

Analytical Simulation of the Thermal Behaviour of Parallel-Series Configurations of Helical Steel Piles

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Abstract— The high initial cost of boreholes required for ground source heat pump (GSHP) systems is prompting a search for more cost-effective alternatives. This paper explores the use of helical steel piles (HSPs) as dual-purpose geo-exchange systems, offering both structural support and thermal energy exchange capabilities, and presenting a more economical option compared to traditional boreholes. In this study, an analytical approach utilizing the finite line source (FLS) model is developed to examine and compare the thermal performance of HSPs in various configurations—parallel, series, and mixed. For each configuration, the model dynamically computes the heat transfer rate of each pile as well as the overall inlet fluid temperature of the pile configuration. The proposed method has been validated against literature-reported test cases, demonstrating its accuracy and efficiency. The results indicate that the thermal efficiency of HSPs is similar to that of boreholes, and in the studied test case, HSPs were shown to perform better when the configurations were identical. However, comparisons may become unfair and are not advisable when the heights of the boreholes and piles are different. Moreover, the efficiency of various pile configurations varies significantly with their series or parallel arrangements as well as their flow rates, suggesting that the selection of pile arrangement and operational parameters should be carefully considered to optimize performance.

Keywords—*helical steel pile; borehole heat exchanger; series-parallel configuration; non-dimensional inlet temperature; geothermal energy; ground source heat pump*

I. INTRODUCTION

Geothermal systems represent a significant advancement in renewable energy technologies by harnessing the stable temperatures of the earth to efficiently heat and cool buildings. Traditionally, these systems have relied on boreholes, which involve drilling deep into the earth to reach stable thermal zones. While effective, these boreholes, ranging from 50 to 250 metres in depth, incur significant upfront costs due to the specialized equipment and extensive labor required for installation. Recognizing the limitations of this method, particularly in urban

or land-constrained environments, the industry has seen the emergence of HSPs [1] as a transformative alternative. Originally designed for structural support, HSPs have been repurposed to also function as ground heat exchangers. This dual-purpose design not only maximizes the utility of installation space but also offers a shallower, less resource-intensive installation, typically ranging from 6 to 30 metres in depth [2], [3].

HSPs [1], consisting of a hollow steel or concrete casing with off-centre inlet and outlet pipes, optimize thermal performance through their unique, asymmetrical design, as illustrated in Fig. 1.a. A helical screw at the base allows easy driving into the ground, eliminating the need for pre-drilling and reducing environmental impact and installation costs. In this paper, as illustrated in Fig. 1.b, the design of HSPs is simplified to two concentric pipes. It is assumed that the fluid enters through the inner pipe, flows to the bottom, and then moves upward through the space between the outer wall of the inner pipe and the wall of the outer shell, maintaining a constant mass flow rate. Additionally, the outlet pipe and the helical screw at the base of the shell are omitted from the model, as several numerical experiments have shown that the simplified HSP's thermal performance closely matches the full design while significantly reducing computational time and effort.

Due to significant construction costs and the necessity to enhance operational efficiency, simulating the thermal performance of HSPs through numerical and analytical models is crucial. These models assess the thermal interactions between the piles and the surrounding soil, evaluating heat transfer rates, temperature distribution, and fluid temperature variations inside the piles. The simulations, incorporating input parameters like geometric properties and thermal properties of the fluid, pipes, and soil, account for hourly and seasonal variations in ground loads, fluid flow rate changes, and the arrangement of helical piles, thereby providing precise predictions of both short-term and long-term performance. Employing such models is essential for maximizing system performance and efficiency, as well as minimizing energy consumption and operational costs of helical piles. In the next section, some models used to simulate the thermal performance of boreholes and HSPs are discussed.

