APEM journal
Advances in Production Engineering & Management
Volume 19 | Number 4 | December 2024 | pp 512–526
https://doi.org/10.14743/apem2024.4.521
ISSN 1854-6250
Journal home: apem-journal.org
Original scientific paper

Integrated production and maintenance policy for manufacturing systems prone to products' quality degradation

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A B S T R A C T

This paper proposes an integrated production planning and preventive maintenance strategy for manufacturing systems prone to quality degradation. The production planning focus on the regulation of production rates and the sizing of finished product's safety stock to meet costumer's demand. The safety stock is built to palliate shortages when the manufacturing operation begins generating non-conforming products and is shutdown to perform restoration action. In the other hand, preventive maintenance activities are also planned to minimise the quantity of non-conforming products. Mathematical models are proposed and consider all sub-policies and scenarios contingent on the production control policy as well as the entire range of possible values for the safety stock level. A numerical procedure has been established to ascertain the optimal integrated policy, aiming to minimize the total accrued cost per time unit along an infinite horizon. A simulation model has also been created to check and validate the analytical results. Finally, a comparative analysis is presented to prove that the proposed joint policy outperforms other strategies considered in the literature and practice and can result in substantial economic gains.

A R T I C L E   I N F O

Keywords:
Production planning;
Stock control;
Preventive maintenance;
Quality degradation;
Hedging point policy;
Analytical modelling;
Simulation analysis

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Article history:
Received 12 December 2023
Revised 1 December 2024
Accepted 15 December 2024

1. Introduction

Production, quality, and maintenance management play an essential role to improve efficiency, profitability, competitiveness as well as sustainability of today's industrial companies affected by the technical developments of Industry 4.0 [1, 2]. Manufacturing businesses are becoming more and more concerned with producing high-quality goods in a sustainable way which establishes the basis for Quality 4.0 [3, 4]. As equipment maintenance, at a level which can produce an appropriate quality product is essential for keeping them operating, sustainable quality products cannot be produced unless they are subject to efficient maintenance [5].

The Growing adherence to Industry 4.0 proposes a novel framework that links the different facets of industrial systems supervision: production, inventory, maintenance, and quality. In general, the main objective is to reduce operating costs and to increase business opportunities which can only be achieved when production, maintenance and quality strategies are jointly addressed. The integration of these aspects leads to important improvement of the system's performance, especially the significant growing of the industrial profit which can be increased up to 40 % [6]. In a recent study, this joint consideration allows a reduction in overall production cost of up to 44 % [7].

However, more efforts must be made to propose mutual production, maintenance, and quality policies according to the multiple industrial environments and to their own specificities [8, 9]. In this situation, production control is closely associated with inventory management and particularly the safety stock. The majority of researchers do not consider all potential value ranges of security stock capacity. and focus only on high levels ignoring possible optimal solutions related to middle security stock level (i.e. low value of repair time) or to zero-stock strategy (i.e. expensive stock cost). This reflection often leads to the identification of suboptimal policies and implies excessive costs.

This paper studies the combined optimization of production, stock, and maintenance strategy for industrial systems prone to quality degradation. We propose a multi-model framework and a broad holistic strategy that considers all potential security stock value ranges. Mathematical models are developed studying all sub-policies and scenarios that my raise based on the production control policy, level of security stock, and the moment when the system moves into the 'out-of-control state'. A simulation model is also designed to validate mathematical models.

In next Section, a literature review covering integrated production and preventive maintenance policies for industrial systems prone to quality degradation is provided. The system synopsis, dynamic behaviour, and notations are stated in Section 3. Section 4 proposes analytical models for the generated sub-policies and associated scenarios according to all potential security stock value ranges. Section 5 presents a numerical procedure to determine the optimal combined policy. A simulation model is designed to validate analytical models. The beneficial effect of the suggested combined policy is then demonstrated through numerical examples and a comparison analysis involving various control policies drawn from the literature and implemented in practice. Conclusions are finally reported in Section 6.

2. Literature revue

During the last decades, several contributions have been proposed integrating production, inventory, maintenance and quality in production planning and control. These contributions can be classified according to several criteria; mainly, the production control policy, the degree of integration, and the industrial context.

Production control focus on the regulation of the production rate to respond to a continuous demand. Several control policies have been considered in the literature [10, 11]. In their pioneer paper, Akella and Kumar [10] introduce the Hedging Point Policy for systems prone to failures and subject to corrective maintenance and prove its optimality. The HPP policy entails to build a security stock to reduce the impact of breakdowns on demand satisfaction.

The basic HPP policy has been extended by several authors to palliate against the negative impact of random failures using reserve resources [12], outsourcing [13] and preventive maintenance [14]. Other researchers have extended these models to consider the quality degradation of produced items. Some authors studied the specific case of perishable products [15]. Several works consider inspection policies to evaluate the quality of produced items [16]. These works have been extended by integrating static and dynamic sampling plans [7, 9] where the manufacturing system is prone to operation-dependent degradations [17].

The majority of preceding research assumes that industrial systems are prone to failures and that the produced quantity of non-conforming items depends on the equipment age; thus, the quality degradation is correlated with the degradation of the equipment reliability. But several researchers consider that the degradation of product quality is not correlated with the aging of the manufacturing equipment. Therefore, the production system can transit to the 'out-of-control' state and starts generating a proportion of non-conforming parts, according to a probability distribution, [18-30].

Pandey *et al*. [18] propose an analytical model to jointly optimize the production scheduling, quality and maintenance in a one-machine industrial system that produces identical batches. Other researchers dealt mainly with the joint consideration of the economic production quantity, product quality degradation, and preventive maintenance policies [19-25].

A combined production-maintenance policy is suggested by Chelbi *et al*. for single-machine systems that generates items that comply as well as those that don't [26]. The presented model aims to determine the optimum lot size and preventive maintenance plan. Colledani and Tolio developed an analytical approach for mutually optimizing control charts and production parameters for unreliable multi-stage transfer lines. The control policy is based on a kanban system [27]. Dhouib *et al*. considered a joint production/maintenance strategy for systems subject to quality degradation [28]. Bahria *et al*. investigate the problem of the assimilation of production planning and the design of control charts [29]. This work has been extended to involve a preventive maintenance policy [30].

These studies, dealing with the one-product single-machine production systems, focus only on high levels of the safety stock, ignoring low levels which can result in non-optimal policies involving excessive incurred costs. In the other hand, these works do not proceed to the validation of analytical results and limit this important step in the modelling methodology to a sensitivity analysis based on the variation of some parameters.

This study addresses the problem of building stochastic analytical models for the combined optimization of production rate control and PM schedule for manufacturing systems prone to quality degradation. The proposed approach allows addressing all possible security stock value ranges.

The primary contributions include: i) Combined control of production, inventory, and maintenance for systems prone to quality degradation; ii) Multi-model approach according to potential security stock value ranges; iii) Resolution of complex stochastic mathematical models via numerical procedure; iv) Dynamic-stochastic simulation model to validate the analytical results; v) Proposed approach outperforms strategies examined in the literature and put into practice.

3. Manufacturing cell description, dynamic behaviour, and notations

3.1 System description and dynamics

The system subject to this study is an automatic one-machine cell devoted to make and inspect one product type to respond to a constant and ongoing demand (d) (Fig. 1). The equipment, representing an aggregation of several machines, is prone to random quality degradation during the production phase.

