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# Joint production and setup control policies of unreliable manufacturing systems minimizing the incurred total cost and taking into account other criteria

M. Assid [a], A. Gharbi [a] , A. Hajji [b]

[a] *Automated Production Engineering Department, École de technologie supérieure, Production System Design and Control Laboratory, University of Québec, 1100 Notre Dame Street West, Montreal, QC, Canada H3C 1K3*

[b] *Department of Operations and Decision Systems & CIRRELT. Laval University. Pavillon Palasis-Prince 2325, rue de la Terrasse Québec (Québec) G1V 0A6*

## Abstract

This paper addresses the production and setup control problem in unreliable multi-products manufacturing system. Several decision criteria are considered in order to conduct an exhaustive comparative study of the two most complete control policies in the literature. The objective is to propose a production and setup control policy for the system under review.

The first part of this work consists in analyzing the effect of the system parameters variation on the difference between the total costs of the two policies studied. The best control policy in term of cost will then be determined. In the second part, two keys performance indicator (KPI): the cost and the customer satisfaction rate are simultaneously considered. The goal is to optimize the parameters of the policies studied, which minimize the total cost incurred while respecting the customer satisfaction constraint. A discussion on the best control policy is conducted based on the cost and the customer satisfaction. An experimental resolution approach is used. It integrates combined discrete-continuous simulation models with statistical techniques of optimization such as design of experiments, analysis of variance and response surface methodology. Finally, a discussion is conducted on the effects of other quantitative and qualitative criteria in order to determine the best control policy and to reach the best concerns of the company's decision makers. These decision criteria are generally related to the storage space required constraint, the setups complexity, the implementation issue, the complexity of the optimal control problem.

**Keywords:** Production control, setup, service level, multiple criteria, simulation modeling, optimization, response surface methodology.



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## 1. Introduction

In the research field of unreliable manufacturing systems control, the concept of hedging point policy (HPP) is one important basis on which several approaches have been developed. For the one machine - one product ($M_1P_1$) unreliable manufacturing systems, Kimemia and Gershwin [1] and Akella and Kumar [2] showed that HPP is optimal. This policy consists of building up a safety stock (threshold) during periods of excess machine capacity to hedge against future possible inventory shortage caused by machine failures. For systems with multiple machines and multiple product types, Sajadi et al. [3] proposed a production control model which determines the production rate taking into account several restrictions of the feeding materials. Caramanis and Sharifnia [4] used a decomposition method in order to propose a suboptimal control policy. They transformed a complex control problem with several part types ($M_1P_n$) into several simpler control problems with one part type ($M_1P_1$) which can be treated analytically. Sethi and Zhang [5] presented an explicit formulation of the optimal control problem of a production system which consists of a single machine capable of producing several part types with negligible setup times and costs. However, this class of manufacturing system isn't representative and systems which require setup time and cost still exist in many industrial sectors. Among the authors who have addressed this category of manufacturing systems in a dynamic stochastic context, Yan and Zhang [6] developed the optimality conditions described by the Hamilton-Jacobi Bellman (HJB) equations for a single machine two part types manufacturing system. Given that an analytical solution of these equations is generally complex to find, the authors have applied a numerical method based on the Kushner approach [7] in order to develop a structure of the suboptimal control policy. Many other authors used this approach to determine the formulation of control policies in deferent industrial contexts, as in Rivera-Gomez et al. [8]. The latter develop a numerical methods based on the Kushner approach in order to determine a new control model in the case of an unreliable manufacturing system subject to quality and reliability deterioration. The system consists of a single machine subject to random failures and deterioration.

In the literature, few authors have addressed the significant impact that a non negligible setup time and cost have on the manufacturing company competitiveness. Among others, Elhafsi and Bai [9] developed and solved a mathematical model of a one-machine, two-product manufacturing system. The study's goal is to control the production rate and the setup time depending on the system state. In the industrial context, the setup actions generate significant losses that should be minimized such as those related to the machine operating cost, to the time recorded by the human resources and to the indispensable time between the start of a setup operation and the start of the production. The objective is to develop a global and effective structure of setup operations which consider all the interactions between the partial system costs, such as the production cost, inventory cost, backlog cost, etc.

All research quoted above use the cost as the sole deciding criterion. However, other criteria may influence the manager decisions when it comes to develop a production strategy in an environment governed by quantitative and / or qualitative constraints. For example, for systems considering maintenance strategies, Barlow and Hunter [10] presented two preventive maintenance policies: the age replacement policy (ARP) and the block replacement policy (BRP). From an economic standpoint, Barlow and Proschan [11] have shown that ARP is better than BRP. Furthermore, BRP is wasteful given the possibility of replacing new components, when the PM actions occur just after the repair actions. However, BRP is widely used because it is more practical to implement and to manage in an industrial context than that age type (ARP), since it does not require continuous tracking of the equipment utilization time.

Other authors have considered the production equipments availability as a performance criterion [12, 13]. Boulet et al. [14] considered a corrective and preventive maintenance model, which take into account two decision criteria: the availability and the cost. Using a multi-criteria optimization approach based on a desirability function, they optimized simultaneously the total cost incurred and the production equipment availability for a system subject to random breakdowns and repairs. Yulan et al. [15] used a Multi-objective genetic algorithm in order to optimize the integrated problem of PM and production scheduling for a single machine. Five objectives were simultaneously considered, including minimizing the maintenance cost, makespan, total weighted completion time of jobs, total weighted tardiness, and maximizing machine availability. In the same way, Dror et al. [16] addressed a multi-objective problem of production and subcontracting planning. They proposed a production control policy that helps to minimize safety stock and storage needs while ensuring high levels of customer service. It is much more interesting to consider several criteria all at once in order to deal with the concerns of decision makers. They are constantly seeking efficiency improvements for their manufacturing processes, so as to minimize costs and to be more responsive to market needs in terms of customer service, delivery deadlines, quality, etc.

Based on the scientific literature, we adopt the most complete production and setup control policies which were developed for unreliable manufacturing systems characterized by non-negligible setup time and cost. The aim is to recommend and / or to improve a production and setup policy which may be applicable in a real industrial context. It must also be efficient in terms of cost, customer service and other criteria that are considered important by decision makers. The first control policy adopted is proposed by Bai and Elhafsi [17]. It's called the Hedging Corridor Policy (HCP). The second one is the solution proposed by Gharbi et al. [18] who extended the results of Bai and Elhafsi [17] by proposing a near-optimal control policy called the Modified Hedging Corridor Policy (MHCP).

This article represents a straight continuation of the work of Bai and Elhafsi [17], Hajji et al. [19] and Gharbi et al. [18]. The latter showed that MHCP is better than HCP in terms of cost. However, only five configurations were considered. In addition, the improvement obtained by MHCP in comparison with HCP is subject to fluctuation. This situation gives no details concerning the influence of the system parameters on the control policies studied, and leads us to wonder if the total cost of MHCP can never actually exceed HCP cost incurred. We believe that these experiences are not sufficient and it is clear that other comparative analysis would be needed to draw meaningful conclusions. After the description of the system and the considered control policies in the Section 2, Section 3 presents a comparative study between the control policies HCP and MHCP based on the cost criterion. This study analyzes the effect of a wide range of system configurations on the cost difference of the two policies. Section 4 deals with two decision criteria simultaneously: the cost and the customer satisfaction in order to enable decision makers to make more informed decisions. To the best of our knowledge, no other research has considered these two criteria simultaneously with non-negligible setup time and cost. The relationship between the customer service level and the cost difference of the control policies studied is also analyzed. In section 5, a discussion is conducted on the influence of certain quantitative and qualitative criteria on the choice of the best joint production and setup policy. The paper is concluded in section 6.

## 2. Description of the system and the control policies studied

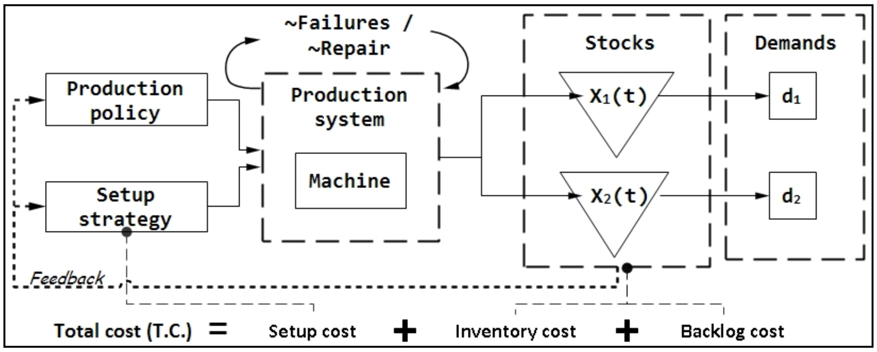



Figure 1. The considered manufacturing system

The considered manufacturing system consists of a non-flexible machine capable of producing two part types. This machine is subject to random breakdowns and repairs that can generate stock outs. To change the production from one product type to another a non-negligible setup time and cost are required. Figure 1 describes the structure of the manufacturing system considered.

### 2.1. Notation

For any $i \in I = \{1,2\}$, we use the following notation throughout this paper.

$\mathbf{P_i}$ : Type of product $i \in I$

$\mathbf{x_i(t)}$ : Inventory level (or backlog) of product $P_i$, $i \in I$, at time t

$\mathbf{d_i}$ : Constant demand rate of the product $P_i$, $i \in I$

$\mathbf{u_i}$ : Production rate of the product $P_i$, $i \in I$

$\mathbf{U_i^{max}}$ : Maximum production rate of the product $P_i$, $i \in I$

$\mathbf{Z_i}$ : Storage space capacity of the product $P_i$, $i \in I$

$\mathbf{T_{ij}^s}$ : Setup time required to switch from the production of $P_i$ to $P_j$, $i, j \in \{1,2\}, i \neq j$

$\mathbf{N_{ij}^s}$ : Setup operations number to switch from the production of $P_i$ to $P_j$, $i, j \in \{1,2\}, i \neq j$ during a period of time

$\mathbf{S_{ij}}$ : Operation of setup in order to switch from the production of $P_i$ to $P_j$, $i, j \in \{1,2\}, i \neq j$

$\mathbf{c_i^+}$ : Product type i ($i \in I$) inventory cost

$\mathbf{c_i^-}$ : Product type i ($i \in I$) backlog cost

$\mathbf{c_{ij}^{s}}$ : Setup cost to switch from the production of $P_i$ to $P_j$, $i, j \in \{1.2\}, i \neq j$

$\mathbf{C_{HCP}}$ : Total cost incurred when the Hedging Corridor Policy (HCP) is used

$\mathbf{C_{MHCP}}$ : Total cost incurred when the Modified Hedging Corridor Policy (MHCP) is used

### 2.2. Description of the control policies

Two joint production and setup control policies are considered in this work. They are defined in the following sections. The selection of the two policies is based on the completeness. Indeed, the studied manufacturing system consists of a one-machine, two-product with non-negligible setup times and costs and random fluctuations of breakdowns and repairs.

