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Efficient Quadrature-Based Source-Term Evaluation for the Gaussian Moment Model Applied to Multiphase Flows

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Abstract

Particle-laden multiphase flows are ubiquitous in various applications, yet they remain computationally challenging, especially with complex processes like evaporation or combustion. Lagrangian models, for instance, remain unable to model particles over the entire domain, as predicting the evolution of every individual particle becomes prohibitively expensive and computationally infeasible at large scales. Eulerian models are conversely less taxing, though significant errors arise from mathematical artifacts when treating even monodispersed multiphase flows with traditional models. However, higher-order moment-closure methods, inspired by gas kinetic theory, provide a compelling alternative. These methods, by directly modeling higher-order particle velocity statistics (such as variances and covariances), have been shown to achieve greater accuracy than traditional Eulerian models without the convergence cost of Lagrangian approaches.

More recently, the ten-moment maximum-entropy closure from gas-kinetic theory has demonstrated success in the recovery particle-laden multiphase flows. However, complex interphase interactions in the source terms of the governing equation, often represented by empirical correlations (e.g., for drag), have limited its applicability due to the lack of closed-form expressions. This limitation motivates the investigation of quadrature rules to expand the applicability of the ten-moment model. While the mathematical connection between moments and Gauss quadrature is well established, the relationship between Gauss quadrature points and the eigenstructure of the ten-moment maximum-entropy closure has received limited attention. Specifically, the eigenvalues of the one-dimensional system correspond to the N Gauss quadrature points that exactly capture the zeroth to $(2N-1)$th moments of the distribution function. However, this connection remains largely unexplored in multiple dimensions. This presentation investigates the relationship between the eigenstructure of the multidimensional closure and the diagonalization of its Hankel matrix. We identify the embedded conditional moment structure within the right eigenvectors, which yields a natural multidimensional Gauss quadrature rule.

The accuracy of this quadrature is assessed for various previously intractable source terms. As a key example, we integrate an empirical drag law for high-speed, compressible, particle-laden flows, extending the accessible Stokes number range beyond previous limitations of the ten-moment closure. We implement this new methodology within a three-dimensional, large-scale, massively parallel, third-order accurate, discontinuous-Galerkin-Hancock (DGH1) framework. We present solutions for particle-laden flows in previously inaccessible regimes, demonstrating the impact of the improved source term integration on the overall flow dynamics. Validation against direct numerical simulations of the kinetic equation, where computationally feasible, showcases the potential of this approach for developing more robust and accurate closure for particle-laden multiphase flows.