Initially, it starts in an 'in-control' situation, generating good products. The duration of this 'in-control' situation is a stochastic variable (τ) characterized by a general probability distribution having a density (cumulative) function $f(\tau)$ ($F(\tau)$) with a mean time within 'in-control' state ($MTIC$). After that, the cell may move to an 'out-of-control' situation, manufacturing non-conforming products with a proportion (α).

Preventive maintenance interventions are scheduled during the in-control period to decrease the shift frequency to the 'out-of-control' situation. They obey to an age maintenance policy (AMP), planned at age (T), and allow to recover the system to an 'as-good as-new' condition. After a situation becomes out of control, a *L*ogistic *D*elay *P*eriod (LDP) is required ensuing the switch to an out-of-control condition to organize total essential resources (both human and material) for the restoration procedure. To guarantee that the demand is met during the LDP, the machine keeps manufacturing. Then, the production process is aborted to undergo restoration which brings the system to the 'as-good as-new' condition, and then starts again generating good products. The delay of the restoration is a r.v. (t_r) defined by a general probability distribution having a density (cumulative) function $h(t_r)$ ($H(t_r)$) with a mean time to restore ($MTTR$).

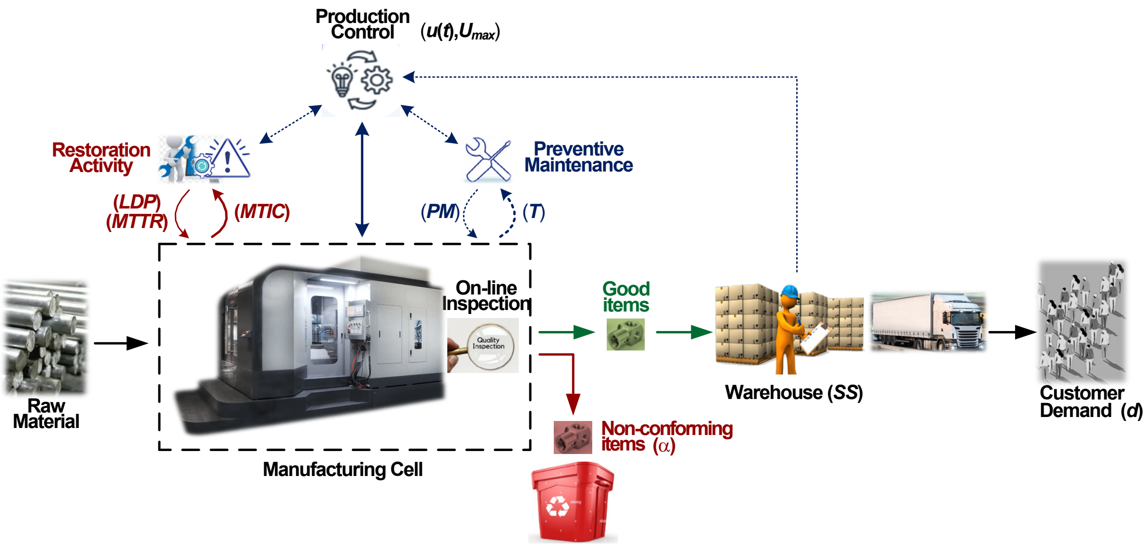

Fig. 1 Manufacturing system dynamics

Throughout the production phase, a reserve of good products is established to address demand and avoid shortages throughout the restoration period. Only after the available inventory runs out can production resume, at which point a setup action is initiated. If a shortage is observed after the restoration, the necessary quantities are not supplied during a period of scarcity; these are seen as penalty costs for missed demand.

The manufacturing process is managed across time through a unified policy governing production, inventory, and maintenance, according to HPP policy. The main aim is to determine the optimal production rates, the level of safety stock, and the preventive maintenance schedule. It aims to minimize the overall incurred cost per time unit over the long range, encompassing costs related to setup, quality, preventive maintenance, restoration, inventory holding, and shortages.

According to the HPP plan, the production cell runs at maximum rate (U_{max}) until the safety stock is built (SS). After then, it lowers its production rate ($u(t)$) to match the demand. A deep analysis of the manufacturing system dynamics has shown different behaviours depending on the value of the safety stock level. In fact, this control strategy, which permits the production of non-conforming goods during LDP while the safety stock is built, leads to three joint control sub-policies, each having a distinct scenario, depending on the many conceivable value ranges of SS:

- Sub-Policy I: HPP and zero-inventory policy ($SS = 0$) (Fig. 2).

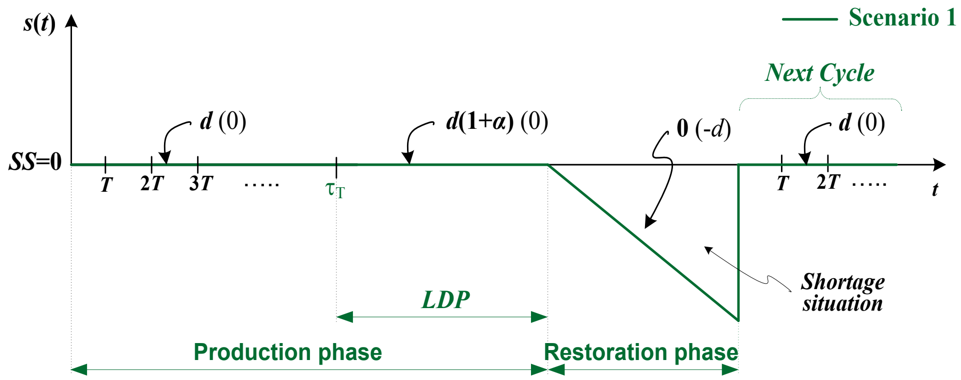

Fig. 2 Inventory level evolution under joint Sub-Policy I

- Sub-Policy II: HPP and stock policy $SS < S_{LDP}$ (middle level of security stock) (Fig. 3). This policy is characterized by the safety stock threshold S_{LDP} (Eq. 1); S_{LDP} is the number of compliant items places in the security stock during LDP.

$$S_{LDP} = LDP(U_{max}(1-\alpha)-d) \tag{1}$$

This crucial threshold signifies that the preparations for restoration conclude upon the completion of constructing the safety stock *SS*. So, and like sub-policy I, non-conforming item manufacture is carried out also during *LDP*.

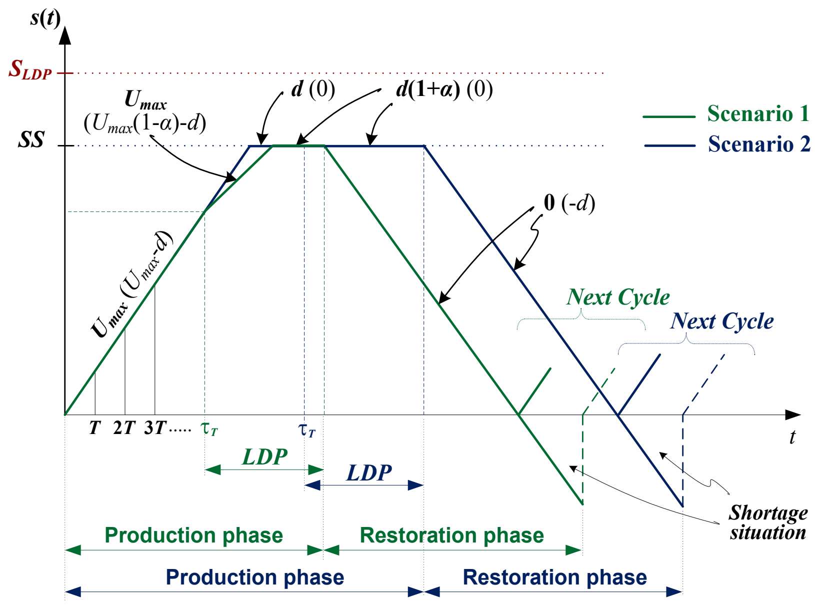

Fig. 3 Inventory level evolution under joint Sub-Policy II

- Sub-Policy III: HPP and stock policy $SS \geq S_{LDP}$ (high-security stock) (Fig. 4). Unlike Sub-Policies I and II, the accomplishment of the preparation action's planning may be achieved prior to the construction of the safety stock *SS* (Scenario 1). In agreement with Sub-policy III, the production of non-compliant goods can extend for a period greater than *LDP*.