#### 2.2.1. Hedging Corridor Policy (HCP)

HCP consists of building positive safety stock in order to hedge against future capacity shortages caused by machine failures and setup times. It is characterized by a single threshold $Z_i, i \in \{1,2\}$ for each part type expressed by an inventory level $x_i$. Figure 2 illustrates an example of the stocks trajectory using HCP. HCP guides the surplus trajectory to target positive stock thresholds ($Z_1$ and $Z_2$). Thus, the machine operates at maximum capacity throughout the availability period and the set-up actions are performed when the stock level of the concerned part type $P_i, i \in \{1,2\}$ reaches $Z_i$ (see figure 2). The decision flowchart of HCP described in figure 3 represents the different situations encountered and the possible setups.

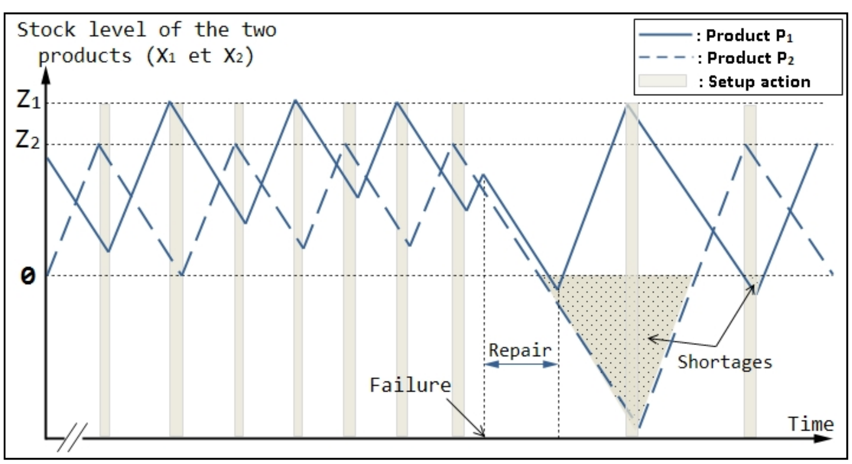



Figure 2. Example of the two-product stock trajectory using HCP

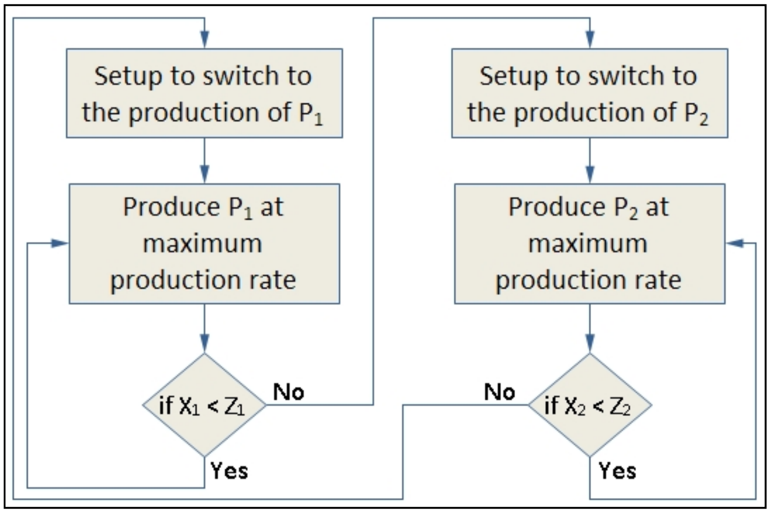



Figure 3. Organization Chart decision of HCP

HCP is defined by the following equations:

$$u_1(.) = \begin{cases} U_1^{max} * \mathrm{Ind}\{S_{21} = 1\} & \text{if } x_1 < Z_1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$u_2(.) = \begin{cases} U_2^{max} * \mathrm{Ind}\{S_{12} = 1\} & \text{if } x_2 < Z_2 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$S_{21} = \begin{cases} 1 & \text{if } (x_2 = Z_2) \text{ and } (x_1 < Z_1) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$S_{12} = \begin{cases} 1 & \text{if } (x_1 = Z_1) \text{ and } (x_2 < Z_2) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

### 2.2.2. Improved Modified Hedging Corridor Policy ($MHCP_2$)

The second policy considered in this article is an improvement of MHCP presented by Gharbi et al. [18]. The latter is characterized by two thresholds ($Z_i$ and $a_i, i \in \{1,2\}$ with $a_i \le Z_i$) connected to the inventory for each product type level. The main advantage of MHCP compared to HCP is to reduce the number of setups. $MHCP_2$ reduces the risk of shortages of MHCP by conducting setups before the stocks of the other product reaches the zero value.

Figure 4 shows an example of the inventories level variation using the $MHCP_2$. Among the major changes introduced, when the inventory level of a product type reaches the threshold $Z_i, i \in \{1,2\}$. Two possibilities are then considered. If the inventory level of the other product $x_i$ is less than the parameter $b_i$ ($b_i > 0$, $i \in \{1,2\}$), then setup actions are performed ①, otherwise, the production rate is adapted to the demand rate ②. As a result, the new parameter $b_i$ is integrated into the structure of the $\mathrm{MHCP_2}$ in order to represent the time necessary to perform the setup operation before shortages. In addition to the thresholds $Z_i$, $i \in \{1,2\}$ and the parameters $b_i$, setup operations depend for each product type on a second threshold $\mathrm{a_i}$ that we call setup threshold. This setup threshold $(a_i)$ triggers the setup action when $x_i \geq a_i$, and the inventory level of the other product type $P_j, j \in \{1,2\}$ is less than or equal to $b_j$ ③.

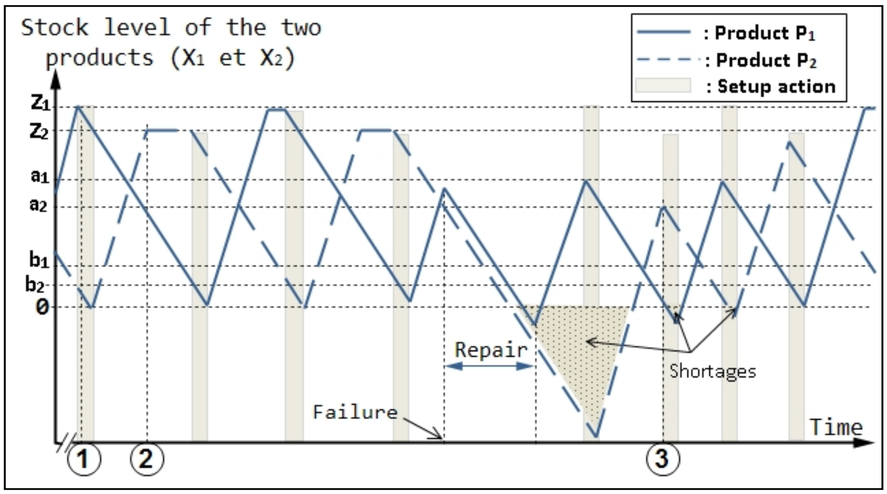



Figure 4. Example of the two-product stock trajectory using $\mathrm{MHCP_2}$

The decision flowchart of the $\mathrm{MHCP_2}$ presented in Figure 5 shows the setup actions to be undertaken according to the situations encountered. The following equations define the structure of the improved $\mathrm{MHCP_2}$. It is characterized by six control parameters $Z_i, a_i$ and $b_i, i \in \{1,2\}$.

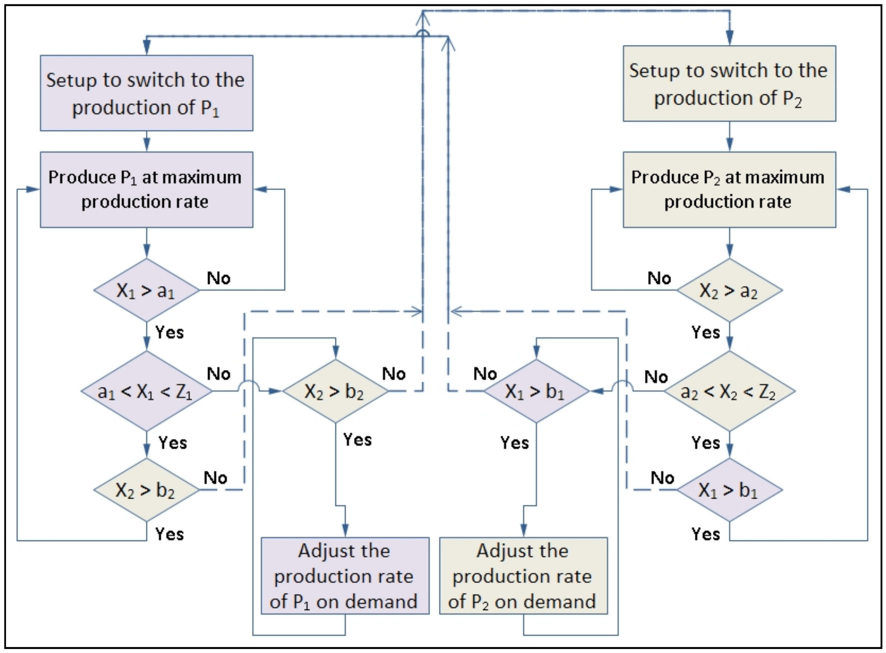



Figure 5. Organization Chart decision of $MHCP_2$

$$u_1(.) = \begin{cases} U_1^{max} * Ind\{S_{21} = 1\} & \text{if } x_1 < Z_1 \\ d_1 * Ind\{S_{21} = 1\} & \text{if } x_1 = Z_1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$u_2(.) = \begin{cases} U_2^{max} * Ind\{S_{12} = 1\} & \text{if } x_2 < Z_2 \\ d_2 * Ind\{S_{12} = 1\} & \text{if } x_2 = Z_2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$S_{21} = \begin{cases} 1 & \text{if } (x_2 > a_2) \text{ and } (x_1 < b_1) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$S_{12} = \begin{cases} 1 & \text{if } (x_1 > a_1) \text{ and } (x_2 < b_2) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

We conclude this section by presenting the assumptions made in [18] and in [17] and which will also be adopted in this work in order to preserve the main characteristics of the manufacturing system:

- The detection of the machine failure is instantaneous and the intervention repairs are started immediately;
- The backlog cost is a cost of delivery delays, and the unmet demand is never lost;
- Customer demand rate is assumed to be constant;

## 3. Comparative study between HCP and $MHCP_2$ based on costs

The objective of this section is to conduct an in depth comparative study in order to determine the best control policy in terms of cost. The effect of each system variable variation on the parameters of both control policies and on the total cost difference is widely studied and analyzed. The two policies are first optimized and then compared. In this sense, an experimental approach integrating simulation, design of experiment (DOE) and response surface methodology is used. The main steps of the experimental resolution approach are discussed further in the next section

### 3.1. Resolution approach

The experimental approach adopted to solve this problem is inspired by that proposed by Gharbi et al. [20]. It consists of a combination of simulation model and statistical methods. Below are the main steps of this approach.

