

3D Stress Analysis of a Loaded Birefringent Sphere by Photoelastic Experiment and Finite Elements method

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This paper deals with a contact problem developed in a birefringent sphere loaded by a plan along its diameter. In mechanical systems, contacts between moving elements can give rise to high stresses that can cause damage. Several authors [1–5] have contributed to the understanding of contact problems. To improve the design and the durability, it is necessary to determine accurately the stress fields particularly in the neighborhood of the contact zones. The analyzed model consists of a birefringent deformable sphere loaded along its diameter by birefringent rigid plans. Stress fields are analyzed experimentally with plan polarized light and circularly polarized light; photoelastic fringes are used to calculate stresses. A finite elements analysis with Castem package allows calculating the stress fields. Comparison between the experimental solution and the finite element one shows good agreements.

Keywords: photoelasticity, birefringent, isochromatic, isoclinic, contact, stress

EXPERIMENTAL ANALYSIS

The birefringent sphere is machined from a birefringent parallelepiped on a high speed numerically controlled machine. The model is then loaded inside an oven (figure 1left) at the stress freezing temperature (120°C). A thermal cycle is used to freeze stresses within the volume of the model. The model is then mechanically sliced in a high speed rotating machine to prevent residual stresses. The birefringent slice is then positioned in the light path of

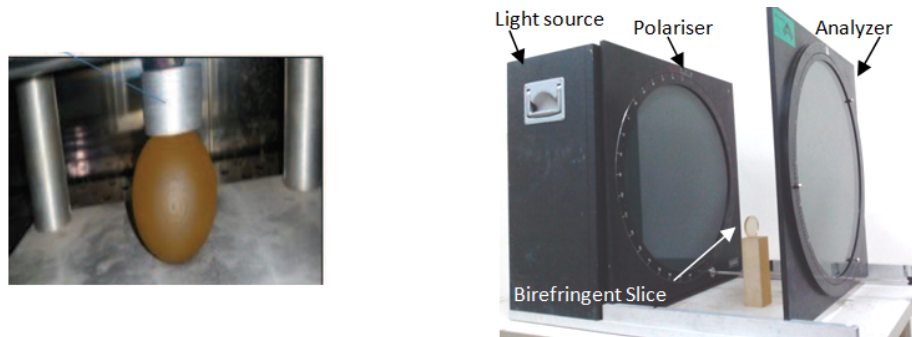


Figure 1: sphere inside the oven (left), a slice analyzed in a polariscope (right)

a polariscope (figure 1 right) to obtain the photoelastic fringes. The light intensity after the analyzer is given by Eq. (1). The terms $\sin^2 2\alpha$ and $\sin^2 \varphi/2$ give respectively the isoclinic fringe pattern and the isochromatic fringe pattern where α and φ are respectively the isoclinic parameter and the isochromatic parameter [6].

$$I = a^2 \sin^2 2\alpha \sin^2 \varphi/2 \quad (1)$$

The experimental isochromatic fringes are used to determine the values of the principal stresses difference in the model by using the well known Eq. (2).

$$\sigma_1 - \sigma_2 = \frac{Nf}{e} \quad (2)$$

Where N is the fringe order, f is the photoelastic fringe value, and e is the model thickness. The values of the fringe order N are determined experimentally.

A 10mm thickness slice along the load direction (figure 2) is analyzed with plane polarized light on a regular polariscope. One can see clearly the isochromatics and the isoclinics developed on the model particularly in the neighborhood of the contact zones where stresses are higher (zone of maximum shear stress).

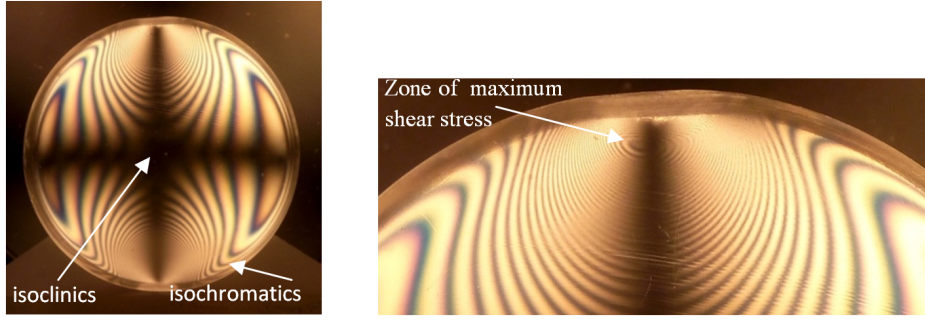


Figure 2: Photoelastic fringes obtained with plan polarized light

NUMERICAL ANALYSES

A finite element analysis is used to determine the stress fields developed in the models particularly in the neighborhood of the contact zones; a program developed under *castem package* allowed us to obtain stress values as well as numerical photoelastic fringes that can be compared to the experimental photoelastic fringes. The analysis is performed in the elastic domain. The meshing is refined in the neighborhood of the contact zone for a better simulation (figure 3).

The isoclinic fringe pattern is calculated with eq. (3) where α is the isoclinic parameter [6]. Once α is obtained the value of $\sin^2 2\alpha$ gives directly the isoclinic fringe pattern.

$$\alpha = \arctan(2\tau_{xy}/(\sigma_x - \sigma_y)) \quad (3)$$

The simulated isochromatic fringe patterns are obtained with eq. (4). The different values of $\sin^2 \varphi/2$ give then easily the numerical isochromatic fringes.

$$\varphi = \frac{2\pi e}{f} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad (4)$$

The graph of variation of the principal stresses difference (Fig. 3) is obtained along the vertical axis. Stresses increase up to approximately 0.6MPa and then decrease as we move away from the contact zone. We can see relatively good agreement between the two solutions.

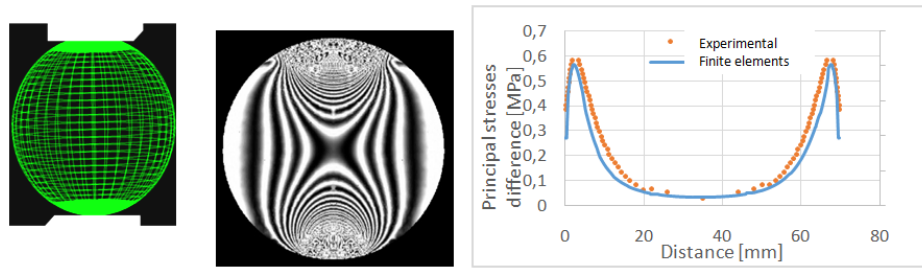


Figure 3: Model meshing and isochromatic fringes for a slice along the load direction

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