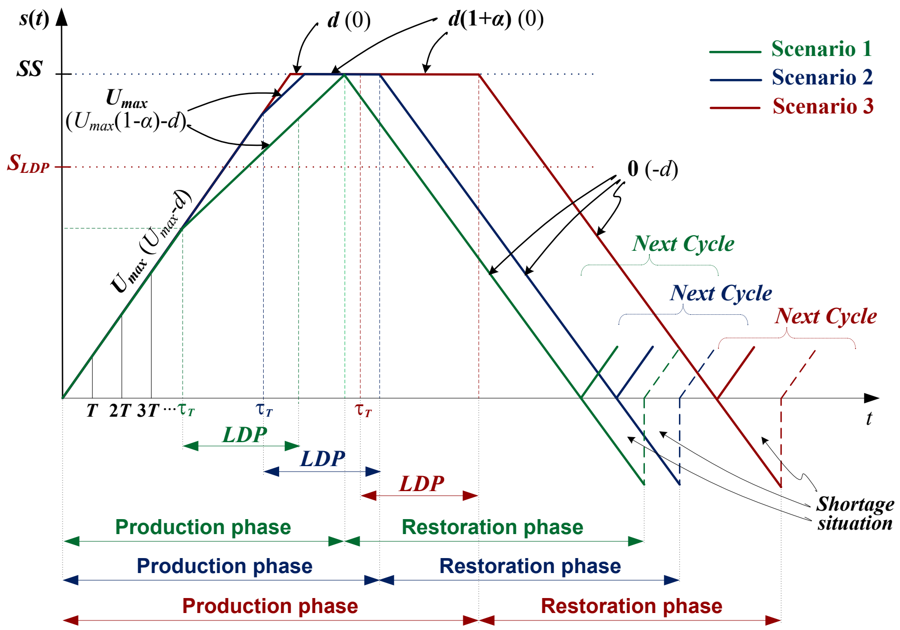

Fig. 4 Inventory level evolution under joint Sun-Policy III

3.2 Notations

In this paper, we consider the following notations, where indices *j* and *i* denote a specific scenario (j = 1, 2, 3) in the sub-policy (i = I, II, III). Additional notations are defined in the text.

n_i	Number of scenarios associated with the sub-policy i (i = I, II, III).
$s(t)$	Inventory level at instant t.
τ_T	Random instant to transit to the 'out-of-control' situation according to the AMP.
$E_{ij}(\tau_T)$	Mean duration of residence in control situation associated to sub-policy i-scenario j.
$E_{ij}(t_r)$	Anticipated restoration delay while scarcity case associated to sub-policy i-scenario j.
Pr_{ij}	For sub-policy i, Probability of falling into scenario j.
Pr_{Hij} (Pr_{Sij})	Probability of experiencing an excess inventory (scarcity) condition for sub-policy i, following the occurrence of scenario j.
CL_{Hij} (CL_{Sij})	Expected length of the production/restoration cycle in an excess inventory (scarcity) condition for sub-policy i, following the occurrence of scenario j.
$\overline{IH_{ij}}$	Average stock level held during the production phase for sub-policy i, in scenario j.
$PPIC_{ij}$	Expected Inventory holding cost incurred during Production Phase associated to sub-policy i, following the occurrence of scenario j.
NCC_{ij}	Cost of Non-Compliant parts for sub-policy i, scenario j .
$NbPM_{ij}$	Mean number of Preventive Maintenance interventions for sub-policy i, in scenario j.
PMC_{ij}	Cost of Preventive Maintenance for sub-policy i, in scenario j.
PPC_{ij}	Anticipated cost accrued through the Production Phase associated to sub-policy i-scenario j.
$\overline{IH_{Rij}}$	Average inventory kept during restoration phase for sub-policy i, in scenario j.
$RPIC_{ij}$	Expected Inventory holding cost incurred during Restoration Phase associated to sub-policy i, following the occurrence of scenario j.
$NbLD_{ij}$	Expected amount of Missing Demand for sub-policy i, in scenario j.
SC_{ij}	Expected Shortage cost for sub-policy i, in scenario j.
RPC_{Hij} (RPC_{Sij})	Anticipated cost accrued through the Restoration Phase in an excess (lack) circumstance for sub-policy i-scenario j.
TC_i	Overall Expected cost associated with the joint sub-policy i.

4. Modelling joint control policies

To model the behaviour of the proposed joint control policies, the cycle of the studied system is divided into two phases: the production phase, during which the cell is producing, and the phase of restoration following the shutdown of the manufacturing cell. The machine transits, cyclically, from 'operating' to 'shutdown' and from 'shutdown' to 'operating' state. Figs. 2, 3, and 4 show that each cycle starts and ends with an empty stock. Based on these observations, we are certain that the system dynamics is a renewal process since cycles are independent of one another. Therefore, 'the elementary renewal theorem' [31] can be employed to calculate the mean value of any KPI per time unit beyond an infinite time span (TC, WIP, preventive actions number, Amount of non-conforming items, etc.).

The global cost for sub-policy i (i = I, II, III) is assessed by Eq. 2. It consists of the expenses paid both during manufacturing and restoration stages whether there is excess or lack of inventory, weighted by corresponding probability of scenario.

$$TC_i = \frac{\sum_{j=1}^{n_i}\Big(\big(PPC_{ij} + RPC_{Hij}\big)\text{Pr}_{Hij} + \big(PPC_{ij} + RPC_{Sij}\big)\text{Pr}_{Sij}\Big)\text{Pr}_{ij}}{\sum_{j=1}^{n_i}\big(CL_{Hij}\text{Pr}_{Hij} + CL_{Sij}\text{Pr}_{Sij}\big)\text{Pr}_{ij}} \tag{2}$$

The production phase cost comprises the expenses due to setup (C_{SU}), non-compliant items, inventory holding, and preventive maintenance activities (Eq. 3).

$$PPC_{ij} = C_{SU} + PPIC_{ij} + NCC_{ij} + PMC_{ij} \tag{3}$$

The inventory keeping cost is computed by multiplying the cost of an item held in stock per time unit (C_H) with the average quantity seized in the safety stock (Eq. 4).

$$PPIC_{ij} = C_H\,\overline{IH_{ij}} \tag{4}$$

The non-conforming items expense includes the costs of raw material (C_{RM}) and operating of the manufacturing cell per time unit (C_{MCO}).

