 \le \overline{\omega} \le k\omega_{max}} \left\{ \min_{\overline{u} \in K(k)} \{(\overline{u}_1 - d_1)\upsilon_{x1}(\tilde{x}) + (\overline{u}_2 - d_2)\upsilon_{x2}(\tilde{x}) \right.$$
$$\left. + \frac{1}{2}(k_1 d_1 + k_2 d_2)\upsilon_{a_M}(\tilde{x}) + \overline{G}(x, a_M, \overline{\omega}) \} \right\} \tag{37}$$

Let us now apply numerical methods to equation (37) in order to define the control policy of the limiting problem $(i.e., (\overline{u}_1, \overline{u}_2, \overline{\omega}))$.

By approximating partial derivatives in equation (37), we obtain, after a couple of manipulations, the following equation :

$$\upsilon_h(x_1, x_2, a_m) = \min_{0 \le \overline{\omega} \le k\omega_{max}} \left\{ \min_{\overline{u} \in G_{\overline{k}}^h} \left\{ \begin{bmatrix} \dfrac{c^+ x^+ c^- x^- + \sum_{\alpha=0}^{2} \mu_\alpha c^\alpha}{Q_h'(.)\left[1 + \dfrac{\rho}{Q_h'(.)}\right]} + \dfrac{1}{\left[1 + \dfrac{\rho}{Q_h'(.)}\right]} \\ \begin{bmatrix} P_{h_{x1}}(\tilde{x}, x_1 + h_{x1})\upsilon_h(x_1 + h_{x1}, x_2, a_M) \\ P_{h_{x2}}(\tilde{x}, x_2 + h_{x2})\upsilon_h(x_1, x_2 + h_{x2}, a_M) \\ + P_{h_{x1}}(\tilde{x}, x_1 - h_{x1})\upsilon_h(x_1 - h_{x1}, x_2, a_M) \\ + P_{h_{x2}}(\tilde{x}, x_2 + h_{x2})\upsilon_h(x_1, x_2 - h_{x2}, a_M) \\ + P_{h_a}(\tilde{x}, a_M + h_a)\upsilon_h(x_1, x_2, a_M + h_a) \end{bmatrix} \end{bmatrix} \right\} \right\} \tag{38}$$

where h_{x1}, h_{x2} and h_a are discrete increments associated to state variables x_1, x_2 and a_m respectively.

In addition, we have :

$$Q'_h(.) = \frac{k_1 d_1 + k_2 d_2}{2h_a} + \frac{|\bar{u}_1 - d_1|}{h_{x1}} + \frac{|\bar{u}_2 - d_2|}{h_{x2}}$$

$$P_{h_{x1}}(\tilde{x}, x_1 + h_{x1}) = \begin{cases} \dfrac{\bar{u}_1 - d_1}{h_{x1} Q'_h(.)} & si\ \bar{u}_1 - d_1 \geq 0 \\ 0 & otherwise \end{cases}$$

$$P_{h_{x2}}(\tilde{x}, x_2 + h_{x2}) = \begin{cases} \dfrac{\bar{u}_2 - d_2}{h_{x2} Q'_h(.)} & si\ \bar{u}_2 - d_2 \geq 0 \\ 0 & otherwise \end{cases}$$

$$P_{h_{x1}}(\tilde{x}, x_1 - h_{x1}) = \begin{cases} \dfrac{d_1 - \bar{u}_1}{h_{x1} Q'_h(.)} & si\ \bar{u}_1 - d_1 < 0 \\ 0 & otherwise \end{cases}$$

$$P_{h_{x2}}(\tilde{x}, x_2 - h_{x2}) = \begin{cases} \dfrac{d_2 - \bar{u}_2}{h_{x2} Q'_h(.)} & si\ \bar{u}_2 - d_2 < 0 \\ 0 & otherwise \end{cases}$$

$$P_{h_a}(\tilde{x}, a_M + h_a) = \frac{k_1 d_1 + k_2 d_2}{2h_a Q'_h(.)}$$

Equation (38) is solved numerically using the following values :

1) $h_{x1} = h_{x2} = h_a = 0{,}2$, $x_1, x_2 \in [-2, 2]$, $a_M \in [0, 6]$ and $u_{max}^{ij} = 1$.