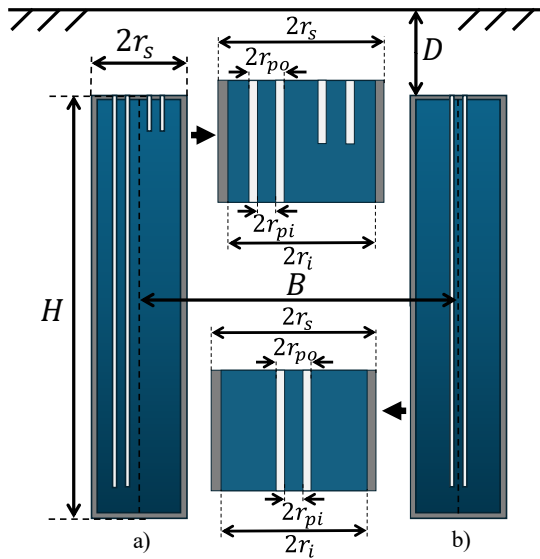


Figure 1: Comparison of real (a) and simplified (b) geometries of HSPs

II. LITERATURE REVIEW

Simulating the thermal performance of ground heat exchangers, such as boreholes or piles, is a complex task due to the involvement of numerous factors, including thermal and geometric parameters. These simulations must account for hourly variations in ground heat loads and fluid flow rates while also simulating the performance of heat exchangers over extended periods, such as months or years. Furthermore, the slender, narrow, and long geometry of boreholes and piles necessitates the use of fine grids near the heat exchanger interfaces, transitioning to progressively coarser grids further away. This requirement is compounded by considerations such as short-circuiting effects between borehole legs, thermal interactions between adjacent boreholes, or the influence of groundwater, underscoring the necessity for simulation models that can accurately represent these complex interactions.

One approach is to use numerical models. Although accurate, they are time-consuming and computationally intensive. Alternatively, analytical models are often preferred in the literature because they provide quicker and more efficient, yet still precise, long-term performance predictions compared to numerical methods. However, these analytical models employ simplifications that, while generally effective, may not be applicable in real test scenarios. Analytical methods vary in scope; some consider only the thermal effects in the radial direction of heat exchangers, while others account for both radial and axial directions, incorporating physical dimensions such as the radius, length, or buried depth of these heat exchangers.

No work in the literature has yet applied an analytical methodology to simulate the performance of HSPs. Instead, HSP performance is typically analyzed using numerical methods such as finite element (FEM) and finite volume methods (FVM) within software such as COMSOL or ANSYS Fluent [3]–[5]. These simulations usually assume zero thermal interactions between the piles, simplifying the problem to a single pile in order to save on computational cost. Additionally, they ignore series and parallel connections of piles and normally consider

laminar rather than turbulent fluid regimes as it is simpler to simulate and more common in energy piles. Moreover, the simulations are often short-term, covering only a few years rather than extended periods (e.g., ten or twenty years) due to computational constraints [2], [6], [7]. As a result, they may not accurately represent real-world performance, since these commonly ignored factors can have significant effects on the system. Therefore, for HSPs, more efficient models, particularly analytical ones, are needed.

Among the analytical models, the FLS model stands out due to its comprehensive approach that includes both radial and axial heat transfer in boreholes. This model accounts for the depth, radius, and height of heat exchangers, enabling it to accurately calculate both the heat transfer rate and the average wall temperature of the heat exchangers. In the literature, the FLS model is primarily used for boreholes, largely due to its applicability to high-aspect ratio cases. Within boreholes, the sectional area and volumetric flow rate of U-pipes remains constant, and typically in a turbulent flow regime. This consistency allows for the assumption that the mean average temperature between the inlet and outlet occurs at the bottom of the borehole. By using the effective borehole resistance, designers can relate this mean average temperature to the borehole's wall temperature, as evaluated by the FLS model [8], thereby facilitating the simulation of fluid temperature variations within boreholes [9].

To accurately evaluate borehole wall temperature, it is crucial to incorporate hourly load variations. This requirement can be addressed through heat load aggregation or by employing modified methods that utilize blocks of loads of varying sizes, including recent hourly loads, to expedite the process [10]. More efficient models, such as the fast Fourier transform (FFT) [11] or Laplace transform (LT) [8], [12] methods, offer alternative approaches. The LT model, introduced by Lamarche [12], calculates the heat transfer rate and the average wall temperature at each time step by taking an inverse Laplace transform of the results. The FFT model is another alternative that is faster and simpler but requires that the heat transfer rate at each borehole be known at every time step. This presents a challenge, as the heat transfer rate of each borehole is unknown *a priori* since it depends on both its position within the bore field and the overall ground heat loads that may vary in time.

To overcome this limitation, Pasquier and Marcotte [13] developed a methodology that begins with an initially assumed distribution of heat transfer rates for all time steps, which is then iteratively adjusted to align with imposed temperature signals on the borehole wall. They later refined this approach by incorporating a sequential method based on Laplace techniques [14] to dynamically calculate heat transfer rates during the simulation. This innovative model evaluates both the heat transfer rate of boreholes and their overall non-dimensional inlet temperature in response to a unit heat extraction rate, allowing for the simulation of various configurations of boreholes [15].

In addition to the FLS model, the thermal performance of boreholes, particularly in series-parallel configurations, can also be effectively simulated using thermal resistance and capacity models (TRCMs) and g-functions. The TRCM employs a network of thermal resistances and capacitances to model the

thermal interactions between borehole components, including the heat carrier fluid, pipe, and grout. This approach enables the simulation of short-term borehole behaviour with high accuracy, producing results comparable to numerical models [16]. Using TRCM helps prevent the overestimation of energy consumption by heat pumps [17] and mitigates the oversizing of boreholes [9]. Another method for simulating series-parallel borehole configurations involves the use of g-functions, as demonstrated by Cimmino, who introduced advanced g-functions specifically designed for series- and parallel-connected boreholes with variable fluid mass flow rates and reversible flow directions [18].