The preventive maintenance charge is determined by multiplying the expense of a preventive activity (C_{PM}) with the anticipated number of performed preventive interventions (Eq. 5).

$$PMC_{ij} = C_{PM}\,NbPM_{ij} \tag{5}$$

The restoration phase cost comprises, in case of surplus, the inventory holding and the restoration cost (C_R) (Eq. 6). In scarcity situation, it also includes the unit shortage expense (C_S) multiplied by the non-delivered parts (Eqs. 7 and 9).

$$RPC_{Hij} = C_R + RPIC_{ij} \tag{6}$$

$$RPC_{Sij} = C_R + RPIC_{ij} + SC_{ij} \tag{7}$$

where

$$RPIC_{ij} = C_H\,\overline{IH_{Rij}} \tag{8}$$

and

$$SC_{ij} = C_S\,NbLD_{ij} \tag{9}$$

The cycle length equals the mean time to transit to 'out-of-control' condition under an AMP policy, the *LDP* while reaching *SS*, and the *MTTR* in a situation of a shortage or the delay to consume all the safety inventory in instance of excess inventory.

4.1 Sub-policy I: HPP and zero-inventory policy

If a zero-stock policy is required based on production parameters (*SS* = 0), the production control sub-policy can be expressed by Eq. 10.

$$u(t) = \begin{cases} d & \text{If the cell is producing in the 'in control' state} \\ d(1+\alpha) & \text{If the cell is producing during } LDP \\ 0 & \text{alternatively} \end{cases} \tag{10}$$

The progression of the stock level is presented in Fig. 2, where shortage situation is inevitable; in fact, only one scenario can occur according to Sub-Policy I ($n_{\mathrm{I}} = 1$). Fig. 2 also indicates the production and the inventory construction/depletion rates during production and restorations phases. The global accrued cost TC_{I} is assessed by Eq. 2, where:

- $\mathrm{Pr}_{\mathrm{I}1} = 1$, $\mathrm{Pr}_{H\mathrm{I}1} = 0$, and $\mathrm{Pr}_{S\mathrm{I}1} = 1$.
- The cycle length is calculated using Eq. 11.

$$CL_{S\mathrm{I}1} = E_{\mathrm{I}1}(\tau_T) + LDP + MTTR \tag{11}$$

  $E_{\mathrm{I}1}(\tau_T)$ is the expected time to move to 'out-of-control' condition beneath a *T*-age AMP policy (Eq. 12), and *MTTR* is calculated by Eq. 13.

$$E_{\mathrm{I}1}(\tau_T) = \sum_{k=0}^{\infty} R(T)^k \int_{kT}^{(k+1)T} \tau\, f(\tau - kT)\,d\tau \tag{12}$$

$$MTTR = \int_0^{\infty} t_r\, h(t_r)\,dt_r \tag{13}$$

- $PPC_{\mathrm{I}1}$ can be assessed with Eq. 3, with $PPIC_{\mathrm{I}1}$ is 0 as no stock building is allowed (Fig. 2). $NCC_{\mathrm{I}1}$ and $PMC_{\mathrm{I}1}$ costs can be assessed by Eqs. 14 and 5, respectively, where $NbPM_{\mathrm{I}1}$ is given by Eq. 15.

$$NCC_{\mathrm{I}1} = \alpha\, LDP\left(d\, C_{RM} + \frac{1}{1+\alpha}\, C_{MCO}\right) \tag{14}$$

$$NbPM_{\text{I}1} = \sum_{k=0}^{\infty} k\, R(T)^k F(T) \tag{15}$$

- $RPC_{S\,\text{I}1}$ is given by Eq. 7; $RPIC_{\text{I}1}$ is 0 and $SC_{\text{I}1}$ is expressed with Eq. 9, where:

$$NbLD_{\text{I}1} = d \int_0^{\infty} t_r\, h(t_r) dt_r \tag{16}$$

4.2 Sub-policy II: HPP and the stock policy *SS* < S_{LDP}

When mandatory security stock quantity is fewer than S_{LDP} based on production parameters (middle level), the production control sub-policy will be described by Eq. 17.

$$u(t) = \begin{cases} U_{max} & \text{If } s(t) < SS \\ d & \text{If } s(t) = SS \text{ where the cell is 'in control' state} \\ d(1+\alpha) & \text{If } s(t) = SS \text{ where the cell is 'out of control' state} \\ 0 & \text{alternatively} \end{cases} \tag{17}$$

In the production phase, the system may exist in one of two scenarios ($n_{\text{II}} = 2$) contingent on the shift instant (τ_T) to the 'out-of-control' condition (Fig. 3).

Scenario 1: Takes place if the cell go into the 'out-of-control' condition before attaining *SS*. Constraint $\tau_T < SS/(U_{max} - d)$ imposes the occurrence of scenario 1.

Scenario 2: Conditioned by the constraint $\tau_T \geq SS/(U_{max} - d)$, it takes place if the security inventory is at present filled.

The probability to be into Scenario 1 (Scenario 2) can be expressed by Eq. 18 (Eq. 19).

$$\text{Pr}_{\text{II}1} = \sum_{k=0}^{NM_S - 1} R(T)^k\, F(T) + R(T)^{NM_S}\, F\left(\frac{SS}{U_{max} - d} \bmod T\right) \tag{18}$$

$$\text{Pr}_{\text{II}2} = 1 - \text{Pr}_{\text{II}1} \tag{19}$$

NM_S denotes the highest preventive maintenance interventions performed prior to the attainment of the inventory level *SS* ($NM_S = \frac{SS}{U_{max}-d} \operatorname{div} T$).

According to sub-policy II, the occurrence probability of an inventory excess (scarcity) circumstance does not depend on manifestation of Scenario 1 (Scenario 2). Thus, these probabilities are provided by Eqs. 20 and 21, for *i* = II and *j* = 1, 2.

$$\text{Pr}_{Hij} = H(SS/d) \tag{20}$$

$$\text{Pr}_{Sij} = 1 - H(SS/d) \tag{21}$$

The total expense CT_{II} is given by Eq. 2, with cycle lengths and production/restoration phase costs are assessed in the following paragraphs.

The average renewal cycle length in a surplus (shortage) situation is given by Eq. 22 (Eq. 23), for *i* = II and *j* = 1, 2.

$$CL_{Hij} = E_{ij}(\tau_T) + LDP + SS/d \tag{22}$$

$$CL_{Sij} = E_{ij}(\tau_T) + LDP + E_{ij}(t_r) \tag{23}$$

$E_{ij}(\tau_T)$ and $E_{ij}(t_r)$ are given by Eqs. 24, 25, and 26, respectively (*i* = II, *j* = 1, 2).

$$E_{\text{II}1}(\tau_T) = \left(\sum_{k=0}^{NM_S - 1} R(T)^k \int_{k\,T}^{(k+1)T} \tau\, f(\tau - k\,T) d\tau + R(T)^{NM_S} \int_{NM_S\,T}^{\frac{SS}{U_{max}-d}} \tau\, f(\tau - NM_S\,T) d\tau \right) \Big/ \text{Pr}_{\text{II}1} \tag{24}$$

$$E_{\text{II}2}(\tau_T) = \left(\sum_{k=NM_S}^{\infty} R(T)^k \int_{k\,T}^{(k+1)T} \tau\, f(\tau - kT) d\tau - R(T)^{NM_S} \int_{NM_S\,T}^{\frac{SS}{U_{max}-d}} \tau\, f(\tau - NM_S\,T) d\tau \right) \Big/ \text{Pr}_{\text{II}2} \tag{25}$$

$$E_{ij}(t_r) = \int_{SS/d}^{\infty} t_r\, h(t_r) dt_r / \text{Pr}_{Sij} \tag{26}$$