2)   Instantaneous cost (see table II)

Table II. Instantaneous cost parameters (two machines, two products)

c_1^+	c_1^-	c_2^+	c_2^-	$\bar{c}^0$	$\bar{c}^1$	$\bar{c}^2$	$\bar{c}^3$	$\bar{c}^4, \bar{c}^5$	$\bar{c}^6$	$\bar{c}^7$	$\bar{c}^8$	$\bar{c}^9$	k_1	k_2
1	10	1.1	10.2	0	10	1	20	10	1	11	11	2	2	2

3) $d_1 = d_2 = 1{,}6,\ \rho = 0{,}005,\ q_{12}(a_i(t)) = A_0 + K_a a_i(t),$

       $i = 1,\ 2$ with $A_0 = 0{,}0001$ and $K_a = 0{,}005$

4) $q_{21} = 0{,}1$ and $q_{31} = 0{,}25$

With those data, the obtained control policy $\bar{u}_1, \bar{u}_2, \bar{\omega}$ is presented by figures 3, 4 and 5.

It is interesting to observe from the production policy that when $(c_2^+ > c_1^+$ and $c_2^- > c_1^-)$, machines are assigned to product P_1 if $x_2 \geq 0$ (see figure 3). From figure 4, the production of P_2 is maximal for $x_2 < 0$. A production at the maximal rate is recommended for $x_2 \geq 0$. This is also valid near $x_2 = 0, x_1 > 0$ in order to built an inventory. The production rate of P_2 is then set to the demand rate for $x_1 \geq 1$ and $x_2 = 0{,}2$.

Note that for $a_m = 3.2$ and $\bar{\omega} = 0{,}04,$ the average of the maximal capacity of the system is 1,84 (see figures 3 and 4). Figure (5) shows that the preventive maintenance rate of the limiting problem is different from zero for significant stock levels (i.e., $x_1 > 0{,}6$ et $x_2 > 0{,}6$).

Let us now apply the heuristic method presented in section 5 to construct a machine age dependent control policy for the limiting problem. Figures 6 and 7 show that, for new machines, there is no need to built inventory. Conversely, significant stock levels are recommended for old machines (see figures 8 and 9).

From figure 10, note that the preventive maintenance rate is set to zero for small machine age values such as $a_1 = 1$ (i.e., the machine is supposed to be new). The situation is different for $a = 5$, given that a preventive maintenance must be done for

31

comfortable stock levels such as $x_2 > 0{,}8$ et $x_1 > 0{,}6$. This is clearly explained by figure 11.

The previous age dependent control policy of the limiting problem has been successfully used to construct the stochastic optimal control of the original problem through the procedure described in section 5. We provide here a set of figures to illustrate the main properties of the control policy related to the state variables a_1, a_2, x_1 and x_2 (see figures 12 to 17). Such a policy is the expansion of the machine age dependent limiting control policy obtained from the proposed heuristical control approach. It appears from the obtained control policy (figure 12 to 17) that, for a machine age greater than A_i, the structure of the optimal production policy could be given by :

$$u_j(\widetilde{x}) = \begin{cases} u^j_{max} & si \quad x_j(t) < X_j \\ d_j & si \quad x_j(t) = X_j \\ 0 & si \quad x_j(t) > X_j \end{cases} \tag{39}$$

