Complement to standard method for measuring normal incidence sound transmission loss with three microphones

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Abstract: Complement to standard E2611-09 of the American Society for Testing and Materials [*Standard Test Method for Measurement of Normal Incidence Sound Transmission of Acoustical Materials Based on the Transfer Matrix Method* (American Society for Testing and Materials, New York, 2009)] is proposed in order to measure normal incidence sound transmission loss of materials in a modified impedance tube using a three-microphone two-load or one-load method. The modified tube is a standard two-microphone impedance tube, where a third microphone is mounted on a movable hard termination. This method is conceptually identical to the four-microphone two-load or one-load method described in the standard; however, it requires fewer transfer functions and one microphone less. The method is validated on (1) symmetrical homogeneous and (2) non-symmetrical non-homogeneous specimens.

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1. Introduction

The normal incidence sound transmission loss (nSTL) is an important indicator to assess the sound insulation property of acoustic materials. This measurement of the nSTL has been standardized in ASTM E2611-09.[1] It uses a plane wave tube instrumented with four microphones and a termination of adaptable acoustic load. For any specimen, the four-microphone two-load (4M2L) standard method can be applied for measuring its transfer matrix and nSTL. The two loads are typically a minimally reflecting termination (e.g., anechoic termination), and a termination reflecting a portion of incident wave (e.g., open termination). For a geometrically symmetrical specimen (presenting the same physical properties to the sound field on either side), the four-microphone one-load (4M1L) standard method can be used. In this case, only one load is used, preferably the minimally reflecting termination. In the former method, a minimum of six transfer functions need to be measured, while a minimum of three are required in the latter method.

For these standard methods, if the surface acoustic impedances of the loads are known, the fourth microphone is useless (this was underlined elsewhere[2] but not formally explored). In this case, one can propose an additional configuration to those described in standard ASTM E2611-09 (see Table II of the standard[1]). This configuration would use three microphones, fewer transfer function measurements (four in the general case, and two in the symmetrical case) than the standard method, and would enable the use of a slightly modified classical impedance tube, where the third microphone is flush mounted on a movable hard termination. As proposed in the standard ASTM E2611-09, the three-microphone two-load (3M2L) method presented in this paper can be reduced to a three-microphone one-load (3M1L) method for geometrically symmetrical specimens. It is worth noting that a similar configuration was recently used in conjunction with a two-calibration method.[3] The purpose of this express letter is to present a generalization of a previous work,[4] and how this additional configuration can be used in conjunction with (or in complement to) standard ASTM E2611-09.

2. Theory

A schematic view of the modified impedance tube is shown in Fig. 1. The apparatus consists of a finite-length hard walled impedance tube with uniform inner cross-section. The tube features a loudspeaker (source) at one end and a movable hard termination at the other end. The loudspeaker is used to generate a plane wave field in the impedance tube. There are two microphones flush mounted upstream the test sample and one microphone flush mounted on the hard termination. Downstream the sample, an air cavity is added. The thickness of the cavity is adjusted with the movable hard termination.

Now, suppose a unit amplitude incident plane wave with time dependence of the form $\exp(j\omega t)$, where $j=\sqrt{1}$, ω is the angular frequency, and t is the time. The acoustic pressure $p_i(x)$ and particle velocity $u_i(x)$ upstream ($x \le 0$) and downstream ($x \ge d$) the test sample are, respectively, given by

$$p_i(x) = e^{-jkx} + R_i e^{jkx}, u_i(x) = \frac{e^{-jkx} - R_i e^{jkx}}{Z_s} \tag{1}$$

and

$$p_i(x) = 2A_i e^{-jkL_i}\cos(k(x-L_i)),\ \ u_i(x) = -j2\frac{A_i}{Z_s}e^{-jkL_i}\sin(k(x-L_i)), \tag{2}$$