Eq. 3 assesses the encountered charge throughout the production stage where the inventory expense is evaluated by Eq. 4 and the average amount seized into security stock for Scenario 1 (Scenario 2) is calculated by Eq. 27 (Eq. 28).

$$\overline{IH_{\text{II}1}} = (U_{max} - d)\left(LDP\ E_{\text{II}1}(\tau_T) + \int_0^{\frac{SS}{U_{max}-d}} \tau^2 f(\tau)d\tau/2\ \text{Pr}_{\text{II}1}\right)$$
$$+\left(SS^2 - 2SS\ E_{\text{II}1}(\tau_T)(U_{max} - d) + (U_{max} - d)^2 \int_0^{\frac{SS}{U_{max}-d}} \tau^2 f(\tau)d\tau \Big/ \text{Pr}_{\text{II}1}\right)\Big/ 2(U_{max}(1-\alpha) - d)$$
$$+\big(SS - E_{\text{II}1}(\tau_T)\ (U_{max} - d)\big)\big(LDP - \big(SS - E_{\text{II}1}(\tau_T)\ (U_{max} - d)\big)/(U_{max}(1-\alpha) - d)\big) \tag{27}$$

$$\overline{IH_{\text{II}2}} = SS\ (E_{\text{II}2}(\tau_T) + LDP) - SS^2/2(U_{max} - d) \tag{28}$$

The NCC_{ij} are given by Eqs. 29 and 30 and the PMC_{ij} by Eq. 5, where $NbPM_{ij}$ are given by Eqs. 31 and 32, for i = II and j = 1, 2.

$$NCC_{\text{II}1} = (C_{RM}U_{max} + C_{MCO})\alpha\big(SS - E_{\text{II}1}(\tau_T)(U_{max} - d)\big)/(U_{max}(1-\alpha) - d) +$$
$$(C_{RM}d + C_{MCO}/(1+\alpha))\alpha\big(LDP - \big(SS - E_{\text{II}1}(\tau_T)(U_{max} - d)\big)/(U_{max}(1-\alpha) - d)\big) \tag{29}$$

$$NCC_{\text{II}2} = \alpha\ (C_{RM}d + C_{MCO}/(1+\alpha))\ LDP \tag{30}$$

$$NbPM_{\text{II}1} = \left(\sum_{k=0}^{NM_S-1} k\ R(T)^k\ F(T) + NM_S\ R(T)^{NM_S}\ F\left(\frac{SS}{U_{max} - d} \bmod T\right)\right)\Big/ \text{Pr}_{\text{II}1} \tag{31}$$

$$NbPM_{\text{II}2} = \left(NM_S\ R(T)^{NM_S}\left(F(T) - F\left(\frac{SS}{U_{max} - d} \bmod T\right)\right) + \sum_{k=NM_S+1}^{\infty} k\ R(T)^k\ F(T)\right)\Big/ \text{Pr}_{\text{II}2} \tag{32}$$

The incurred expense throughout the restoration stage can be expressed by Eq. 6 (Eq. 7) in a surplus (shortage) situation, and Eq. 8, with the mean amount detained in inventory and the sum of lost demands can be appraised by Eqs. 33 and 34, respectively, for i = II and j = 1, 2.

$$\overline{IH_{Rij}} = SS^2/2d \tag{33}$$

$$NbLD_{ij} = d\ \big(E_{ij}(t_r) - SS/d\big) \tag{34}$$

4.3 Sub-policy III: HPP and the stock policy $SS \geq S_{LDP}$

When mandatory security stock quantity is superior to S_{LDP} based on production parameters (high level), the production rate will also be governed by Eq. 17.

Throughout the stage of production, the system may occur in either of three following scenarios (n_{III} = 3) dependent on the change instant to the 'out-of-control' condition (Fig. 4).

Scenario 1: Takes place if the producing cell enters 'out-of-control' condition before attaining S_{LDP}. This is the only scenario in which non-conforming items are manufactured for a period exceeding the *LDP*. The proposed sub-policy explicitly recommends continuing manufacturing until reaching the *SS* to mitigate the possibility of deficiencies. Constraint $\tau_T < \frac{SS - S_{LDP}}{U_{max} - d}$ imposes the occurrence of Scenario 1.

Scenario 2: Trained by the constraint $\frac{SS - S_{LDP}}{U_{max} - d} \leq \tau_T < \frac{SS}{U_{max} - d}$, it happens when the cell enters out-of-control state as its inventory is split between S_{LDP} and *SS*.

Scenario 3: Conditioned by the constraint $\tau_T \geq SS/(U_{max} - d)$, it occurs once the security stock *SS* level is by now reached.

The probability of being in Scenarios 1, 2, and 3 are given by Eqs. 35, 36, and 37.

$$\text{Pr}_{\text{III}1} = \sum_{k=0}^{NM_L-1} R(T)^k\ F(T) + R(T)^{NM_L}\ F\left(\frac{S_{LDP}}{U_{max} - d} \bmod T\right) \tag{35}$$

$$\text{Pr}_{\text{III}2} = \sum_{k=NM_L}^{NM_S-1} R(T)^k\ F(T) - R(T)^{NM_L}\ F\left(\frac{S_{LDP}}{U_{max} - d} \bmod T\right) + R(T)^{NM_S}\ F\left(\frac{SS}{U_{max} - d} \bmod T\right) \tag{36}$$

$$\mathrm{Pr}_{\mathrm{III3}} = 1 - (\mathrm{Pr}_{\mathrm{III1}} + \mathrm{Pr}_{\mathrm{III2}}) \tag{37}$$

NM_L expresses the highest preventive maintenance interventions executed prior to the completion of the inventory level S_{LDP} ($NM_L = \frac{S_{LDP}}{U_{max}-d} \operatorname{div} T$).

The probability of being in excess (lack) circumstances is given by Eq. 20 (Eq. 21).

The total charge CT_{III} is given by Eq. 2, with cycle lengths and production/restoration phase costs are assessed as follows.

The average renewal cycle length for Scenario 1 in a holding (shortage) case is given by Eq. 38 (Eq. 39), and by Eq. 22 (Eq. 23) for Scenarios 2 and 3, where *i* = III and *j* = 2, 3.

$$CL_{H\mathrm{III1}} = (SS - \alpha\, E_{\mathrm{III1}}(\tau_T)\, U_{max})/(U_{max}(1-\alpha) - d) + SS/d \tag{38}$$

$$CL_{S\mathrm{III1}} = (SS - \alpha\, E_{\mathrm{III1}}(\tau_T)\, U_{max})/(U_{max}(1-\alpha) - d) + E_{\mathrm{III1}}(t_r) \tag{39}$$

$E_{ij}(\tau_T)$ is given by Eqs. 40, 41, and 42, and $E_{ij}(t_r)$ is given by Eq. 26, for *i* = III and *j* = 1, 2, 3.