where X_j is the threshold value of product j and $u^j_{max} = \sum_{i=1}^{m} u^{ij}_{max}$ with u^{ij}_{max} describing the maximal production rate of product j on machine i. Recall that A_i is the age of the machine I at which it is necessary to stock parts. The optimal machine age dependent preventive maintenance policy depends on a machine a B_i at which machine I is sent to preventive maintenance when threshold values X_j, $j = 1, \cdots, n$, are achieved.
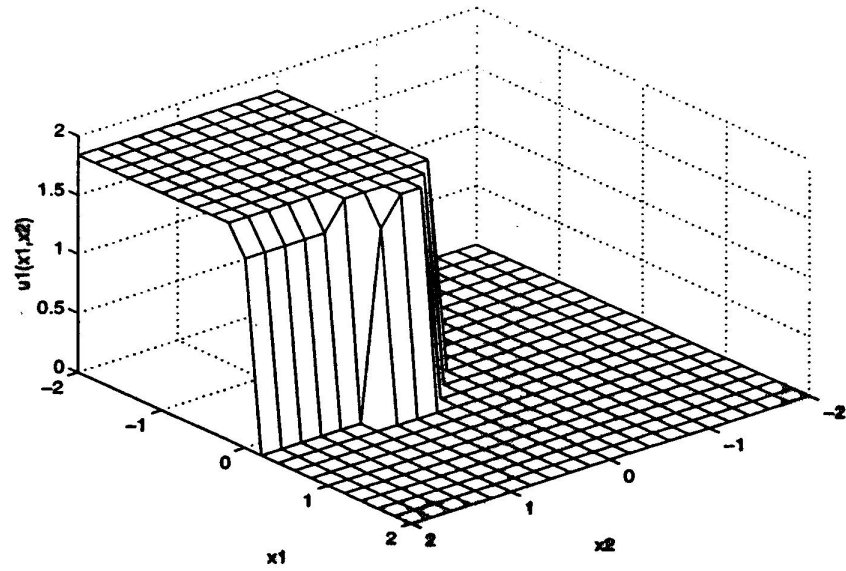
Figure 3: Production rate $\bar{u}_1(x_1, x_2)$ for product 1
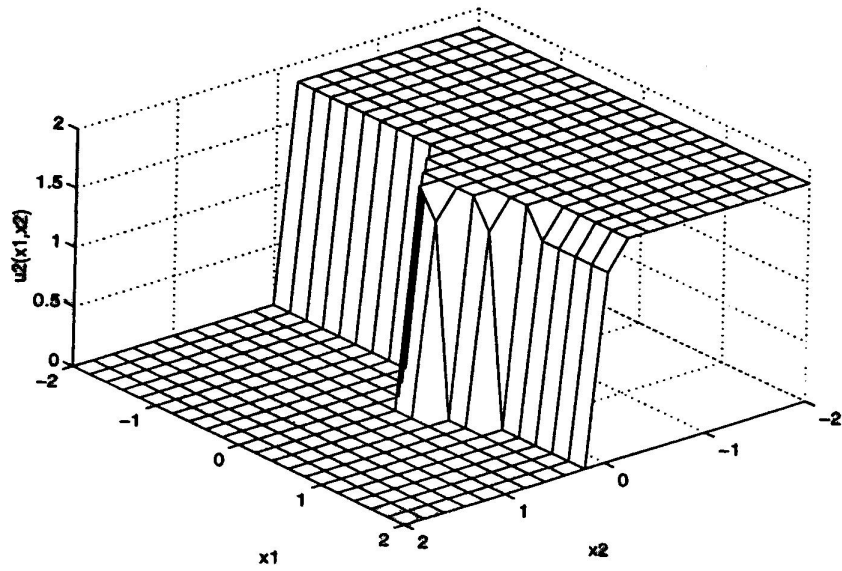
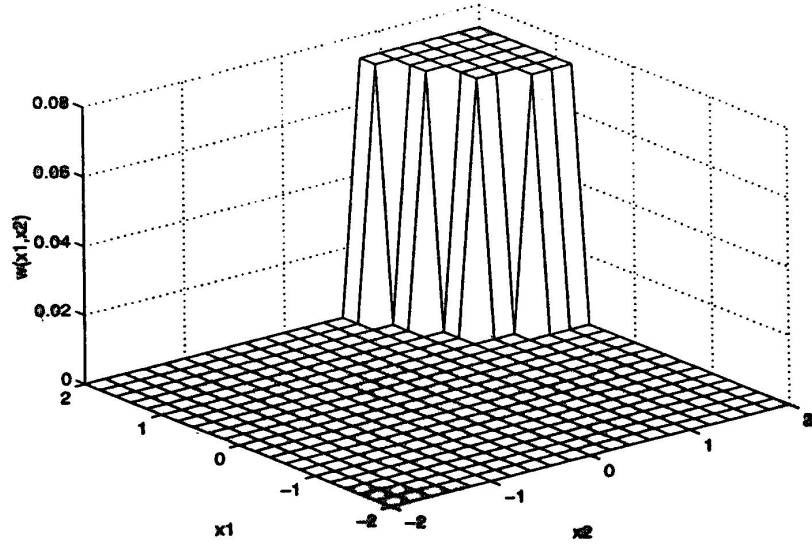Figure 4: Production rate $\bar{u}_2(x_1, x_2)$ for product 2