In response to the need for more efficient simulation models for HSPs, this paper applies the FLS model along with the modified FFT model described in [15] to simulate the thermal performance of HSPs. The proposed approach considers time-dependent heat load variations and determines non-dimensional overall inlet fluid temperature variations of HSPs by considering their series, parallel, or mixed configurations. This model is versatile, can accommodate both laminar and turbulent fluid regimes, and can account for thermal interactions among piles, making it an efficient and comprehensive tool for simulating various HSP configurations. The model is more suitable for simulation of long time periods than short time periods, as the short-term capacity effects inside the piles are not considered by the FLS model. Further, the developed model can account for ground temperature variations by time but it is unable to consider the ground temperature variations by depth. In the following sections, the mathematical background of the proposed model is presented. The model is then validated against test cases reported in the literature [15] and applied to simulate the performance of four different helical pile configurations with various series-parallel arrangements.

III. MATHEMATICAL MODEL

Based on the methodology introduced in [15], by assuming steady-state conditions inside the heat exchangers (piles or boreholes) and considering a linear variation of the fluid temperature along the heat exchanger pipes, the inlet and outlet fluid temperatures can be determined using an analytical FLS-type model as follows:

$$T_{in}(t) = T_{f,ave}(t) + q/(2\dot{m}C_p) \quad (1)$$

$$T_{out}(t) = T_{f,ave}(t) - q/(2\dot{m}C_p) \quad (2)$$

$$T_{f,ave}(t) = (q/H)R_s + \bar{T}_s(t) \quad (3)$$

where T_{in} and T_{out} are the inlet and outlet fluid temperatures of the pile, $T_{f,ave}$ is the average fluid temperature at the bottom of the pile, q is the heat exchange rate of the pile, \dot{m} is the fluid mass flow rate, C_p is the specific heat of the fluid, H is the pile height, R_s is the effective thermal resistance of the pile, and \bar{T}_s is the average wall temperature of the pile. In a field consisting of n piles, the wall temperature change of each pile i is determined by adding the thermal interactions from all other piles j , including pile i , to pile i , as illustrated by (4):

$$\bar{T}_{s,i}(t) = T_g(t) + \sum_{j=1}^n \Delta T_{i,j}(t) \quad (4)$$

$$\Delta T_{i,j}(t) = h_{i,j}(t) + q_j(t)f_{\Delta t}(r_{i,j}) \quad (5)$$

$$h_{i,j}(m\Delta t) = (\tilde{\mathbf{q}}_j * \mathbf{f})(m\Delta t) \quad (6)$$

where T_g is the ground temperature (which may vary by time), and $\Delta T_{i,j}$ is the thermal effect of pile j on pile i due to a heat load q_j , both of which are assumed to be a function of time step Δt . If the time t is split into m intervals of Δt , the thermal interaction of pile j on pile i can be evaluated by considering the *historical heat effects*, $h_{i,j}$, that accounts for heat transfer effects of pile j from 0 to $(m-1)\Delta t$ and the *present heat effect*, $q_j(m\Delta t)f_{\Delta t}(r_{i,j})$, which accounts for current heat transfer effect of pile j due to its current heat transfer rate $q_j(m\Delta t)$.

In (5), $f_{\Delta t}(r_{i,j})$ is the unit temperature response function computed for one time step with the FLS model, $r_{i,j}$ is the distance between piles i and j . When $i = j$, $r_{i,j} = r_s$, where r_s is the pile radius. The historical term can be evaluated by a discrete convolution product as represented in (6). In this equation, the vector $\tilde{\mathbf{q}}_j$ is $[q_{j,1}, q_{j,2} - q_{j,1}, \dots, q_{j,m-1} - q_{j,m-2}, -q_{j,m-1}]$, and \mathbf{f} is the temperature response function evaluated by FLS model for all the $m\Delta t$ steps for a distance $r_{i,j}$ by assuming a unit heat load at each pile. For evaluating the convolution, $m-1$ zeros should be added to the vectors $\tilde{\mathbf{q}}_j$ and \mathbf{f} to account for the non-periodicity of these functions. By evaluating the convolution vector $\tilde{\mathbf{q}}_j * \mathbf{f}$ using FFT, the $h_{i,j}(m\Delta t)$ is evaluated as the m th element of the convolution vector. The convolution should be evaluated for each pair of piles, and repeated at each time step to account for the historical effects. However, it is observed that the results do not depend significantly on the size of time steps, and as a result, for a constant heat exchange load rate, the time steps can get larger as the simulation progresses.