$$E_{\mathrm{III1}}(\tau_T) = \left(\sum_{k=0}^{NM_L-1} R(T)^k \int_{kT}^{(k+1)T} \tau\, f(\tau - kT)d\tau + R(T)^{NM_L} \int_{NM_L T}^{\frac{S_{LDP}}{U_{max}-d}} \tau\, f(\tau - NM_L\, T)d\tau\right) \Big/ \mathrm{Pr}_{\mathrm{III1}} \tag{40}$$

$$E_{\mathrm{III2}}(\tau_T) = \left(\begin{matrix} R(T)^{NM_S} \int_{NM_S T}^{\frac{SS}{U_{max}-d}} \tau\, f(\tau - NM_S\, T)d\tau + \sum_{k=NM_L}^{NM_L-1} R(T)^k \int_{kT}^{(k+1)T} \tau\, f(\tau - kT)d\tau \\ -R(T)^{NM_L} \int_{NM_L T}^{\frac{S_{LDP}}{U_{max}-d}} \tau\, f(\tau - NM_L\, T)d\tau \end{matrix}\right) \Big/ \mathrm{Pr}_{\mathrm{III2}} \tag{41}$$

$$E_{\mathrm{III3}}(\tau_T) = \left(\sum_{k=NM_S}^{\infty} R(T)^k \int_{k\,T}^{(k+1)T} \tau\, f(\tau - kT)d\tau - R(T)^{NM_S} \int_{NM_S T}^{\frac{SS}{U_{max}-d}} \tau\, f(\tau - NM_S\, T)d\tau\right) \Big/ \mathrm{Pr}_{\mathrm{III3}} \tag{42}$$

The recorded cost during the fabrication stage is evaluated by Eq. 3. The cost of inventory held can be assessed by Eq. 4 with the expected amount seized at occurrence of Scenarios 1, 2, and 3 can be evaluated by Eqs. 43, 44, and 45, respectively.

$$\begin{gathered}\overline{IH_{\mathrm{III1}}} = (U_{max} - d) \int_0^{S_{LDP}/U_{max}-d} \tau^2\, f(\tau)d\tau/2\, \mathrm{Pr}_{\mathrm{III1}} + \\ \left(SS^2 - (U_{max} - d)^2 \int_0^{S_{LDP}/U_{max}-d} \tau^2\, f(\tau)d\tau/\mathrm{Pr}_{\mathrm{III1}}\right)/2(U_{max}(1-\alpha) - d)\end{gathered} \tag{43}$$

$$\begin{gathered}\overline{IH_{\mathrm{III2}}} = (U_{max} - d) \int_{S_{LDP}/U_{max}-d}^{SS/U_{max}-d} \tau^2\, f(\tau)d\tau/2\, \mathrm{Pr}_{\mathrm{III2}} + SS\, LDP - \\ \left(SS - E_{\mathrm{III2}}(\tau_T)(U_{max} - d)\right)^2/2(U_{max}(1-\alpha) - d)\end{gathered} \tag{44}$$

$$\overline{IH_{\mathrm{III3}}} = SS\, (E_{\mathrm{III3}}(\tau_T) + LDP - SS/(U_{max} - d)) + SS^2/2(U_{max} - d) \tag{45}$$

The NCC_{ij} are given by Eqs. 46, 47, and 48 and the PMC_{ij} by Eq. 5, where $NbPM_{ij}$ are given by Eqs. 49, 50, and 51, for *i* = III and *j* = 1, 2, 3.

$$NCC_{\mathrm{III1}} = (C_{RM}U_{max} + C_{MCO})\alpha\big(SS - E_{\mathrm{III1}}(\tau_T)(U_{max} - d)\big)/(U_{max}(1-\alpha) - d) \tag{46}$$

$$\begin{gathered}NCC_{\mathrm{III2}} = (C_{RM}U_{max} + C_{MCO})\alpha\big(SS - E_{\mathrm{III2}}(\tau_T)(U_{max} - d)\big)/(U_{max}(1-\alpha) - d) + \\ (C_{RM}d + C_{MCO}/(1+\alpha))\alpha\Big(LDP - \big(SS - E_{\mathrm{III2}}(\tau_T)(U_{max} - d)\big)/(U_{max}(1-\alpha) - d)\Big)\end{gathered} \tag{47}$$

$$NCC_{\mathrm{III3}} = \alpha\, (C_{RM}d + C_{MCO}/(1+\alpha))\, LDP \tag{48}$$

$$NbPM_{\mathrm{III1}} = \left(\sum_{k=0}^{NM_L-1} k\, R(T)^k\, F(T) + NM_L\, R(T)^{NM_L}\, F\left(\frac{S_{LDP}}{U_{max}-d} \bmod T\right)\right) \Big/ \mathrm{Pr}_{\mathrm{III1}} \tag{49}$$

$$NbPM_{\mathrm{III2}} = \left(\begin{matrix} NM_L\, R(T)^{NM_L}\left(F(T) - F\left(\frac{S_{LDP}}{U_{max}-d} \bmod T\right)\right) + \\ \sum_{k=NM_L+1}^{NM_S-1} k\, R(T)^k\, F(T) + NM_S\, R(T)^{NM_S}\, F\left(\frac{SS}{U_{max}-d} \bmod T\right) \end{matrix}\right) \Big/ \mathrm{Pr}_{\mathrm{III2}} \tag{50}$$

$$NbPM_{\text{III3}} = \left(NM_S\, R(T)^{NM_S} \left(F(T) - F\left(\frac{SS}{U_{max} - d} \bmod T \right) \right) + \sum_{k=NM_S+1}^{\infty} k\, R(T)^k\, F(T) \right) \Big/ \text{Pr}_{\text{III3}} \quad (51)$$

In an excess (deficiency) condition, the accrued restoration phase charge is given by Eq. 6 (Eq. 7) and Eq. 8 with the expected amount detained in inventory and the amount of lost demands can be given by Eqs. 33 and 34, respectively, for *i* = III and *j* = 1, 2, 3.

5. Numerical resolution methodology, results and comparative analysis

5.1 Optimization numerical procedure

Two decision variables specify the optimal solution: the security stock capacity *SS*, and the preventive maintenance age *T*. A numerical resolution procedure has been designed and coded based on the programming language 'Fortran', to determine the optimal solution (SS^*, T^*) minimizing the overall expected cost (CT^*). This algorithm, which is illustrated in Fig. 5, combines a number of subroutines to calculate the probability of each scenario, predicted cycle durations, several *KPIs*, and expenses suffered during the production and restoration cycles for the three sub-policies.

5.2 Numerical results and analytical model validation

Consider first a base case describing the production cell. All costs and operating, demand, quality, and maintenance parameters are provided in Table 1. The time in-control state (to restore) has a Weibull (Gamma) distribution. After executing the numerical program, we established that the minimum total cost (CT^*), as determined by the optimal decision variables SS^* = 1,314 items & T^* = 0.14 months, is equal to 96,621.37 \$/month. Fig. 6 presents the evolution of *CT* depending on decision variables *SS* and *T*. One can note the convex character of the surface confirming the presence of the optimum value.

Table 1 Sample of 6 manufacturing cell configurations with randomly generated parameters

Case	Cost Parameters							Operating Parameters					
	C_H	C_S	C_{RM}	C_{MCO}	C_{SU}	C_R	C_{PM}	*d*	U_{max}	*MTIC*	*α* (%)	*LDP*	*MTTR*
Base	40	400	500	150,000	5,000	10,000	2,000	20,160	32,400	0.9027	1	0.10	0.05
1	80	385	150	200,500	2,800	9,000	3,800	22,350	40,000	1.2638	19	0.11	0.022
2	50	200	250	250,000	3,000	15,000	2,800	18,000	35,000	1.8054	5	0.08	0.10
3	90	500	300	220,000	5,500	20,000	4,000	13,000	20,000	0.9027	20	0.10	0.02
4	25	430	240	160,000	6,200	7,000	4,200	21,540	38,500	1.1735	11	0.03	0.1
5	120	420	290	350,000	6,000	14,000	2,500	20,000	35,100	1.0832	12	0.12	0.067
6	15	630	140	90,000	3,000	4,000	2,700	28,000	44,000	0.6319	8	0.09	0.033

To validate the proposed approach, the analytical stochastic models and the numerical resolution procedure, a simulation model is constructed with ARENA simulation package. It imitates the production cell dynamic and stochastic behaviour. Table 2 presents first the validation of the base case. Hundreds of cell configurations were randomly generated, evaluated for optimal solutions, and then analytical results were validated through simulation. Each simulation is executed during 1,000,000 cycles (Production-Restoration) with warmup delay of 100,000 cycles to assure the steadiness of performance measures. Ten replications were carried out for every cell configuration.