Figure 5: Preventive maintenance rate $\bar{\omega}(x_1, x_2)$ for the limiting problem
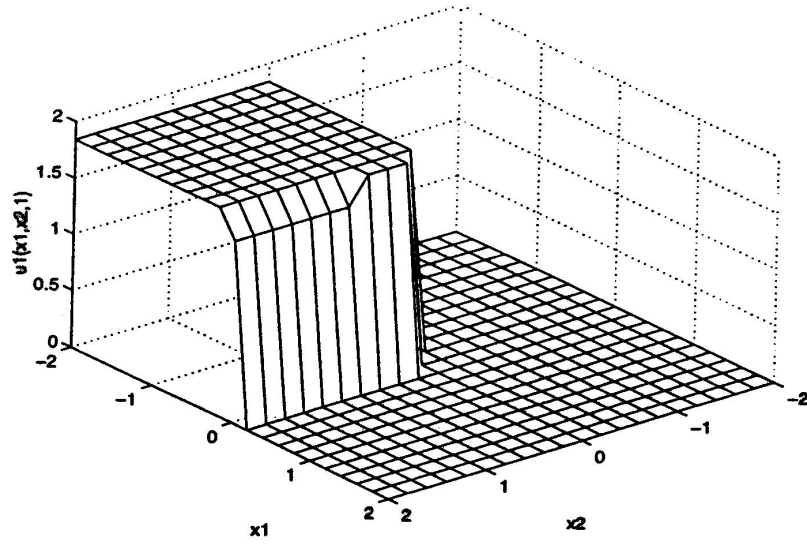

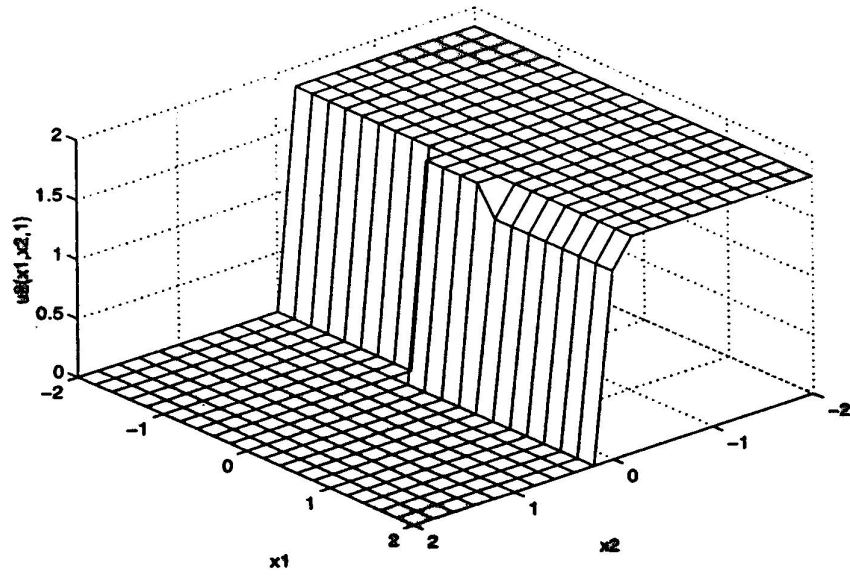Figure 6: Production $\bar{u}_1(x_1, x_2, 1)$ for product 1

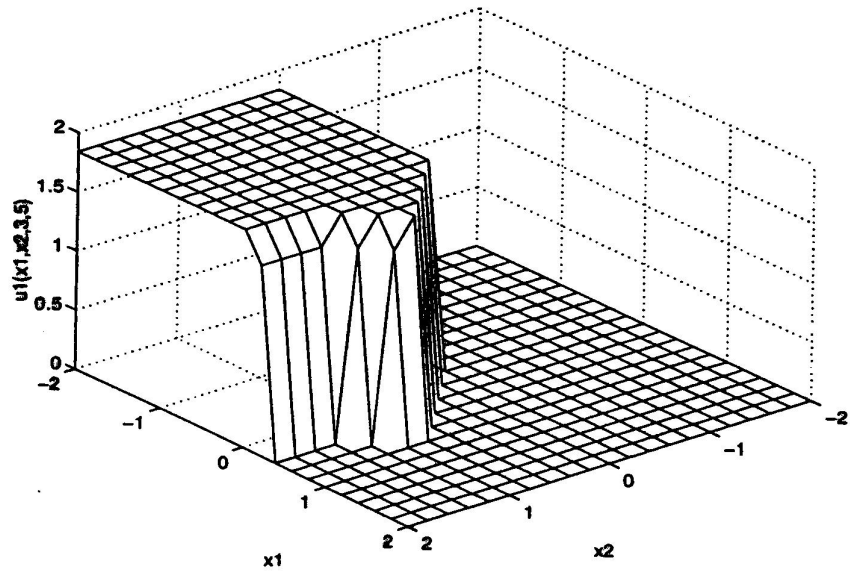Figure 7: Production rate $\bar{u}_2(x_1, x_2, 1)$ for product 2