where subscript $i=a,b$ refers to a value obtained with an air cavity of thickness D_i, $L_i = d + D_i$, d is the thickness of the sample, Z_s and k are the complex specific acoustic impedance and complex wave number of the air in the tube, R_i is the complex sound reflection coefficient at the surface of the sample (i.e., at $x=0$), and $2A_i$ is the maximum pressure amplitude of the standing wave downstream the sample. The two air cavities D_a and D_b will be the two loads of the proposed 3M2L method. The geometrical variables are also defined in Fig. 1. Note that Z_s and k account for viscous and thermal dissipation effects at the tube walls using the expression proposed elsewhere.[5] For the setup shown in Fig. 1, the reflection coefficient is given by

$$R_i = \frac{H_{12}(D_i)e^{jks} - 1}{1 - H_{12}(D_i)e^{-jks}}e^{2jkl}, \tag{3}$$

where $H_{12}(D_i)$ is the measured transfer function between microphones 2 and 1 [$p_i(\mu_2)/p_i(\mu_1)$] with an air cavity of thickness D_i, s is the spacing between microphones 1 and 2,

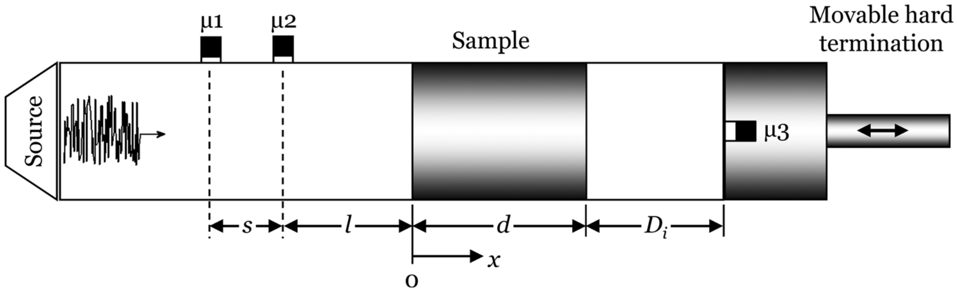

Fig. 1. Measurement configuration of the three-microphone two-cavity (3M2L) method.

and l is the distance between microphone 2 and the front face of the sample. In a similar manner, one can deduce coefficient A_i by introducing transfer function $H_{13}(D_i)$ between microphones 3 and 1 $[p_i(\mu_3)/p_i(\mu_1)]$. This yields

$$2A_i e^{-jkL_i} = H_{13}(D_i)\left(e^{jk(l+s)} + R_i e^{-jk(l+s)}\right). \tag{4}$$

According to Eqs. (1) and (2), calculating R_i and A_i from Eqs. (3) and (4) allows a complete description of the sound field in the air inside the impedance tube presented in Fig. 1.

For any specimen, Eq. (22) of standard ASTM E2611-09 gives the transfer matrix $\mathbf{T}$ of the specimen in terms of the pressure and particle velocity on its both faces (at $x=0$ and at $x=d$). For the sake of clarity, this equation is repeated here with the used notations:

$$\mathbf{T} = \frac{1}{p_a(d)u_b(d) - p_b(d)u_a(d)}\begin{bmatrix} p_a(0)u_b(d) - p_b(0)u_a(d) & p_b(0)p_a(d) - p_a(0)p_b(d) \\ u_a(0)u_b(d) - u_b(0)u_a(d) & p_a(d)u_b(0) - p_b(d)u_a(0) \end{bmatrix}, \tag{5}$$

where pressures and velocities are derived from Eqs. (1)–(4) for two different loads. For cavity load $i=a,b$, they are given by

$$\begin{aligned} p_i(0) &= -2je^{jkl}\frac{H_{12}(D_i)\sin(k(l+s)) - \sin(kl)}{H_{12}(D_i)e^{-jks} - 1}, & p_i(d) &= -2je^{jkl}\frac{H_{13}(D_i)\sin(ks)\cos(kD_i)}{H_{12}(D_i)e^{-jks} - 1}, \\ u_i(0) &= \frac{2e^{jkl}}{Z_S}\frac{H_{12}(D_i)\cos(k(l+s)) - \cos(kl)}{H_{12}(D_i)e^{-jks} - 1}, & u_i(d) &= \frac{2e^{jkl}}{Z_S}\frac{H_{13}(D_i)\sin(ks)\sin(kD_i)}{H_{12}(D_i)e^{-jks} - 1}. \end{aligned} \tag{6}$$

Consequently, the application of the 3M2L method to obtain the transfer matrix $\mathbf{T}$ of any specimen will require only 4 transfer function measurements, which is two measurements less than the 4M2L standard method.