Table 2 also presents a sample of 6 cell configurations (cases 1 to 6) with randomly generated parameters validated through simulation model (Tab. 1); the results show that all analytically computed *KPI*s fell inside the simulation's 95 % confidence interval. The key performance indicators analysed in this paper are the expected values of the following: cycle length (*CL*), work in process (*WIP*), number of lost demand (*NbLD*), number of non-conforming items (*NbNC*), number of preventive maintenance interventions (*NbPM*), and total incurred cost (TC^*).

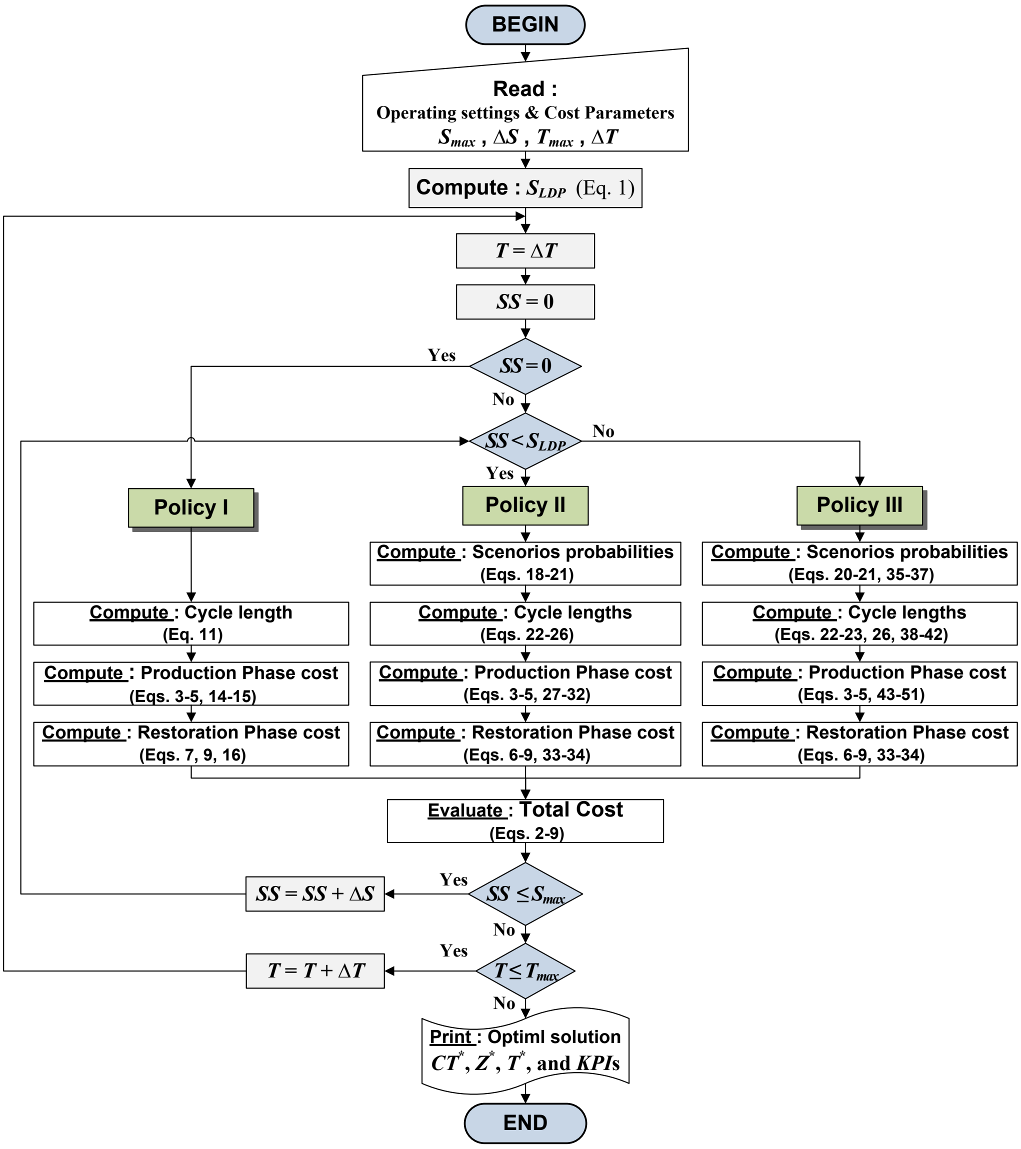

Fig. 5 Optimal policy resolution algorithm

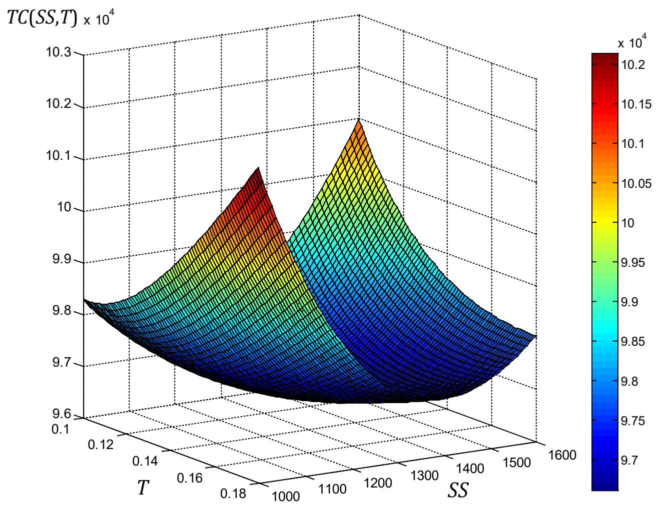

Fig. 6 Evolution of the total incurred cost $CT(SS,T)$

Table 2 Analytical results validation by simulation for manufacturing cell configurations defined in Table 1