- Step 1: Description of the control policies

In section 2.2, the structure of the joint production and setup control policies is analyzed then expressed by mathematical equations. These policies will govern our simulation models. The assumptions adopted are defined at the end of section 2.2.2.

- Step 2: Simulation model

Each simulation model is designed to reflect the system dynamics governed by one of the control policies considered. These policies, presented in section 2.2, are used as an input to conduct several experiments and thus evaluate the system performance. The simulation modeling is developed using the SIMAN language (ARENA simulation software) with a C++ subroutine. Section 3.2 provides more details on our simulation models.

- Step 3: Experimental design and response surface methodology

The experimental design approach defines the experiments number, the experimental space of the input factors (independent variables) considered and the variation extent of each factor. The analysis of variance is subsequently used to determine the main factors and their interactions which have a significant effect on the cost (dependent variable). Then, the response surface methodology allows obtaining the relationship between the dependent variable (cost) and significant main factors and their significant interactions given by the variance analysis. The resulting model is then optimized in order to determine the best combination of the control parameters which minimize the total cost incurred.

### 3.2. Simulation models

The simulation models developed consist of several networks interacting with each other. Each network describes specific tasks and events in the system (production activities, failures and repair interventions, etc.). Thus, the simulation model can accurately imitate the production system behavior. In this context, two combined discrete-continuous models are developed using the SIMAN language (ARENA simulation software) with a C++ subroutine [21]. The choice of this combined modeling approach relies mainly on the impulsive nature of the system dynamics as well as its ability to greatly reduce the execution time compared to the purely discrete models [22]. Our approach provides an advantage over the one developed by Gharbi et al. [18] using a purely discrete modeling.

The first model reproduces the classical production and setup control policy, as proposed in the literature by Elhafsi and Bai [17]. It is characterized by a single threshold ($Z_i, i \in \{1,2\}$) for each product. While the second model, is $MHCP_2$. This policy is characterized by three thresholds ($Z_i, a_i$ and $b_i, i \in \{1,2\}$) for each product type. The overall model structure is relatively the same except when the modified model reaches a threshold ($Z_i, a_i$ or $b_i$). This phenomenon triggers a series of events designed to check inventories level before executing a setup operation ((5)-(8)). Regarding the simulation time, several preliminary experiments have determined the necessary time to attain the steady state. Each simulation model operates on the basis of a production and setup control policy (section 2.2) and executed through the ARENA simulation software in order to reproduce the system dynamics and evaluate its performance. Figure 6 presents the block-diagram schema of our simulation models.

- Block ①: Initialization

It initializes the model variables (production rate, demand rate, machine capacity, etc...) as well as the simulation time. In this step, we consider also the minimum and maximum step time and the warm-up time, from which the statistics are collected.

- Block ②: Customer demand

It directly affects the inventory level of both products at each unit of time. Indeed, we continually control these inventories in order to check the availability of stocks. We note that customer demand rates are used as an input to the differential equations describing the inventories variation.

- Block ③: Control policy

It applies the control rules of the policies considered (HCP in Section 2.2.1 or $MHCP_2$ in Section 2.2.2). Thus the choice of the production rate and the product type to be manufactured, are determined according to the buffer stocks level and the system availability.

- Block ④: Production units

It represents the production machine activity according to control policy adopted (sections 2.2.1 and 2.2.2). The finished products represent the output of this block.

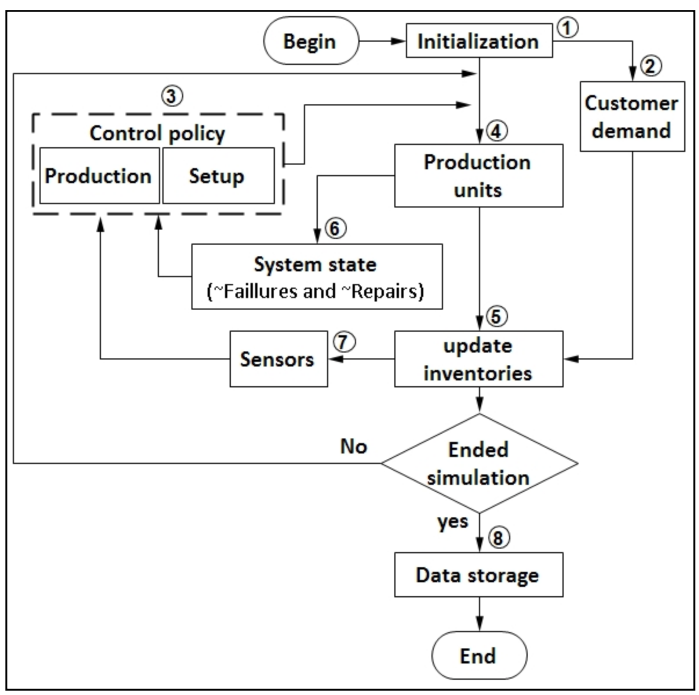



Figure 6. Diagram of the simulation model

- Block ⑤: Update inventories

It updates the inventories level of both products type at each unit of time. The inventories variation depends on production and customer demand rates. When a product type is not out of stock, its production increases the inventory level, otherwise, it merely satisfies the overdue demand, which decreases the stock-out.

- Block ⑥: System state

It defines the time to repair and the time to failure of the machine. Our simulation models are developed to accept any possible probability distribution (~Failure and ~Repair). At any time, the machine availability is checked in order to determine the production rate which is expressed as an equation of state in the C++ program.

- Block ⑦: Sensors

It continuously monitors inventories of both products and sends signals whenever a threshold is crossed ③. The thresholds correspond to the control parameters $Z_i$, $a_i$ and $b_i, i \in \{1,2\}$.

- Block ⑧: Data storage

It starts just before the simulation end and automatically records all the relevant information for calculating the value of the total cost incurred.

### 3.3. Validation of the simulation model

To validate the simulation model, a graphical representation of the buffer stocks variation for $MHCP_2$ was generated (Figure 7). The system modeling is performed for Z1= Z2=30, a1=a2=15 and b1=b2=1. Figure 7 shows that when a product type $P_i$, $i \in \{1,2\}$ may be out of stock ($X_i = b_i$) and the inventory level of the other type of products $P_j$, $j \neq i$ is higher than its setup threshold ($X_j \geq a_j$), then the setup action are automatically executed ①. In this context, the same conditions are checked as soon as a threshold setup $a_i, i \in \{1,2\}$ is reached. Furthermore, when the inventory level of a product type $P_i$, $i \in \{1,2\}$ reaches the $Z_i$, the production rate is adapted to the demand rate while: $X_j > b_j$, $j \neq i$ ②. During the machine failure ③, a decrease in inventory levels of both product types is observed (Only buffer stocks are used to fulfill customer needs) until the end of repairs operations. Note that in order to restore the inventory level of a product type already in production; the machine uses a maximum production rate. Such behaviors occur until the end of the simulation, they show that the simulation model accurately represents the dynamic of the system described in section 2.2.2.

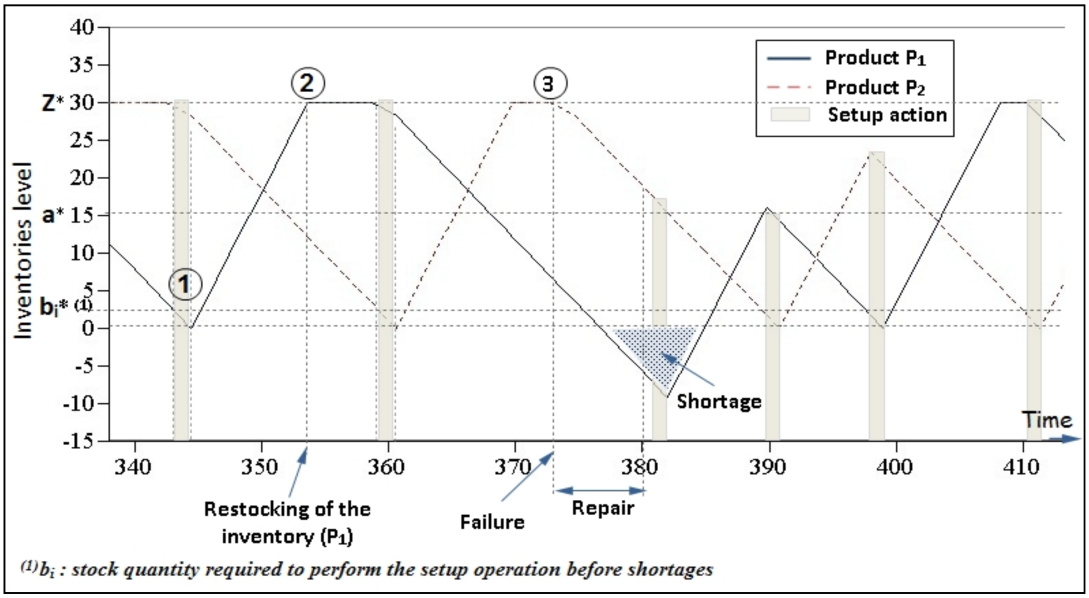



Figure 7. Inventories variation of both products ($Z_1$=$Z_2$=30, $a_1$=$a_2$=15 and $b_1$=$b_2$=1)