In a similar manner than the 4M2L standard method reduces to the 4M1L standard method,[1] if a specimen is geometrically symmetric, the 3M2L reduces to a three-microphone one-load (3M1L) method. This is done by invoking reciprocity and symmetry which force the determinant of transfer matrix to be unity (i.e., $T_{11}T_{22} - T_{21}T_{12} = 1$) and $T_{11} = T_{22}$, respectively. Now, using these two constraints, only one load is needed (e.g., $i=a$ with cavity D_a) and the transfer matrix is given by Eq. (24) of ASTM E2611-09. For the sake of clarity, this equation is repeated here with the used notations:

$$\mathbf{T}_{\text{sym}} = \frac{1}{p_a(0)u_a(d) + u_a(0)p_a(d)}\begin{bmatrix} p_a(d)u_a(d) + p_a(0)u_a(0) & p_a(0)^2 - p_a(d)^2 \\ u_a(0)^2 - u_a(d)^2 & p_a(d)u_a(d) + p_a(0)u_a(0) \end{bmatrix}. \tag{7}$$

Again, the pressures and velocities are obtained from Eq. (6). This time, the application of the 3M1L method requires only two transfer function measurements compared to three for the 4M1L standard method.

In the proposed method, the two measured transfer functions H_{12} and H_{13} are corrected (amplitude and phase correction) following the sensor-switching technique[6] detailed in ASTM E2611-09. Note that the use of this calibration procedure will be validated in Sec. 4.1 in the case of a symmetric air layer specimen. Then, Eq. (6) can be used in place of Eqs. (17)–(21) of the standard[1] to implement the 3M2L and 3M1L methods under that standard. Then, knowing the four transfer matrix coefficients, one can deduce all the material properties as described in Sec. 8.5.5 of the standard.

Before testing experimentally the three-microphone configuration, a singularity specific to the 3M2L method is underlined. Combining Eqs. (5) and (6), one can show that **T** is not determined when $\cos(kD_a)\sin(kD_b) = \cos(kD_b)\sin(kD_a)$. To avoid this, a condition on the difference between the depths of the cavities must be fulfilled. This condition is $D_a - D_b < |\pi/k| \approx 172/f_u$, where f_u is the upper frequency limit of the tube in Hz to ensure plane wave propagation. Such singularity is not observed for the symmetrical 3M1L method.

3. Experimental setup

To implement the 3M2L method, a 44.45-mm diameter tube is used. The tube has three sections: upstream (235-mm long), sample holder, and downstream. The downstream section is terminated with a 30-mm thick sliding piston—the piston acts as a hard wall and is made of steel. In the upstream section, two microphones are flush mounted as shown in Fig. 1. A third microphone is flush mounted directly on the hard termination, as depicted in Fig. 1. The distance between microphone 1 and 2 is $s = 25.2$ mm and the distance between microphone 2 and the front surface of the test sample is $l = 45.5$ mm. Precise measurements of these distances have been carried out using the method proposed by Katz.[7] This latter method is based on the frequency determination of the nulls of the transfer functions between microphones 1, 2 and microphone 3, respectively, H_{31} and H_{32}. These transfer functions are easily measurable using the proposed setup of Fig. 1 without sample and placing the movable piston at position $x = 0$. The cavity thickness downstream the sample is fixed by moving the hard termination conveniently. With the used setup, the working frequency range for this study is 150–4100 Hz. To implement the 4M2L standard method, the same setup is used; however the downstream section is replaced by a two-microphone instrumented 360-mm long tube. The two downstream microphones are separated by a distance $s = 25.6$ mm and flush mounted on the tube extension. The distance between the back surface of the sample and microphone 3 is l_2 (it depends on the test sample, see Sec. 4). The two termination loads are selected to have relatively different reflection coefficients to yield good results. The first load is a partially anechoic termination. It is constructed using a 1.5-m long cylindrical tube filled with low density wool. The wool is arranged in a way that its density increases gradually as the acoustic wave propagates in the tube. The second load is a 25.4-mm thick melamine foam backed by a rigid cap.

