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Inflation of Balloons: Entropic Elasticity and Bifurcation

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ABSTRACT

A gas undergoes an elastic deformation by changing volume. An elastomer undergoes an elastic deformation by changing shape. Both are of entropic origin. The entropy of a gas increases as its volume increases, but the entropy of an elastomer decreases as its length increases (subject to an uniaxial force). A balloon is a pure play of entropy. The entropy of the gas inside the balloon increases when the balloon expands, but the entropy of the polymer network of the membrane increases when the balloon contracts. The two effects compete to equilibrate the balloon. Although the phenomenon is well known, the entropic origin is not generally appreciated. Here, we present a short overview on the thermodynamics of entropic elasticity, starting with the ideal gas law and an ideal elastomer model. The two types of entropic elasticity act together in an inflated balloon. When blowing a balloon, neither the inner pressure nor the inner volume is controlled. Instead, the number of air molecules, N, is controlled by blowing. A spherical balloon inflates monotonically with an increasing number of gas molecules, while the pressure inside the balloon has a peak. When two identically spherical balloons are connected by a rigid tube, inflation of the balloons leads to a bifurcation, with one balloon inflated more than the other, while the pressure is the same in both balloons. We present the bifurcation diagrams using both the neo-Hookean model and the Gent model for the membrane material of the balloons. With multiple branches of solutions, a snap transition is predicted by the Gent model. Finally, we present a mathematical framework for analyzing inflation of balloons of arbitrary shape.