### 3.4. Optimization

This section presents the resolution approach adopted in order to calculate the optimal total cost followed by a comparison analysis between the two control policies: HCP and $\mathrm{MHCP}_2$. This approach is divided into two main parts. The first one allows to obtain the relationship between the dependent variable of the system (i.e. cost) and its factors ($\mathrm{Z_i}, \mathrm{a_i}$ and $\mathrm{b_i}$, $\mathrm{i} \in \{1,2\}$) and their interactions which have a significant effect on the total cost incurred. While the second part calculates the optimal values of the policy parameters (input factors) which minimize the total cost incurred. In this sense, a numerical example is presented in order to illustrate the strategies of the control policies considered. The consideration of the new parameter $b_i > 0, i \in \{1,2\}$ which represents the stock quantity required to perform a setup operation before shortages, implies a total of six control parameters for $\mathrm{MHCP}_2$: $\mathrm{Z_i}, \mathrm{a_i}$ and $\mathrm{b_i}$, $\mathrm{i} \in \{1,2\}$. For illustrative purposes we assume that the both product types are identical and the setup time ($\mathrm{T^s_{ij}}$) necessary to switch the production from one type to another is constant. That is to say that the system parameters are equal ($c_1^+$=$c_2^+$=$c^+$, $c_1^-$=$c_2^-$=$c^-$, $c^s_{12}$=$c^s_{21}$=$c_s$ , $T^s_{12}$=$T^s_{21}$=$T_s$,

$N_{12}^{s}$=$N_{21}^{s}$=$N_s$, $U_1^{max}$=$U_2^{max}$=$U_{max}$, $d_1$=$d_2$=d). As a result, the value of the dependent variables for both product types is also equivalent ($Z_1 = Z_2 = Z$, $a_1 = a_2 = a$ and $b_1 = b_2 = b$). Furthermore, the parameter b depend on the setup time ($T_s$) and the demand rate (d) which can be: $b = d * T_s$.

In order to ensure that $a \leq Z$ a new variable $\alpha$ is considered so that $a = \alpha * Z$ and $0 \leq \alpha \leq 1$. Thus, in this particular situation, HCP is defined by a single control parameter (Z) against two (Z and $\alpha$) for the $MHCP_2$. Table 1 summarizes the system's data.

Table 1. System parameters

| Parameters | $c^+$ | $c^-$ | $c_s$ | $T_s$ | $U_{max}$ | d | MTBF | MTTR |
|---|---|---|---|---|---|---|---|---|
| **Values** | 5 | 200 | 40 | 1 | 5 | 1,75 | EXP (95) | EXP (2,5) |

Regarding HCP, we use a polynomial regression model in order to optimize its sole parameter (Z). Thus, the model obtained, usually quadratic so as to represent the convexity property of the cost function shows that the adjusted correlation coefficient is equal to 97.63%. This means that more than 97% of the observed variability of the expected total cost is explained by our model [23]. The regression model is given by the following equation:

$$C_{HCP} = 209{,}34 - 7{,}54929 * Z + 0{,}279633 * Z^2 \quad (9)$$

The minimum total cost is observed at point $Z^* = 13{,}49$, with a value of $C_{HCP}^* = 158{,}31$.

On the other hand, due to the convexity property of the cost function of MHCP [18], we select a full factorial design with 2 factors at 3 levels each, which leads to perform nine experimental simulations. The full factorial of such a plan is often used for a model that assigns a small number of factors. It gives more accurate results since each interaction is estimated separately. Five replications were performed for each combination of factors, meaning therefore that a total of 45 ($3^2 * 5$) simulations were performed. For both simulation models, the simulation time is equal to $T_{sim}$= 500,000 units of time. It's long enough to reach the steady state. Several preliminary simulation experiments were performed in order to select the levels of the independent variables. These are presented in Table 2.

Table 2. Independent variables

| Factor | Lower level | Center | Upper level | Descriptions |
|---|---|---|---|---|
| **Z** | 15 | 21 | 27 | Inventory level |
| **$\alpha$** | 0,4 | 0,65 | 0,9 | Setup threshold |

We used the statistical software Statgraphics in order to perform a multi-factorial analysis of variance (ANOVA). Thus, the effects of independent variables (i.e.: Z and α), their interactions, and their quadratic effect on the response variables (i.e.: cost) were obtained. The adjusted correlation coefficients ($R^2$) presented in Table 3 show that more than 98% of the observed variability of the expected total cost is explained by our model [23]. Table 3 show also that all main factors ($Z$ and $\alpha$), interaction ($Z.\alpha$) and quadratic effects ($Z^2$ and $\alpha^2$) are significant at a 95% level of significance (the value of the P-value column is less than 0.05). The third order of interactions and all other effects are neglected or added to the error.

An analysis of the residual normality and of the homogeneity of variance was also carried out to check the conformity of the model. The response surface model is given by the following equation:

$$C_{MHCP_2} = 254{,}641 - 7{,}71663.Z - 149{,}468.\alpha + 0{,}2151.Z^2 + 1{,}7753.Z.\alpha + 97{,}322.\alpha^2 \quad (10)$$

Table 3. multi-factorial analysis of variance table for the total cost ($MHCP_2$)

| Factor | sum of squares | dl.[2] | the mean square | F-Ratio | P-Value |
|---|---|---|---|---|---|
| A: Z | 6603,8 | 1 | 6603,8 | 2000,34 | 0,0000 |
| B: α | 385,208 | 1 | 385,208 | 116,68 | 0,0000 |
| AA | 599,799 | 1 | 599,799 | 181,68 | 0,0000 |
| AB | 141,831 | 1 | 141,831 | 42,96 | 0,0000 |
| BB | 369,988 | 1 | 369,988 | 112,07 | 0,0000 |
| Blocks | 646,569 | 4 | 161,642 | 48,96 | 0,0000 |
| Total error | 115,547 | 35 | 3,30133 | | |
| Total (corr.) | 8862,74 | 44 | | | |
| $R^2$(adjusted) = 98,361 percent | | | | | |
| [2] *dl identifies the degree of freedom* | | | | | |

The response surface equivalent to this function is presented in Figure 8. The minimum total cost obtained is equal to $C^*_{MHCP_2} = 148{,}51$ which correspond to the optimal control parameters: $Z^* = 15{,}34$ and $\alpha^* = 0{,}63$ ($a^* = 9{,}66$).

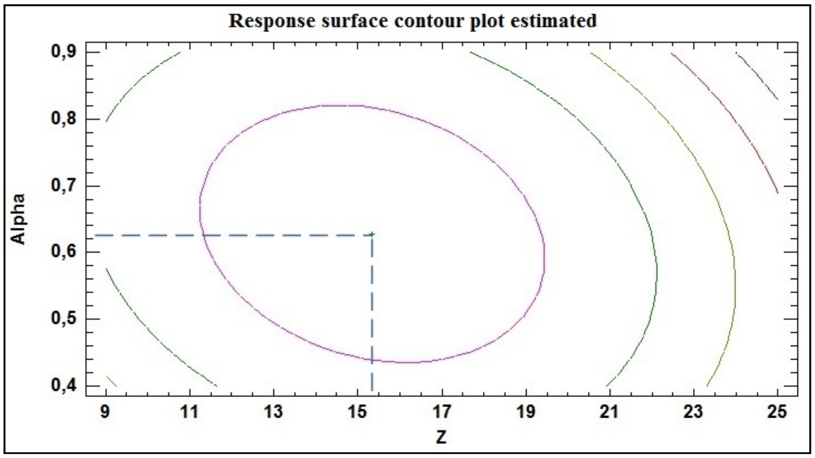



Figure 8. Response surface contour plot for the total cost

Table 4 summarizes the results of the control parameters optimization of both HCP and $MHCP_2$ that minimize the total cost incurred. It shows that for selected system parameters (Table 1), $MHCP_2$ is more advantageous in terms of cost than HCP with a reduction of 6.60%. This improvement is mainly due to the ability of $MHCP_2$ to reduce the number of setups, and therefore the cost of setup, compared to HCP. Note that if the optimal level of the threshold $Z^*$ for the $MHCP_2$, is higher than that for HCP, this does not mean that the inventory cost is greater for the $MHCP_2$. Indeed, the buffer stock level of $MHCP_2$ varies in a larger scale from 0 to $Z^*$ and the product types are more backlogged during system failures than HCP. To cross-check the validity of our models represented by equations (9) and (10), we confirm that the optimal cost for each control policy (Table 4) falls well within the confidence interval at 95% ($\overline{C.T.} \pm t^{n-1}_{1-(\alpha/2)}\sqrt{S^2/n}$) equivalent. This confidence interval obtained using n replications of the simulation model. Where:

| | |
|---|---|
| $\overline{\mathbf{C.T.}}$ | : Average optimal cost |
| $\mathbf{n}$ | : Number of replications (set at 100) |
| $(\mathbf{1}-\boldsymbol{\alpha})$ | : Confidence level (set at 95%) |
| $\mathbf{S}$ | : Sample standard deviation |
| $\mathbf{t^{n-1}_{1-(\frac{\alpha}{2})}}$ | : Student coefficient function |

Table 4. Solutions of both HCP and $MHCP_2$

| Model | $Z^*$ | $a^*$ | Total cost * | Confidence interval (95 %) |
|---|---|---|---|---|
| HCP | 13,49 | - | 158,31 | [153,91 ; 159,01] |
| $MHCP_2$ | 15,34 | 9,66 | 148,51 | [146,50 ; 150,47] |

The confidence intervals of both HCP and $MHCP_2$ presented in Table 4 do not overlap, thereby reflecting the economic benefit of the $MHCP_2$ for selected system parameters.

### 3.5. Influence of the cost and time parameters on the control policies

Sensitivity analyses were performed and the obtained optimal parameters of both control policies evolve in the same directions as those of Gharbi et al. [18] in response to variations in cost parameters. This analysis show that in terms of cost, $MHCP_2$ is more advantageous than HCP. However, the improvement obtained is subject to fluctuation. This leads us to wonder if the average total cost of $MHCP_2$ can exceed that of HCP. In this section, an in-depth comparative study of the effect of the variation of each cost and time parameter on the cost difference (DC[(3)]) of both HCP and $MHCP_2$ is conducted. The results obtained and explanations are thus presented according to the control policies structure and their parameters. Note that we use the DC between HCP and $MHCP_2$ in order to analyze the gap in terms of cost-effectiveness between the two control policies considered.