The same four-channel analysis system and $\frac{1}{4}$-in. MPA416 BSWA microphones are used to conduct all measurements. The analysis system uses a USB Fireface UC sound card driven by a MATLAB script which generates, acquires, and processes the signals. The input signals from the microphones are stored on 24 bits by the sound card with a sampling frequency of 44.1 kHz. The MATLAB script processes the signals to obtain the required transfer functions. The output signal is a white noise and the source is a 4-in. loudspeaker. Each characterization test is the average of 50 repeated measurements, and uses a linear weighting. For the 3M2L method, three microphones (μ_1, μ_2, μ_3) and four channels (ch$_1$, ch$_2$, ch$_3$, ch$_4$) are used for measuring the required four transfer functions [$H_{12}(D_a)$, $H_{13}(D_a)$, $H_{12}(D_b)$, $H_{13}(D_b)$]. Each microphone μ_n ($n = 1, 2, 3$) is connected to channel ch$_n$ to form measurement line $\mu_n$ch$_n$, and ch$_4$ is the output source signal. For correcting the measured transfer functions for amplitude and phase mismatches between the three measurement lines, the sensor-switching technique as described in ASTM E2611-09 is used. Here line $\mu_1$ch$_1$ is the reference line. Consequently the calibration is successively made between $\mu_1$ch$_1$ and $\mu_2$ch$_2$ and between $\mu_1$ch$_1$ and $\mu_3$ch$_3$ using microphone positions 1 and 2. For the 4M2L method, the one-microphone two-channel configuration given in Table II of ASTM E2611-09 is used. In this case, the output source signal on ch$_4$ is the transfer function reference. Here, μ_1 is connected to ch$_1$ and moves successively to microphone locations 1 to 4 for measuring the required eight transfer functions (i.e., four for each load). Since the same microphone is used, no correction on the transfer functions is required.

4. Results and discussions

4.1 Symmetrical and homogeneous specimen: Air layer

With a view to validate both the transfer function correction technique discussed in the previous section and the 3M2L test procedure, an air layer ($d = 80$ mm) is first tested. The air layer is placed at the sample position shown in Fig. 1. The sound transmission loss of an air layer is 0 dB (at least on a small distance when neglecting losses) and its transfer matrix is theoretically known. Its coefficients are $T_{11} = T_{22} = \cos(kd)$, $T_{12} = jZ_s\sin(kd)$, and $T_{21} = j\sin(kd)/Z_s$. Since the air layer is geometrically symmetrical, only one cavity would be sufficient to measure its transfer matrix. However, with a view to validate the 3M2L test procedure, two cavities will be used. Their thicknesses are $D_a = 25.4$ mm and $D_b = 50.8$ mm. These cavities respect the singularity condition of the 3M2L method (i.e., $|D_a - D_b| < 172/f_u \rightarrow 25.4\,\text{mm} < 42\,\text{mm}$). Figure 2 presents the 3M2L measurement results between 150 and 4100 Hz in terms of the real and imaginary parts of the transfer matrix coefficients and the nSTL. With the used microphones, one can note that if the transfer functions are not corrected, poor results are obtained compared to the theoretical results. On the contrary, using the standard sensor-switching correcting technique, a good correlation is obtained between the 3M2L and the theory. Moreover, one can note that the symmetry of the material is preserved with the 3M2L measurements, i.e., $T_{11} = T_{22}$. Nevertheless, a slight divergence can be observed at high frequencies on the nSTL (less than 0.3 dB), and a loss in accuracy when approaching the lower frequency limit of the system due to the small spacing used between microphones 1 and 2.

These first results show that using the standard sensor-switching technique as described earlier is appropriate to correct transfer functions H_{12} and H_{13} of the 3M2L method, even if microphone 3 is used at 90° compared to microphones 1 and 2. Moreover, it was shown that the 3M2L method tested on a geometrically symmetrical specimen yields symmetrical results.