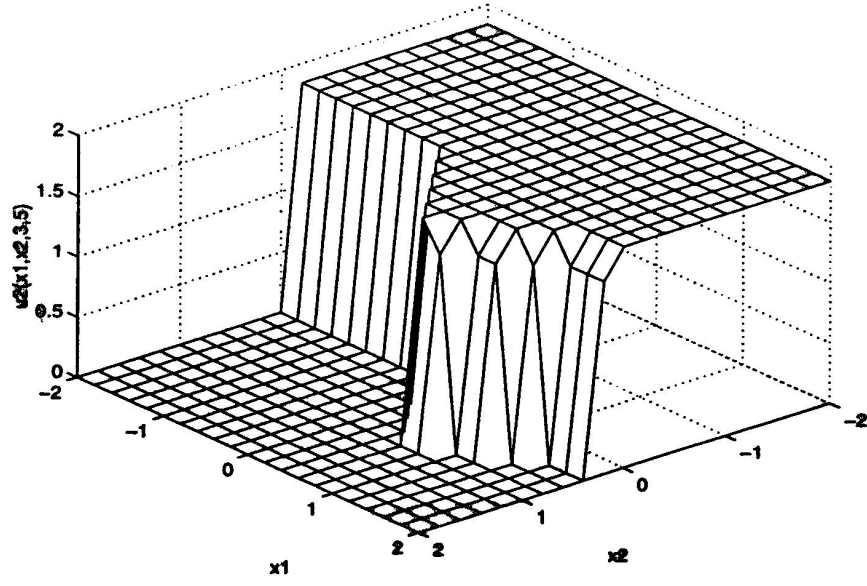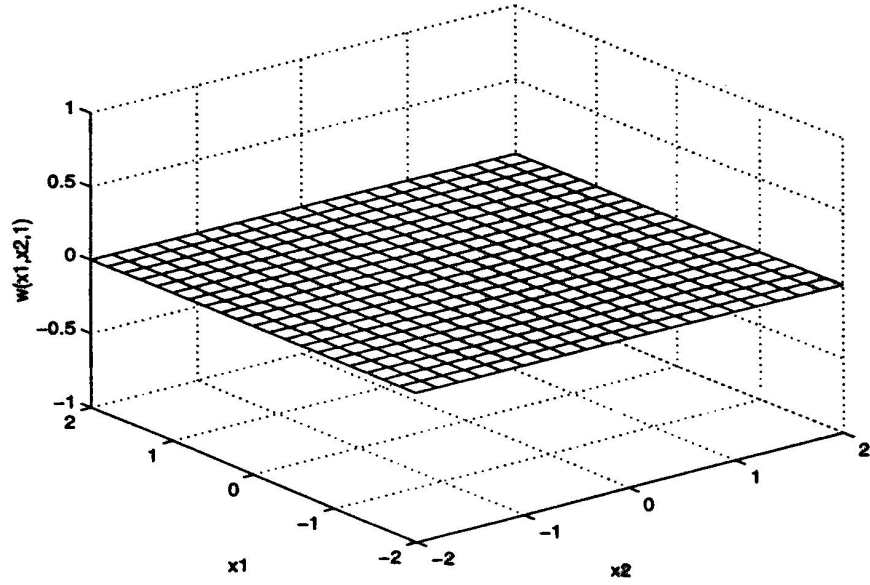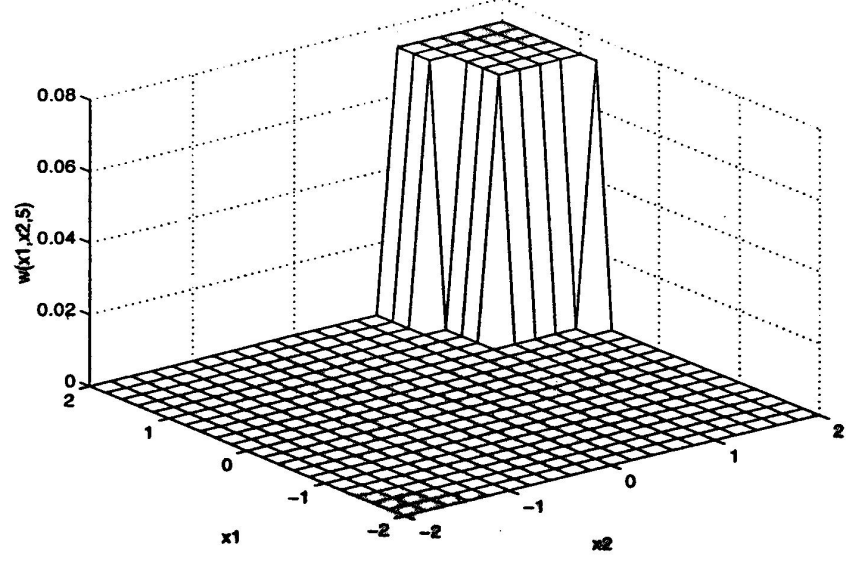Figure 8: Production rate $\bar{u}_1(x_1, x_2, 5)$ for product 1