#### 3.5.1. Influence of the cost parameters

Figure 9 shows for the base case, the total cost difference (DC) between both HCP and $MHCP_2$ when the setup cost (Figure 9 a) and the backlog cost (Figure 9 b) parameters vary respectively from 4 to 20 and from 20 to 120.

$MHCP_2$ was mainly proposed to eliminate unnecessary setup operations. Thus, the growth in setup cost increases the total cost difference between HCP and $MHCP_2$ in favour of the second policy (Figure 9 a). Indeed, this growth is mainly due to the large number of setup actions executed by HCP every time an inventory level ($X_i, i \in \{1,2\}$) reach the threshold ($Z_i$). While

[3] DC: cost difference between HCP and $MHCP_2$, it is expressed by : $DC = \left(C^*_{HCP} - C^*_{MHCP_2}\right) / C^*_{MHCP_2}$

the $MHCP_2$ does not perform any setup operation only if a product could risk becoming out of stock, otherwise, the production rate is adapted to the demand rate.

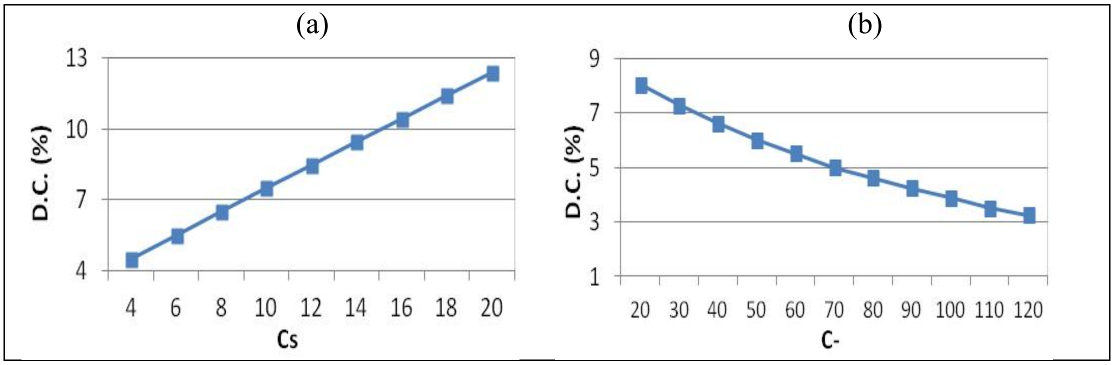



Figure 9. Variation of the cost difference between HCP and $MHCP_2$ according to backlog and setup costs

As shown in Figure 9 (b), the cost difference between HCP and the $MHCP_2$ decreases when the backlog cost ($c^-$) increases. This is due to HCP structure, with which the machine runs continuously throughout its availability period at maximum capacity, so that when the threshold $Z^*$ is reached, a setup operation is automatically performed in order to switch the production from one type to another. Therefore, the random nature of failures allows the $MHCP_2$ to generate more shortage than HCP. This is why DC decreases when $c^-$ increases. However, the total cost of HCP remains higher than that of $MHCP_2$. In order to validate this result, we compared the total cost of both policies for a high value of $c^-$. The DC value calculated for $c^- = 1000$ equals 0.93 % $\geq 0$.

### 3.5.2. Influence of the mean time to repair and the setup time

The setup time variation ($T_s$) affects the thresholds level ($Z^*$ and $a^*$ increase as a result of higher value of $T_s$) and mainly the setup operations number ($N_s$). Indeed, when $T_s$ increases $N_s$ is considerably reduced for HCP. Hence the result shown in Figure 10 (a) namely the DC reduction when $T_s$ increases.

The DC decrease is also observed in the increase of the mean time to repair (MTTR) (Figure 10 (b)). The reason coincides with that given in section 3.5.1 ($c^-$ variation). Indeed, random failure occurrences which are longer (MTTR increases) create more inventory ($Z^*$ and $a^*$

increase) in order to protect the system against shortages. However, since $MHCP_2$ does perform a setup action only when a product type may be out of stock, this policy generates more shortages compared to HCP during machine failures, hence the decrease in DC.

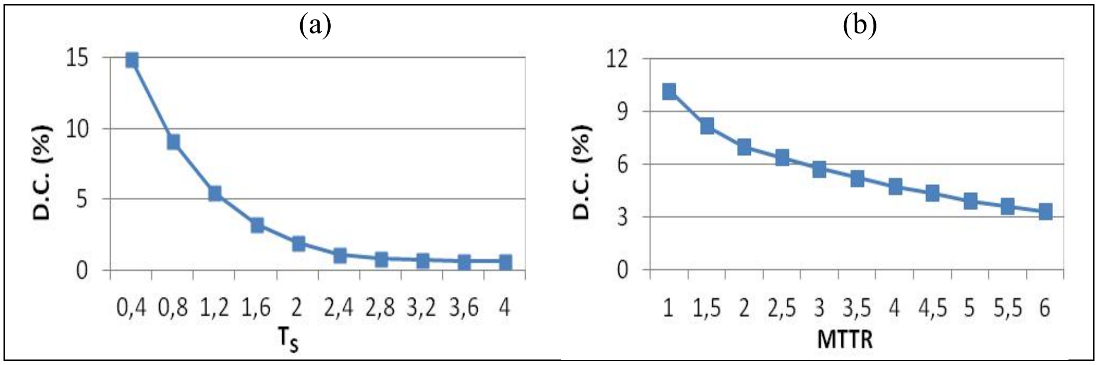



Figure 10. Variation of the cost difference between HCP and $MHCP_2$ according to the mean time to repair and the setup time

In order to confirm the superiority of the $MHCP_2$ in comparison with HCP policy, we select the case where $c_s = 5, c^- = 1000$, $T_s = 4$ and MTTR $= 6$ respecting the feasibility conditions of the production system. Thereby, the DC value obtained is equal to $0{,}08\ \% \geq 0$.

### 3.5.3. Influence of the maximum production rate

Taking into account the feasibility conditions of the system, Figure 11 shows that DC increases when the maximum production rate ($U_{max}$) increases incrementally from $4{,}4\ p/t.u.$ to $7\ p/t.u.$. The other cost and time system parameters and time of the system remain fixed at the same value as the base case. The $U_{max}$ variation directly affects the setup cost, but also affects the inventory level of both product types. Indeed, the increase of $U_{max}$ is equivalent to production capacity increasing of the system, hence reducing the threshold ($Z^*$) and the backlog cost. Moreover, the setup cost of HCP increases significantly since the inventories level reached more quickly $Z^*$ (higher production capacity), which means more setup actions.

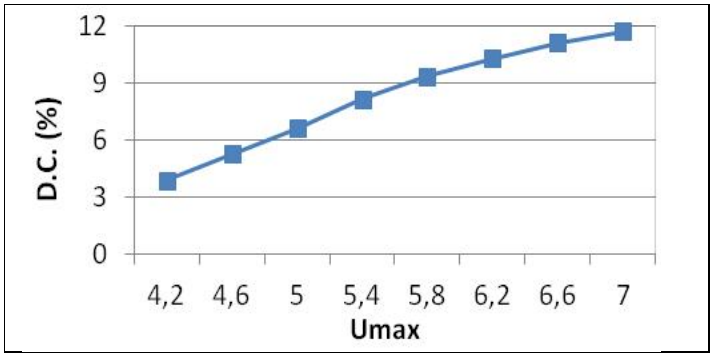



Figure 11. Variation of the cost difference between HCP and $\mathrm{MHCP}_2$ according to $U_{max}$

### 3.5.4. Discussion on the difference between HCP and the $\mathbf{MHCP_2}$ based on cost

The comparative study conducted in section 3.5 shows that the $\mathrm{MHCP}_2$ gives better results than HCP in terms of costs. This advantage is due to its ability to perform setup operations only when the system risks shortages. However, $\mathrm{MHCP}_2$ generates more shortages and requires a higher storage space ($\mathrm{Z}^*$) than HCP. Thus, in some cases (when $\mathrm{c}^-$, MTTR and $\mathrm{T_s}$ increase), the cost difference between both HCP and $\mathrm{MHCP}_2$ (DC) becomes very small. That is why it is important to consider other decision criteria in order to determine the best control policy.

## 4. Simultaneous consideration of the cost and the customer satisfaction criteria

In a competitive context which characterizes several industrial fields as the automobile, the pharmaceutical, the electronics, etc., cost and customer satisfaction factors are very important. Consequently, it is interesting to consider simultaneously the total cost and the customer satisfaction as two decision criteria in our comparative study. In fact customer satisfaction could be influenced by the production rate and the $\mathrm{Z}^*$ value which are different, depending on the considered control policy (HCP or $\mathrm{MHCP}_2$). The customer satisfaction is also related to the product availability. Thus, for every part type $P_i,\ i = \{1,2\}$, the customer satisfaction is calculated as follows:

$$S(P_i) = 1 - \left[ \left(\sum_i T_i^{NS}\right) / T_{Sim} \right] \tag{11}$$

With $T_i^{NS}$ is the time during which the demand $d_i$ is not satisfied at the right time, and $T_{Sim}$ represents the simulation time. Not only the cost ($) and the customer satisfaction (%) have different units of measure, but the direction of the control parameters optimization depends on the dependent variable used (cost or customer satisfaction). Indeed, maximize the level of customer service involves generally of a high inventory cost.

By using the same experimental approach as in the section 3, the purpose of this section is to conduct a further comparative study between both HCP and the $MHCP_2$ based on two decision criteria: cost and customer satisfaction. More specifically, we analyze the effect of wide range system configurations on the optimal control parameters which minimize the total incurred cost while respecting a customer satisfaction constraint. The cost difference between both policies considered is also studied according to the variation of the required customer satisfaction levels. With reference to Section 3.4, both product types $P_1$ and $P_2$ are assumed to be identical. It implies an identical customer service level for both product types during the same given period of time. Here is the functions of the response surface of the customer satisfaction constraint for both control policies: HCP and $MHCP_2$. $\forall i \in \{1,2\}$

$$S(P_i)_{HCP} = 86{,}2893 + 1{,}21775.Z - 0{,}02788.Z^2 \tag{12}$$

$$S(P_i)_{MHCP_2} = 82{,}9296 + 0{,}9068.Z + 11{,}5843.\alpha - 0{,}0133.Z^2 - 0{,}2437.Z.\alpha - 4{,}51733.\alpha^2 \tag{13}$$

These functions are used as constraints respectively to calculate the optimal control parameters which minimize the cost ((9) for HCP and (10) for $MHCP_2$). The figure 12 presents the variation of the threshold Z and of the average total cost of the policies: HCP and $MHCP_2$ according to the customer service level.

