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A multigrid solution strategy based on an explicit time-marching method for the solution of incompressible flows

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ABSTRACT

Multigrid methods are widely used for the solution of partial differential equations in numerous fields. The field of computational fluid dynamics (CFD) makes no exception. Indeed, most general purpose incompressible-flow CFD codes use multigrid methods to solve the linear systems of equations that result from the discretization and whose structure depends on the algorithmic procedure. Since these codes usually deal with unstructured grids, algebraic multigrid methods are generally used. On the other hand, so-called full multigrid methods, that encompass the whole solution algorithm in a full-approximation scheme, are still not widespread in these general-purpose codes. This presentation discusses a multigrid strategy in which an explicit pseudo-time-marching procedure is used to obtain steady-state solutions. The core idea of the method is to compute explicitly the residuals on the fine grid and then restrict and smooth these residuals on coarser grids. Then, the solution undergoes a saw-tooth cycle, advancing in time with a simple first-order scheme on all grid levels before being prolonged back to the finest grid. This method is thus a full-approximation scheme since the residuals are computed directly from the field variables rather than their corrections. Moreover, because the scheme is explicit, the residuals do not need to be computed directly on the coarse grids as opposed to implicit full-approximation schemes. Therefore, coarse grids are not constrained by shape or quality criteria since only cells volumes and centroids are needed to perform restriction and prolongation operations. As such, the method can be classified as a variant of algebraic methods. The pseudo-time step is chosen such that the corresponding stability condition is satisfied on each grid level, allowing larger time steps to be performed on successive coarser grids. A von Neumann stability analysis on a simple one-dimensional case shows that the pseudo-time step can indeed be increased on coarser grids. The algorithm was successfully tested with finite-difference and finite-volume methods on several one-dimensional and two-dimensional problems such as the Laplace equation and Burgers' equation. Implementation challenges for general-purpose finite-volume codes will be discussed and a preliminary implementation for the incompressible Navier-Stokes equations on unstructured grids will be presented.