35

Figure 9: Production rate $\bar{u}_2(x_1, x_2, 5)$ for product 2

Figure 10: Preventive maintenance rate $\bar{\omega}(x_1, x_2, 1)$ for the limiting problem

36

Figure 11: Preventive maintenance rate $\bar{\omega}(x_1, x_2, 5)$ for the limiting problem
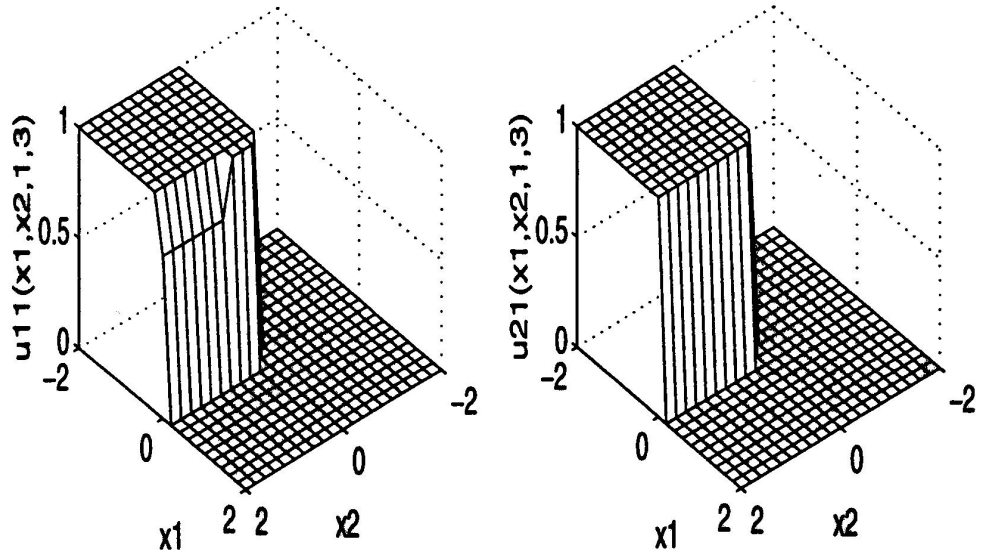