4.2 Non-symmetrical non-homogeneous sample: Step discontinuity

To validate the 3M2L method in the case of a non-symmetrical specimen, a 20-mm thick step discontinuity is tested and compared to the standard 4M2L method. A sketch of the sample is shown in Fig. 3. The sample is made of a rigid thermoplastic (density of 1400 kg/m^3) and has an outside diameter of 44.44 mm. The sample is tight mounted in the impedance tube and grease is used on its circumference to prevent acoustical leaks. On its central axis, it contains a large hole (diameter 11 mm; length 14.2 mm) followed by a small hole (diameter of 5.2 mm; length of 5.8 mm). Since the perforation is small compared to the tube diameter, thicker cavities ($D_a = 89.0$ mm and $D_b = 109.0$ mm) and larger distance $l = 88.9$ mm were used to allow the plane wave field to be well established at microphones 2 and 3. This is in agreement with the minimum spacing between microphone and sample recommended in standard ASTM E2611-09 for specimen with non-homogeneous surface (see Sec. 6.5.5.2 of the standard). Again, the used cavities respect the singularity condition of the 3M2L method. For the 4M2L method, a length $l_2 = 183.1$ mm is used. Figure 3 first compares the proposed 3M2L method to results obtained with a numerical simulation of the experiment. The simulation uses an axisymmetrical finite element model of the 3M2L setup with the step discontinuity. The details on the model are discussed elsewhere.[8] As one can note, these simulation results correlate well with the experimental results on the full range of frequencies. Figure 3 also compares the results obtained with the two experimental methods (3M2L, 4M2L). In general, one can note that similar results are obtained between the 3M2L and 4M2L methods. With the used setup and analysis system, the transfer

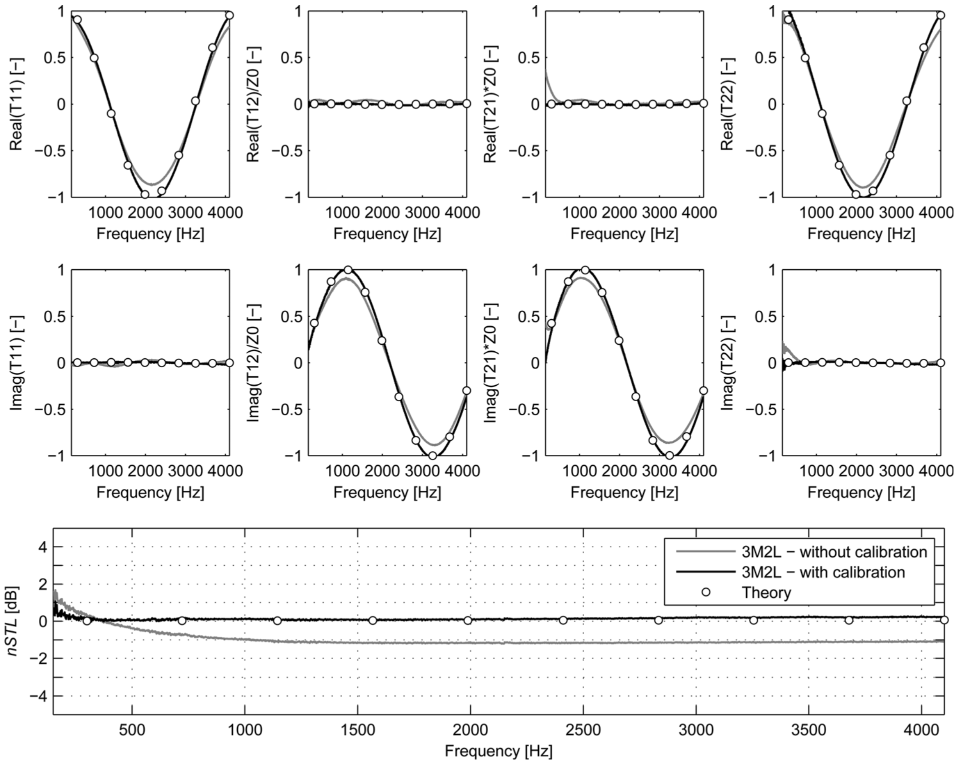

Fig. 2. Transfer matrix coefficients and normal sound transmission loss of a symmetrical specimen (air layer of 80 mm). Comparison between theory and 3M2L method with and without sensor-switching calibration.