Case	Key Performance Indicators					Optimal Solution		
	CL	*WIP*	*NbLD*	*NbNC*	*NbPM*	TC^*	T^*	SS^*
Base	**2.860**	**1,270.52**	**171.24**	**20.35**	**18.59**	**96,621.4**	**0.14**	**1,314**
	[2.853 - 2.863]	[1,270.37 - 1,270.59]	[170.61 - 172.27]	[20.34 - 20.36]	[18.55 - 18.62]	[96,540.1 - 96,761.8]		
1	**4.420**	**120.36**	**379.44**	**476.12**	**28.01**	**86008.1**	**0.15**	**121**
	[4.416 - 4.427]	[120.367 - 120.369]	[379.06 - 379.86]	[467.12 - 467.12]	[27.97 - 28.03]	[85,959.4 - 86,107.7]		
2	**8.719**	**0**	**1,800**	**72**	**77.02**	**70,263.73**	**0.11**	**0**
	[8.707 - 8.728]	[0 - 0]	[1,798.8 - 1,801.5]	[72 - 72]	[76.92 - 77.11]	[70,210.7 - 70,361.9]		
3	**2.720**	**225.351**	**84.47**	**260.25**	**16.71**	**99,821.7**	**0.15**	**228**
	[2.718 - 2.721]	[225.350 - 225.353]	[84.26 - 84.62]	[260.24 - 260.26]	[16.69 - 16.72]	[99,797.8 - 99,871.7]		
4	**3.914**	**2,998.14**	**286.38**	**89.55**	**22.66**	**139,729.7**	**0.16**	**3,142**
	[3.903 - 3.918]	[2,997.75 - 2,998.26]	[285.13 - 288.53]	[89.49 - 89.70]	[22.59 - 22.68]	[139,655.4 - 140,004.7]		
5	**6.070**	**0**	**1,333.33**	**288**	**117.07**	**158,260.3**	**0.05**	**0**
	[6.061 - 6.079]	[0 - 0]	[1332.35 - 1334.46]	[288 - 288]	[116.88 - 117.24]	[158,110.6 - 158,507.9]		
6	**1.480**	**2,275.11**	**16.77**	**215.93**	**5.60**	**77,041.9**	**0.21**	**2,481**
	[1.478 - 1.483]	[2,274.56 – 2,275.36]	[16.54 - 17.01]	[215.83 - 216.07]	[5.59 - 5.62]	[76,937.9 - 77,160.1]		

5.3 Comparative analysis

To highlight the contribution of the suggested approach, a comparative investigation was carried out with strategies considered in the literature and practice:

Strat. 1: The Proposed policy, described by 3 sub-policies and 6 scenarios.

Strat. 2: proposal of Chelbi *et al.* [29], described by 2 scenarios (no sub-policies), and can only handle the case $SS > S_{LDP}$.

Strat. 3: proposal of Dhouib *et al.* [31], described by 3 scenarios (no sub-policies), and can only handle the case $SS > S_{LDP}$.

Strat. 4: The proposed policy (*Strat.* 1) but limited to a range of safety stock level $SS > S_{LDP}$ as considered in *Strat.* 2 and *Strat.* 3.

Strat. 5: The proposed policy (*Strat.* 1) without preventive maintenance.

The comparative study is based on the sample of 7 manufacturing cell configurations with randomly generated parameters presented in Table 1 (Base case and cases 1 to 6).

Table 3 displays first the optimal control solution for the proposed *Strat*. 1 (SS^*,T^*,TC^*) and according to which optimal sub-policy (P^*) it belongs (P^* = I, II or III). It also presents optimal solutions for the other compared strategies. First, we must recall that all the results generated by the proposed approach (*Strat.* 1) are validated by comparing them with those made by simulation, confirming the exactness of the proposed analytic models. The results demonstrate that the recommended approach (*Strat.* 1) outperforms other ones and allowing cost savings of up to 96 % (60 %) compared to the proposal of Chelbi *et al.* (2008) (*Strat.* 2 – case 1) (Dhouib *et al.* (2012) (*Strat.* 3 – case 5)); The primary reason for this discrepancy is because these models do not account for any potential security stock value ranges.

Table 3 Comparison of the proposed policy to ones from literature and practice

Case	*Strat.* 1 - Proposed -				*Strat.* 2				*Strat.* 3				*Strat.* 4				*Strat.* 5			
	SS^*	T^*	TC^*	P^*	SS^*	T^*	TC^*	*Cost Red.* (%)	SS^*	T^*	TC^*	*Cost Red.* (%)	SS^*	T^*	TC^*	*Cost Red.* (%)	SS^*	T^*	TC^*	*Cost Red.* (%)
Base	1,314	0.14	96,621	III	1,202	0.09	116,924	21.1	1,493	0.17	105,955	9.7	1,314	0.14	96,621	0	2,087	-	113,809	17.8
1	121	0.15	86,008	II	1,106	0.14	168,782	96.3	1,106	0.3	131,641	53.1	1,106	0.29	127,107	47.8	638	-	134,278	56.1
2	0	0.11	70,129	I	1,220	0.16	112,269	60.1	1,220	0.20	107,329	53.1	1,220	0.20	102,766	46.5	1,666	-	150,759	114.9
3	228	0.15	99,821	II	521	0.11	154,108	54.4	300	0.16	111,862	12.1	300	0.17	100,646	8.3	426	-	152,680	52.9
4	3,142	0.16	139,729	III	4,218	0.19	172,126	23.2	3,691	0.31	155,617	11.4	3,142	0.16	139,729	0	4,712	-	153,390	9.8
5	0	0.05	158,260	I	1,307	0.06	281,517	77.8	1,307	0.07	253,460	60.2	1,307	0.09	237,428	50.1	1,592	-	346,397	125.2
6	2,481	0.21	77,041	III	3,246	0.16	100,747	30.8	2,532	0.24	89,867	16.7	2,481	0.21	77,041	0	2,869	-	88,905	15.4

Strat. 4, which is a limited version of *Strat.* 1, surpasses *Strat*. 2 and *Strat*. 3 on all studied configurations, but it generates errors of up to 50 % compared to *Strat*. 1 (case 5). Finally, comparing *Strat*. 5 to the proposed *Strat*. 1 show that preventive maintenance allowed reducing the total incurred cost of up to 125 % (case 5). We note that the high-cost reductions are obtained when the zero-stock is the best sub-policy; in fact, this joint policy is based on high inventory holding cost, and since no preventive maintenance is planned with *Strat*. 5, it reacts by increasing the security stock capacity (SS^*), from 0 (*Strat*. 1 – case 5) to 1,592 (*Strat*. 5 – case 5) implying high and costly *WIP*.

6. Conclusion

In this manuscript, we introduce a comprehensive strategy for production planning and preventive maintenance concerning manufacturing systems prone to quality degradation considering every potential range of values for the security stock. An age-based AMP plan is implemented to diminish the amount of non-compliant items. Three joint control sub-policies are developed based on a specific value of the security stock capacity determined by the production system characteristics and the amount added to the security stock during the logistic delay period (S_{LDP}): Sub-policy I – HPP and the zero-inventory policy, Sub-policy II – HPP and the stock policy $SS < S_{LDP}$, and Sub-policy III – HPP and the stock policy $SS \geq S_{LDP}$.

The three inferred sub-policies and related scenarios are addressed using a broad stochastic multi-model method for industrial systems with general distributions of the restoration delay and the alteration period to the 'out-of-control' condition. We develop a numerical approach to address intricate stochastic models and assess the optimal integrated control policy by computing the expected overall cost. A simulation model has also been built and hundreds of system configurations with randomly generated parameters were tested to validate the recommended analytical models showing the quality and the effectiveness of the provided approach. Finally, we examined various control policies derived as of existing literature and applied in practical situations to underline the efficacy of the recommended integrated policy. The findings indicate that, for all randomly generated configurations, the proposed policy outperforms other ones and can achieve substantial economic gains.

This work is intended to inspire additional research on integrated production control and preventive maintenance policies. Indeed, the suggested strategy can be extended to circumstances involving maintenance and restoration efforts which are imperfect. Control charts and sampling inspection programs are two more quality inspection procedures that may be taken into consideration in a future work. In the other hand, further research is needed to analyse more complex production cells with several product kinds and multiple manufacturing workstations.

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