Figure 12: Machines production rates at mode 1 for $a_1 = 1$ and $a_2 = 3$ (product 1)
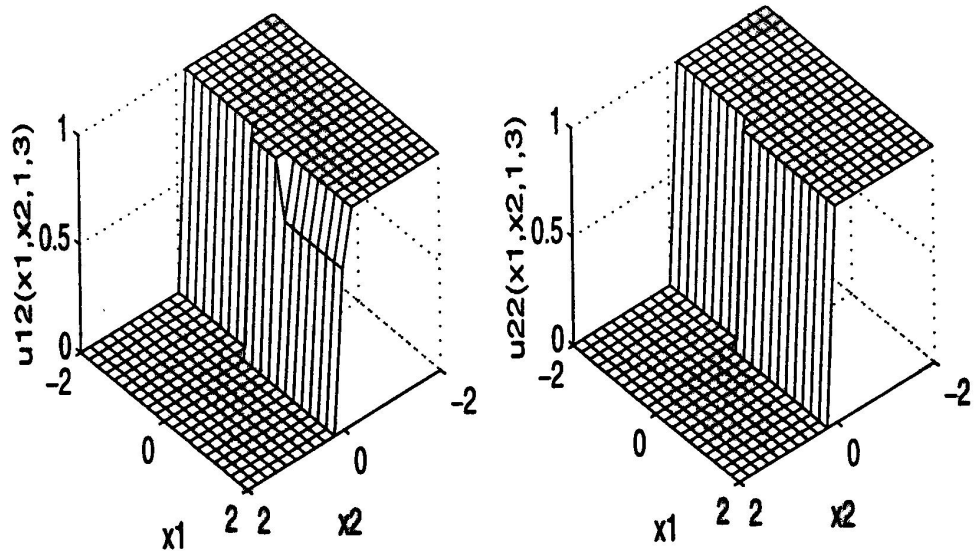
Figure 13: Machines production rates at mode 1 for $a_1 = 1$ and $a_2 = 3$ (product 2)
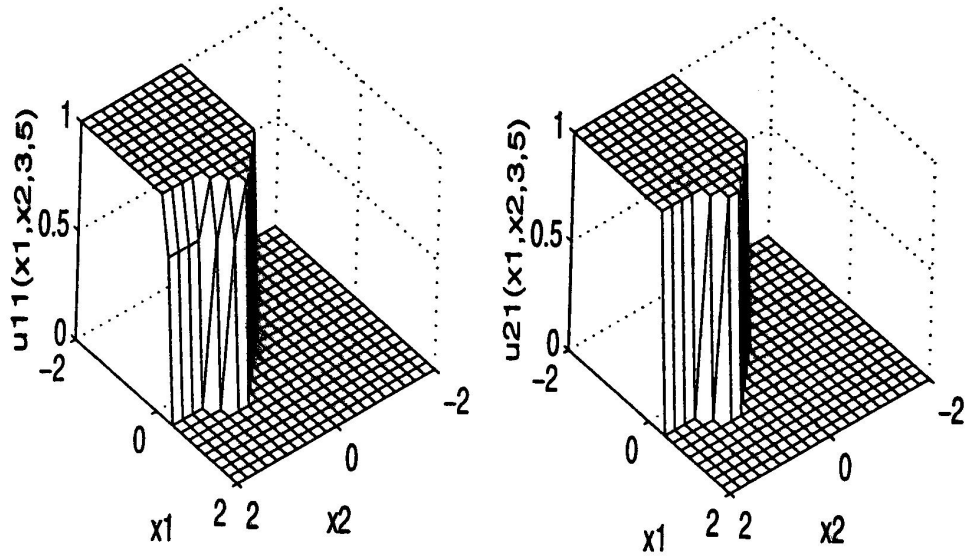

Figure 14: Machines production rates at mode 1 for $a_1 = 3$ and $a_2 = 5$ (product 1)
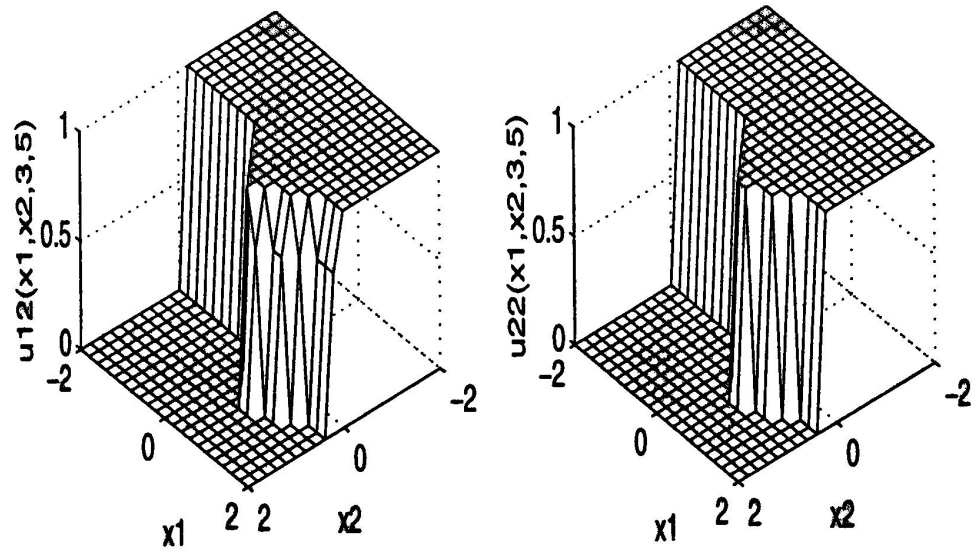
Figure 15: Machines production rates at mode 1 for $a_1 = 3$ and $a_2 = 5$ (product 2)
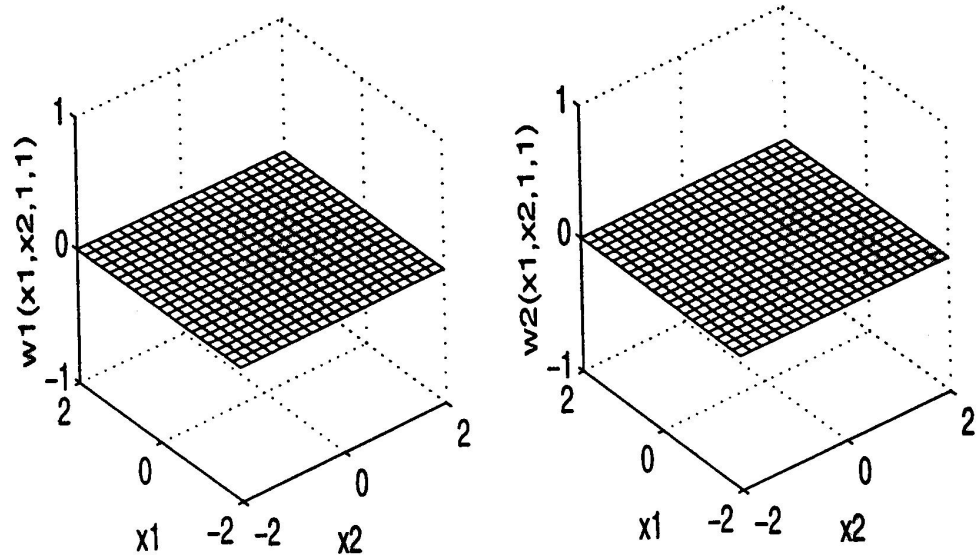