matrix coefficients measured with the 3M2L method are less noisy than those obtained with the 4M2L method. Nevertheless, the 3M2L results are noisier than in the case of the air layer. This is due to the fact that the step discontinuity is quite reflective. In this case, microphones 1 and 2 may coincide with pressure nodes (and this is a common issue when measuring reflective specimens) at some frequencies, and microphone 3 may have a poor signal-to-noise ratio since the amplitude of the transmitted wave may be low. For the 4M2L method, the same problem at microphones 1 and 2 exists. However, for microphone 3 (also 4), the problem may be larger than in the 3M2L method. In fact, microphone 3 is always at a maximum pressure in the 3M2L, which is not the case for microphone 3 (also 4) in the 4M2L.

5. Conclusion

This letter showed how a three-microphone configuration (3M2L or 3M1L) complies with standard ASTM E2611-09 for measuring the sound transmission loss and transfer matrix of acoustical materials. Also, it was shown that the standard sensor-switching technique can be used with this additional configuration without any modification, even if microphone 3 is used at 90° compared to the other 2 microphones. In fact, even if the wave propagates in only one direction, the stress induced by the acoustic pressure in a fluid is the same in all space directions. Consequently, ASTM E2611-09 can be complemented with this additional three-microphone configuration, if its Eqs. (22) and (24) (respectively, for any specimens and symmetrical specimens) are used with Eq. (6) of this letter.

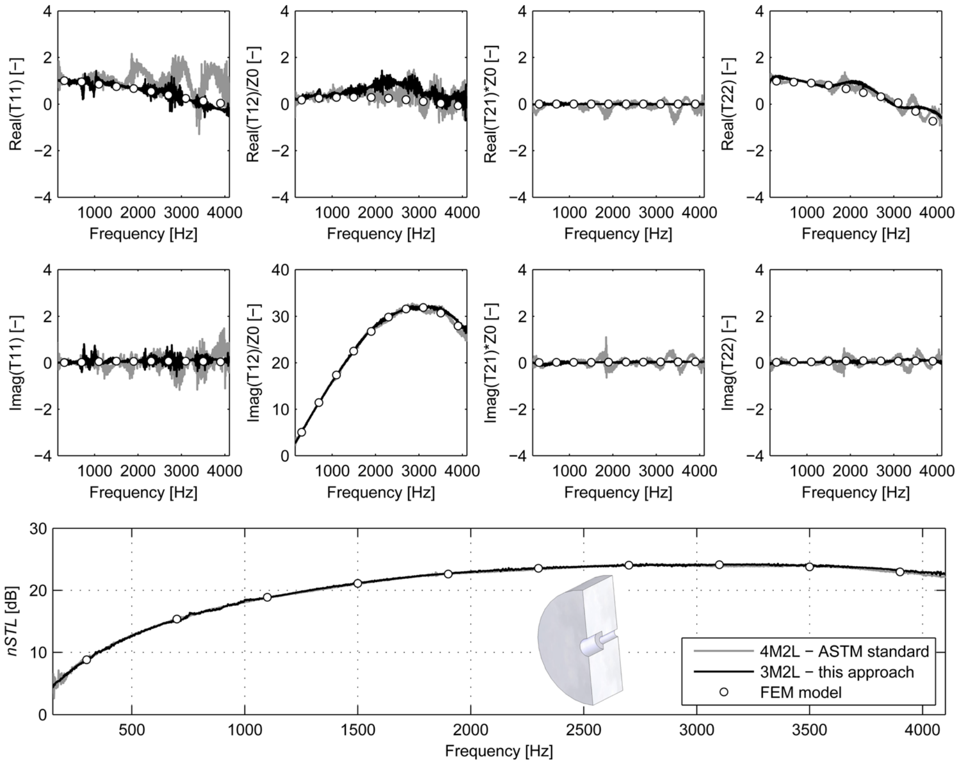

Fig. 3. (Color online) Transfer matrix coefficients and normal sound transmission loss of an asymmetrical specimen (step discontinuity). Comparisons between 3M2L method and 4M2L standard method.

Acknowledgments

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