As shown in figure 12.a, the value of the optimal control parameters ($Z^*_{HCP}$ and $Z^*_{MHCP_2}$) increases with the growth of the required service level. This increase is due to the reaction of the system which seeks to reduce the risk of shortages. It implies the increase of the inventory cost, hence the growth of the average total cost (Figure 12.b).

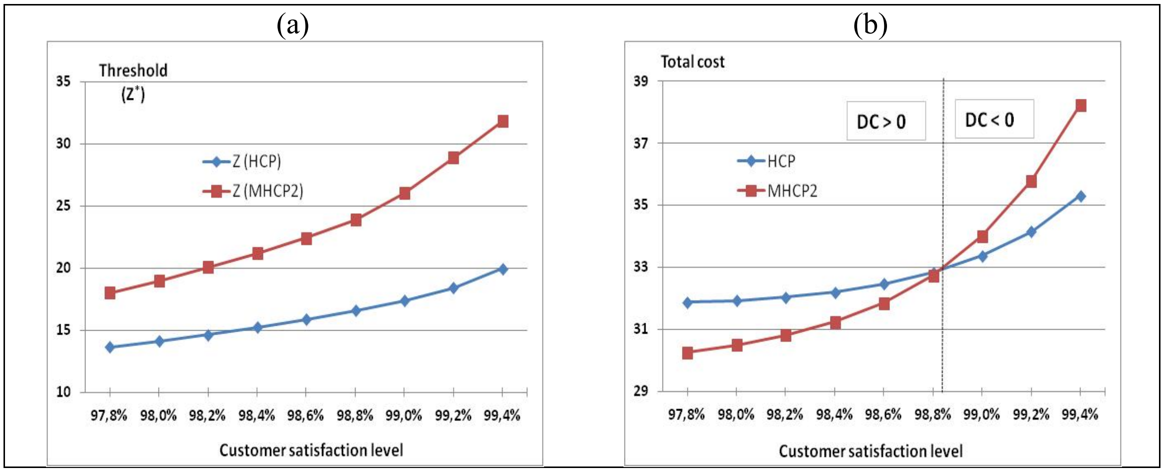



Figure 12. Variation of the threshold Z and of the average total cost of the policies: HCP and $MHCP_2$ according to the customer service level

Figure 12.b shows that the average total cost of the $MHCP_2$ can exceed that of HCP. Indeed, the DC value becomes negative just before reaching the customer service level of 99%, which confirms the superiority of HCP in terms of cost compared to $MHCP_2$, in this area. This phenomenon is due to the faster growth of the $Z^*_{MHCP_2}$ value compared to that of the $Z^*_{HCP}$ since $MHCP_2$ generates more shortage in comparison with HCP during system breakdowns (see section 3.5.1). Thus, the additional inventory costs necessary to increase the customer service level for $MHCP_2$ are higher than those of HCP. As a consequence, the total cost difference between two different service levels calculated by $MHCP_2$ is higher than that obtained by HCP. Table 5 presents the value of these cost differences calculated with and without customer service constraint of 99 %. It shows that the value of $\Delta C_{MHCP_2}$ is higher than that of $\Delta C_{HCP}$. Explaining the intersection of the cost curves of HCP and $MHCP_2$. We note that the choice of a service level of 99 % reflects the fierce competition which characterizes several industry sectors (pharmaceutics, technology, etc.).

Table 5. Costs and thresholds levels difference of both policies with and without customer service constraint of 99%.

| **Parameters** | $Z^*$ | $Z^*_{99\%}$ | $\Delta Z$ [4] | $C^*$ | $C^*_{99\%}$ | $\Delta C$ [5] |
|---|---|---|---|---|---|---|
| **HCP** | 13,55 | 17,25 | **3,70** | 31,88 | 33,38 | **1,50** |
| **$MHCP_2$** | 15,53 | 25,64 | **10,11** | 29,90 | 34,01 | **4,11** |

*[4] $\Delta Z$ : difference between the optimal threshold with ($Z^*_{99\%}$) and without ($Z^*$) customer service constraint of 99%. [5] $\Delta C$ : difference between the total cost calculated with ($C^*_{99\%}$) and without ($C^*$) customer service constraint of 99%*

The intersection point of the cost curves of the policies considered translates their equivalence in terms of cost. Rather this point is registered; greater is the area where HCP is better. Its location is influenced by the value of the system parameters ($c^-$, $c_s$, MTTR, $T_s$, etc.). The objective of the next sections is to explain this influence.

### 4.1. Influence of the customer satisfaction constraint for different $c^-$

Figure 13 shows that the intersection point of the cost curves of both policies when the service level increases, varies according to the value of the backlog cost ($c^-$). Indeed, we notice that the increase of $c^-$ reduces the area where HCP has an advantage compared to $MHCP_2$ ($DC < 0$). This phenomenon is related to the fact that when the value of $c^-$ is very low, the value of $Z^*$ and the incurred total cost are smaller (see section 3.), hence a low service level is recorded. Therefore, the consideration of a customer satisfaction constraint will significantly increase the value of $Z^*$. The same rule applies to the incurred total cost which varies according to the value of $Z^*$. That is why for low values of $c^-$ and for each control policy, the difference between the initial cost which is calculated without considering the customer satisfaction constraint and the cost which corresponds to a given service level ($\Delta C$) is more important (see Table 6). Moreover, the more the value of $\Delta C_{MHCP_2}$ is high in comparison with that of the $\Delta C_{HCP}$, the more likely the total cost curves of the studied control policies cross earlier when the service level increases. Therefore, the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$) increases.

Table 6 summarizes the difference of costs and thresholds levels of both policies with and without customer service constraint of 99 % for different values of $c^-$. It shows that when $c^-$ increases, the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ decreases. Hence the reduction of the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$).

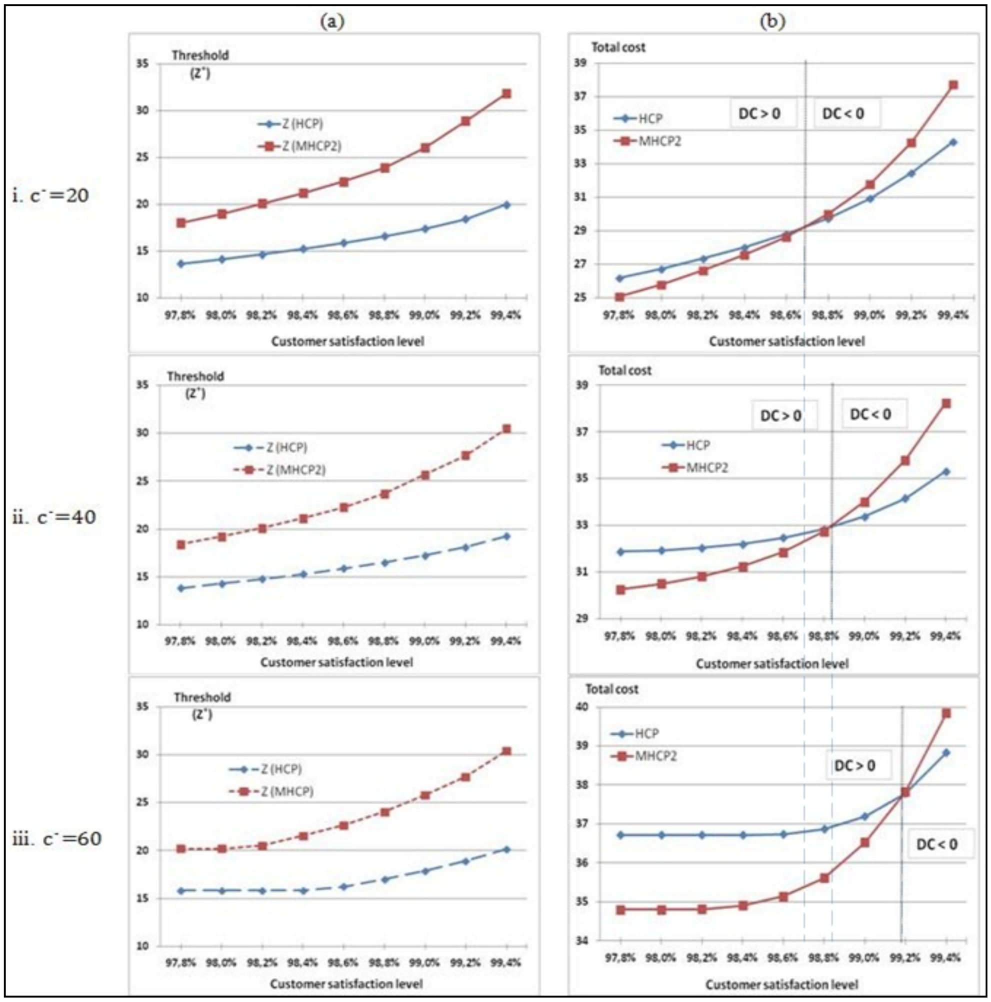



Figure 13. Variation of the threshold Z and of the average total cost of both HCP and $MHCP_2$ according to the customer service level

One interesting fact is that when $c^-$ is equal to 60, the total cost value of both policies remains constant during a given interval. This phenomenon applies to the situations where the value of $Z^*_{HCP}$ and $Z^*_{MHCP_2}$ engenders a higher service level than that required by the customer satisfaction constraint ((12) - (13)). Indeed, when the customer satisfaction constraint is removed, the calculated value of $Z^*_{HCP}$ and $Z^*_{MHCP_2}$ which minimizes the incurred total cost, generates respectively a service level of 98,48 % and 98,12 % for HCP and the $MHCP_2$. Therefore, if we require a service level of 98 % for example, the system will not decrease the thresholds value ($Z^*_{HCP}$ and $Z^*_{MHCP_2}$) in order to avoid the increase of the total cost.