Figure 16: Machines preventive maintenance rates at mode 1 for $a_1 = 1$ et $a_2 = 3$
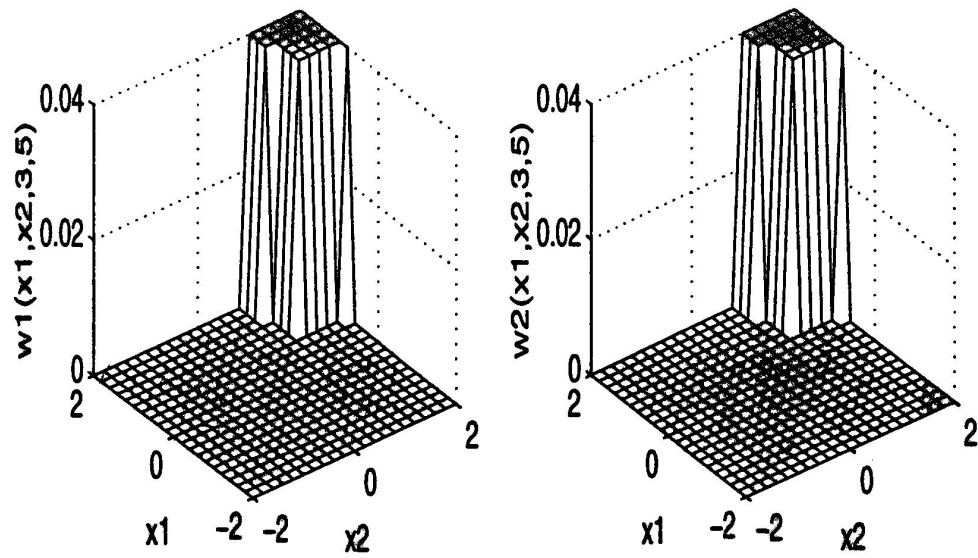
Figure 17: Machines preventive maintenance rates at mode 1 for $a_1 = 3$ et $a_2 = 5$

7 CONCLUSION

In this paper, we present the singular perturbation formulation of the production and preventive maintenance rates planning problem in manufacturing systems. We consider two times scale processes related to the rate at which the cost is discounted and the one at which machines failures occur. The obtained limiting problem has been shown to be more tractable numerically. This result can be very useful because of the large dimension of control problem in real world flexible manufacturing system. In such a system one can use the proposed approach to approximate the solution of the stochastic problem through the limiting problem concept. The quality of the results (their precision) achieved with the proposed heuristical method depends on the choice of the partition. A best approximation of the stochastic optimal control is ideally given by a partition with a large number of sets. But there is a practical limit with such a number. With a suitable number of partition sets, the proposed heuristical method, combined with the proposed hierarchical control approach extend the application of the singular perturbation method to non-homogeneous Markov processes (machine age and control dependent). The usefulness of the proposed approach is illustrated by the numerical example presented in this paper.

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