Table 6. Costs and thresholds levels difference of both policies with and without customer service constraint of 99 % for different values of $c^-$.

| $c^-$ | 20 | | 40 | | 60 | |
|---|---|---|---|---|---|---|
| **Policy** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** |
| $\mathbf{Z^*}$ | 9,47 | 11,24 | 13,55 | 15,53 | 15,83 | 20,20 |
| $\mathbf{Z^*_{99\%}}$ | 17,35 | 26,04 | 17,25 | 25,64 | 17,91 | 25,79 |
| $\mathbf{\Delta Z}$ | **7,89** | **14,80** | **3,70** | **10,11** | **2,08** | **5,59** |
| $\mathbf{C^*}$ | 23,88 | 22,10 | 31,88 | 29,90 | 36,71 | 34,80 |
| $\mathbf{C^*_{99\%}}$ | 30,83 | 31,97 | 33,38 | 34,01 | 37,19 | 36,62 |
| $\mathbf{\Delta C}$ | **6,95** | **9,87** | **1,50** | **4,11** | **0,48** | **1,82** |
| $\mathbf{\Delta C_{MHCP_2} - \Delta C_{HCP}}$ | **2,92** | | **2,61** | | **1,34** | |

## 4.2. Influence of the customer satisfaction constraint for different $c_s$

Faced with the growth of the service level, figure 14 shows that the increase of $c_s$ reduces the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$). This phenomena is similar to that observed in the previous section concerning the backlog cost parameter ($c^-$), since the $Z^*_{MHCP_2}$ value reacts in the same way as the $c_s$ variation. Indeed, a high value of $Z^*_{MHCP_2}$ implies a smaller number of the setup operation. However, the variation of $c_s$ value has almost no effect on $Z_{HCP}$, because when HCP is used, the machine operates constantly at maximum capacity throughout the availability period. This means that the number of setup operations remains constant regardless of the value of $c_s$. As a result, the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ decreases when $c_s$ increases.

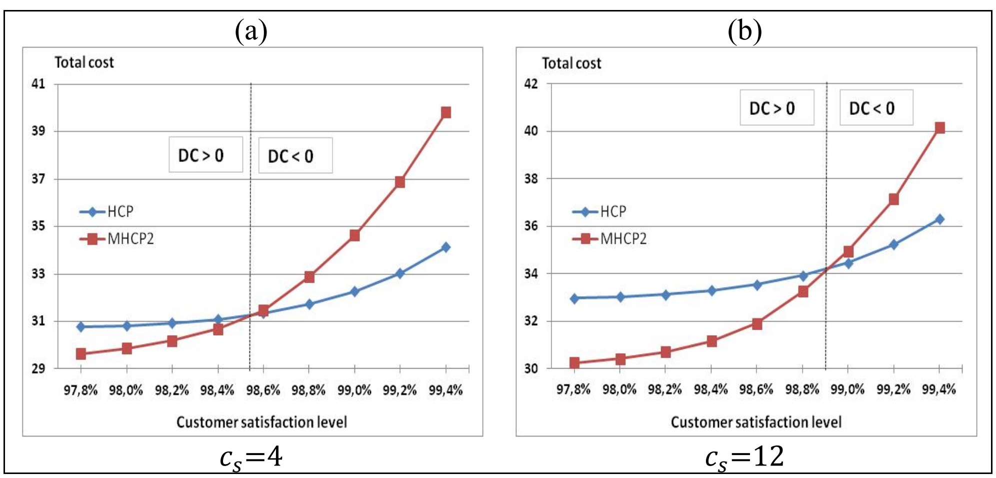



Figure 14. Variation of the average total cost of both HCP and $MHCP_2$ according to the customer service for different $c_s$

Table 7 confirms the results obtained in the case of the backlog cost ($c^-$). It shows that when $c_s$ increases, the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ decreases. This explains the reduction of the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$).

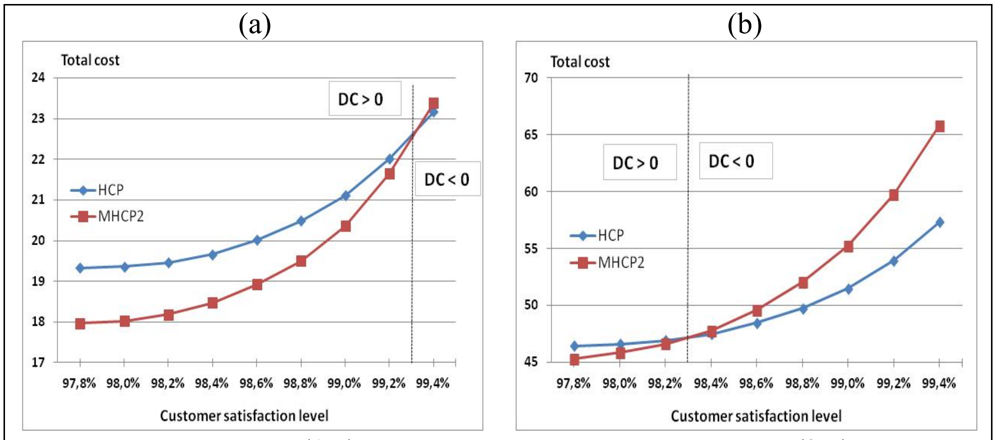
Table 7. Costs and thresholds levels difference of both policies with and without customer service constraint of 99 % for different values of $c_s$

| $c_s$ | 4 | | 8 | | 12 | |
|---|---|---|---|---|---|---|
| **Policy** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** |
| $Z^*$ | 13,50 | 15,17 | 13,55 | 15,53 | 13,61 | 16,78 |
| $Z^*_{99\%}$ | 17,22 | 26,30 | 17,25 | 25,64 | 17,27 | 26,30 |
| $\Delta Z$ | **3,72** | **11,13** | **3,70** | **10,11** | **3,67** | **9,52** |
| $C^*$ | 30,57 | 29,40 | 31,88 | 29,90 | 33,17 | 30,91 |
| $C^*_{99\%}$ | 32,25 | 33,82 | 33,38 | 34,01 | 34,45 | 34,76 |
| $\Delta C$ | **1,68** | **4,42** | **1,50** | **4,11** | **1,28** | **3,85** |
| $\Delta C_{MHCP_2} - \Delta C_{HCP}$ | **2,74** | | **2,61** | | **2,57** | |

### 4.3. Influence of the customer satisfaction constraint for different MTTR

Figure 15 presents the total cost variation of both policies: HCP and $MHCP_2$ according to the customer service level for different mean time to repair (MTTR). Unlike the results concerning the backlog cost ($c^-$) and the setup cost ($c_s$), Figure 15 shows that the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$) gets larger when MTTR increases. To explain this phenomenon, it is necessary to indicate that the increase of MTTR causes the reduction of the system availability. This implies a greater risk of shortages. Therefore, since $MHCP_2$ generates more shortage than HCP. The value of $\Delta Z_{MHCP_2}$ grows more compared with that of the $\Delta Z_{HCP}$ when the required service level increases. This means that the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ increases when MTTR increases. As a consequence, the area where HCP has an advantage compared with the $MHCP_2$ ($DC < 0$) is larger.

For different value of MTTR, Table 8 presents the cost and the thresholds levels difference of both policies with and without customer service constraint of 99 %. The obtained results show that when the MTTR increases, the value of « $\Delta C_{MHCP_2}$ - $\Delta C_{HCP}$ » increases too.




$MTTR$ = exp(1,5) $MTTR$ = exp(3,5)

Figure 15. Variation of the average total cost of both HCP and $MHCP_2$ according to the customer service for different mean time to repair

Table 8. Costs and thresholds levels difference of both policies with and without customer service constraint of 99 % for different values of MTTR

| **MTTR** | **exp(1,5)** | | **exp(2,5)** | | **exp(3,5)** | |
|---|---|---|---|---|---|---|
| **Policy** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** | **HCP** | **$MHCP_2$** |
| $\mathbf{Z^*}$ | 8,92 | 10,43 | 13,55 | 15,53 | 18,06 | 23,61 |
| $\mathbf{Z^*_{99\%}}$ | 11,59 | 15,19 | 17,25 | 25,64 | 25,62 | 42,54 |
| $\mathbf{\Delta Z}$ | **2,67** | **4,76** | **3,70** | **10,11** | **7,56** | **18,92** |
| $\mathbf{C^*}$ | 19,30 | 17,97 | 31,88 | 29,90 | 46,44 | 44,34 |
| $\mathbf{C^*_{99\%}}$ | 21,11 | 20,36 | 33,38 | 34,01 | 51,49 | 55,23 |
| $\mathbf{\Delta C}$ | **1,81** | **2,40** | **1,50** | **4,11** | **5,05** | **10,89** |
| $\Delta C_{MHCP_2} - \Delta C_{HCP}$ | **0,59** | | **2,61** | | **5,84** | |

### 4.4. Influence of the customer satisfaction constraint for different $T_s$

As shown in Figure 16, the variation of the $T_s$ value affects intersection point of the cost curves of the policies: HCP and $MHCP_2$ in the same way as the MTTR parameter (see previous paragraph). Indeed, the higher $T_s$ is, the more the area where HCP has an advantage compared with $MHCP_2$ ($DC < 0$) during the increase of the service level, is larger. As already observed in the section 4.3, this phenomenon is due to the system availability reduction which increases of the thresholds value ($\Delta Z_{MHCP_2}$ and $\Delta Z_{HCP}$) such that the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ increases. Table 9 confirms this conclusion and shows that when $T_s$ increases, the difference between $\Delta C_{MHCP_2}$ and $\Delta C_{HCP}$ increases.

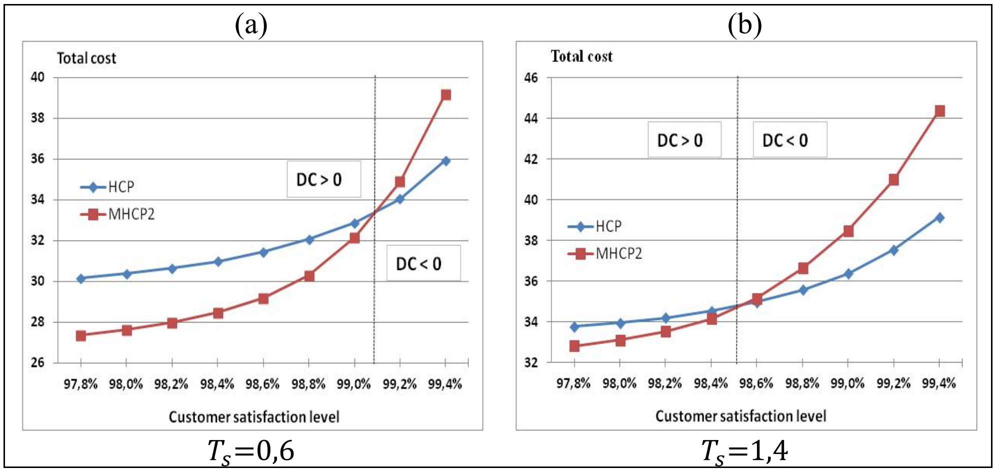



Figure 16. Variation of the average total cost of both HCP and MHCP$_2$ according to the customer service for different setup time

Table 9. Costs and thresholds levels difference of both policies with and without customer service constraint of 99 % for different values of $T_s$

| $T_s$ | 0,6 | | 1,0 | | 1,4 | |
|---|---|---|---|---|---|---|
| **Policy** | **HCP** | **MHCP$_2$** | **HCP** | **MHCP$_2$** | **HCP** | **MHCP$_2$** |
| $\mathbf{Z}^*$ | 8,95 | 12,54 | 13,55 | 15,53 | 14,83 | 18,24 |
| $\mathbf{Z}^*_{99\%}$ | 12,15 | 22,03 | 17,25 | 25,64 | 20,67 | 29,89 |
| $\Delta\mathbf{Z}$ | **3,20** | **9,49** | **3,70** | **10,11** | **5,84** | **11,65** |
| $\mathbf{C}^*$ | 29,92 | 26,98 | 31,88 | 29,90 | 33,55 | 32,42 |
| $\mathbf{C}^*_{99\%}$ | 32,89 | 32,05 | 33,38 | 34,01 | 36,39 | 38,50 |
| $\Delta\mathbf{C}$ | **2,97** | **5,07** | **1,50** | **4,11** | **2,84** | **6,08** |
| $\Delta\mathbf{C}_{\mathrm{MHCP_2}} - \Delta\mathbf{C}_{\mathrm{HCP}}$ | **2,10** | | **2,61** | | **3,25** | |

### 4.5. Discussion on the influence of customer satisfaction

In the section 4, we compared the incurred total cost of both policies HCP and MHCP$_2$ while taking into account the customer satisfaction constraint. The buffer stock between workstations and at the end of the manufacturing cycle has been proved to be an effective way to protect the system against random perturbations and to maintain the customer satisfaction during the periods of the lack of production capacity. However, a high level of buffer stocks has several inconveniences such as the increase of operating and inventory costs. In addition, low buffer stocks increase the risk of shortages and, as a consequence, it generates a lower satisfaction rate. The results obtained show that, contrary to those of the section 3.5, the average total cost of $MHCP_2$ can exceed that of HCP. As presented in Section 4, the values of $Z^*_{HCP}$ and $Z^*_{MHCP_2}$ are influenced by the adopted customer service level. But the magnitude of this influence varies depending on the control policy used (HCP or $MHCP_2$). On the one hand, both $Z^*_{HCP}$ and $Z^*_{MHCP_2}$ increase with the growth of the required service level in order to reduce the risk of shortages. This implies the increase of the inventory cost and also the incurred total cost. On the other hand, the value of $Z^*_{MHCP_2}$ increases faster than that of $Z^*_{HCP}$ since $MHCP_2$ generates more shortage in comparison with HCP. Consequently, the more the customer service level is high, the more HCP becomes advantageous compared to the $MHCP_2$. Therefore, the choice of the best joint production and setup control policy depends on the required customer service level and on the value of system parameters as well as other decision criteria if necessary.

## 5. Comparative study of HCP and $MHCP_2$ based on several quantitative and qualitative criteria

Up to now the cost and the customer service level are the two comparison criteria used to evaluate the performance of both HCP and $MHCP_2$. However, in the unreliable manufacturing environment which involves the production of several part types, other criteria may influence the company's decision makers on the choice of the best production and setup control policy. These decision criteria are generally related to the constraint of the storage space required, the complexity of the setup operations, the complexity of the control policy implementation, the dimension of the problem to optimize, etc. In addition, it is much more interesting that the policies comparison take into account several criteria simultaneously in order to better deal with the concerns of the company's decision-makers. In this regard, the results of the section 3.5 have shown that the $MHCP_2$ is more advantageous than HCP in terms of cost. However, the latter is easier to implement and to manage in an industrial context than the $MHCP_2$. It also generates less shortage and requires a lower storage space. In addition, the consideration of the customer satisfaction constraint in the optimization process of control parameters that minimize the total cost incurred, showed that HCP can become more economical than $MHCP_2$ from a given customer service level. Table 10 summarizes the most advantageous control policy according to various decision criteria which are considered individually.

In the following sections, we discuss the influence of several decision criteria which are considered individually, on the choice of the best joint production and setup control policy.

This comparative study shows that, with the exception of the cost and the number of setups, HCP is more advantageous than the $\mathrm{MHCP}_2$ according to several decision criteria which are based on those used in industry.

Table 10. The choice of the best policy according to various criteria

| Decision criteria | Control policy | |
|---|---|---|
| | **HCP** | **$\mathrm{MHCP}_2$** |
| Total cost | | × [6] |
| Customer satisfaction | × | |
| Storage space | × | |
| Complexity of the control policy implementing | × | |
| Complexity of the problem | × | |
| Complexity of the setup operations | × | |
| Number of setup operations | | × |

[6] × *: Choice of the best control policy with regards to the selected criteria*

### 5.1. Total cost

In Section 3.5, we conduct a comparative study of the effect of each cost and time parameter variation of the system on the cost difference between the two control policies: HCP and $\mathrm{MHCP}_2$. The total cost is considered as the only decision criterion. The obtained results show that the $\mathrm{MHCP}_2$ is more economic than HCP for a wide range of system configurations.

### 5.2. Customer satisfaction

Given its structure, HCP operates at maximum capacity throughout the availability period and generates a lower risk of stock-outs than the $\mathrm{MHCP}_2$. That is why the customer service level realized by HCP is higher than that calculated by the $\mathrm{MHCP}_2$. In conclusion, HCP is more advantageous than the $\mathrm{MHCP}_2$ in terms of customer satisfaction.

### 5.3. Storage space

All the optimization results obtained in this work show that the optimal value of $Z^*_{HCP}$ remains always lower than that of $Z^*_{MHCP_2}$ (see Tables 5, 6, …, 9 and Figure 13.a). It is explained by the fact that HCP generates a lower risk of stock-outs than the $MHCP_2$ for the same system parameters adopted. Therefore, HCP requires a lower storage space than the $MHCP_2$.

### 5.4. Complexity of the control policy implementing

In addition to the staff involvement, the number of variables is an important factor to consider during the implementation of the control policy. Both policies considered in this work require a continuous monitoring of the inventory level variation of both product types in order to detect the time when a given threshold is reached. HCP is characterized by a single control parameter (Z) for every part type. It is more practical to implement and to manage in an industrial environment than $MHCP_2$. Indeed, the latter account three parameters (Z, a and b) for every part type (see section 2.2.2).

### 5.5. Problem complexity

The problem complexity using an experimental approach as presented in the section 3.1, depends on the number of the studied policy parameters. As an example, let n be the product types number. In the case of different product types, the choice of a full design of experiment at three levels with five replications requires a total of $5*3^n$ simulation runs for HCP. At the same time, this number increases to $5*3^{3n}$ for the $MHCP_2$. It is $3^{2n}$ times higher than the simulation runs number required by HCP. For example, for n = 2, $MHCP_2$ requires 81 more simulation runs than HCP. We recall that HCP is characterized by a single control parameter (Z) for each product type. While $MHCP_2$ is characterized by three control parameters (Z, a and b) for each product type. Therefore, it is clear that the problem complexity increases according to the increase of the products type number. For a greater value of n, the use of another design of experiment such as the Central Composite type Design (CCD, Box-Wilson) is recommended (Lavoie et al., 2007). The number of runs for such a design is expressed as following: $2^n + (2*n) + k$ for HCP and $2^{3n} + (2*3n) + k$ for the $MHCP_2$ given that n is the products type number and k is the number of center points. For example, for n = 4 and k = 2, the number of simulation runs number required by $MHCP_2$ is about 158 times higher than that required by HCP. This gives advantage to HCP. For more details, we refer our readers to Montgomery (2008).

### 5.6. Complexity of the setup operations

The setup operations complexity can be measured by the setup time ($T_s$) necessary to switch the production from one type to another. It can be also related to the notion of flexibility of production resources. The more complex the setup action considered is, the more high $T_s$ is. In the section 3.5.2, we explained that the increase of the $T_s$ value reduces the number of setup operations for both control policies. This reduction is more important regarding HCP. Thus the increase of the setup operations complexity reduces the setup actions number and gives an advantage to HCP. Note that the flexibility of the manufacturing machine has an inverse effect on the $T_s$ variation and the number of setup operations in comparison with the complexity of setup operations.

### 5.7. Number of setup operations

HCP generates a higher number of setup operations than the $MHCP_2$. Indeed, the production according to HCP switches from a part type to another every time an inventory level ($X_i$, $i \in \{1,2\}$) reaches its threshold ($Z_i$). On the other hand, the $MHCP_2$ performs no setup operation unless if a product type risks to be out of stock, and otherwise, the production is adapted at the demand rate. Therefore, $MHCP_2$ is more advantageous in terms of number of setup operations compared to HCP.

## 6. Conclusion

In this article, a comparative study of two joint production and setup control policies for an unreliable manufacturing system is conducted. The control policies considered are the Hedging Corridor Policy (HCP) and the Improved Modified Hedging Corridor Policy ($MHCP_2$). This work is realized by adopting an experimental approach which integrates the combined continuous-discrete simulation, and statistical techniques as design of experiment, the analysis of the variance and the response surface methodology.

The first results show that $MHCP_2$ is more advantageous in terms of cost compared to HCP for a wide range of system configurations. In the same comparative context, other studies are conducted which consider simultaneously the cost and the customer satisfaction as two decision criteria with the intent to better joint the concerns of the company's decision-makers. The aim is to optimize the control parameters which minimize the total incurred cost while taking into account the constraint of the customer satisfaction required by the decision-makers. These studies explain how HCP can become better in terms of cost depending on the service level adopted. They also demonstrated that the area where $MHCP_2$ is more advantageous than HCP, is influenced by the system parameters value.

A discussion is conducted afterward on the effects of several quantitative and qualitative criteria on the choice of the best production and setup policy. Thus, we demonstrated that the $MHCP_2$ provides better results in terms of the cost and the number of setup operations than HCP. However, the latter is easier to implement and to manage in an industrial context than the $MHCP_2$. It also requires a lower storage space, generates less shortage and offers a better service. Therefore, the choice of the best control policy to adopt will depend on the company objectives and on the market constraints.

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