By considering the series, parallel, and mixed configurations of piles, a system of equations will be obtained that should be solved for each time step. This system of equations can be summarized for all three configurations as follows:

$$\begin{bmatrix} \mathbf{G} + \mathbf{D} + \mathbf{L} & \mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q}_t \\ -(T_{in} - T_g) \end{bmatrix} = \begin{bmatrix} -\mathbf{h} \\ 1 \end{bmatrix} \quad (7)$$

In this equation, \mathbf{h} , \mathbf{G} , and \mathbf{D} would be evaluated similarly for all three configurations. \mathbf{h} is a vector with n components that accounts for the historical thermal effects of n piles. \mathbf{G} is an $n \times n$ matrix where each element G_{ij} accounts for the thermal effects of pile j with one unit of heat transfer on pile i for the current time step. Note that \mathbf{G} depends on the time step and not the elapsed time, and should be evaluated only one time. \mathbf{D} is a diagonal $n \times n$ matrix such that the value of each of its i th element is $R_{s,i}/H_i + 1/(2\dot{m}_i C_p)$. \mathbf{q}_t is the vector containing the heat transfer rate of all the n piles at the current time step.

Equation (7) involves $n+1$ unknowns at each time step, including n heat transfer rates of piles, \mathbf{q}_t , and the inlet temperature T_{in} . For parallel piles, the T_{in} of all piles is

assumed to be equal, but their mass flow rates may differ. In a series configuration, the outlet temperature of each pile is assumed to be equal to the inlet temperature of the next, and the mass flow rate of all piles is assumed to be equal. This means that for configurations in which the fluid mass flow rate per pile is considered the same for both parallel and series arrangements, parallel configurations must have significantly higher total mass flow rates. For all configurations, the sum of the heat transfer rates of all piles is assumed to be equal to one unit of the heat transfer rate of the pile field.

Considering these points, matrix \mathbf{L} , which considers the piles' series and parallel combinations, can be evaluated. For parallel configurations, \mathbf{L} is a zero matrix. For series configurations, \mathbf{L} is a lower triangular matrix with zeros along the diagonal and the remaining components equal to $1/\dot{m}C_p$. In mixed arrangements, \mathbf{L} is a matrix that considers m parallel branch of piles, where each branch may have one or more piles in series. \mathbf{L} is written as follows:

$$\mathbf{L} = \begin{bmatrix} L_1 & 0 & \dots & 0 \\ 0 & L_2 & 0 & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & L_m \end{bmatrix} \quad (8)$$

where L_i corresponds to the i th parallel branch of piles which has n_i piles in series and a flow rate of \dot{m}_i . L_i is an $n_i \times n_i$ lower triangular matrix with zeros along the diagonal and all other values of $1/\dot{m}_i C_p$.

The left side of (7) is not dependent on time, resulting in a linear system of equations solved easily at each time step. By solving the system, the inlet temperature variation of the entire pile field induced by 1 Watt of heat load can be evaluated at each time step. Using the evaluated inlet temperature, it is possible to evaluate the non-dimensional unit response function as illustrated below:

$$\tilde{T}_{in} = (T_{in} - T_g)2\pi k_g nH \quad (9)$$

The evaluated \tilde{T}_{in} accounts for the series, parallel, and mixed arrangements of piles and may be used with any heat load scenarios. In the next section, representative test cases will be introduced and simulated using these equations to show the applicability of the proposed method.

IV. MODEL VALIDATION, TEST RESULTS, AND DISCUSSION

Test 1: Validation of the Proposed Model

Using the equations discussed in the previous sections, a model has been implemented that can simulate the thermal operation of both boreholes and piles in different configurations. This model accounts for the thermal effects of the internal components of the heat exchangers and assumes a uniform wall temperature to evaluate their effective thermal resistances using the equations suggested by Hellström [19]. The model is validated with a test case reported in [15]. In this example, the thermal performance of nine boreholes arranged in a 3×3 grid

with 3-metre spacing is simulated in both parallel and series configurations. For this analysis, the following parameters are used: $H=30$ m, $r_b=0.08$, $D=2$ m, $R_b=0.12$ (m.K)/W, $T_g=0^\circ\text{C}$, $\dot{m}=0.3$ kg/s, m , $C_p=4180$ J/(kg.K), $k_g=2.5$ W/(m.K), and $\alpha_g=0.108$ m²/day. Here, R_b is the borehole resistance, r_b is its radius, k_g is the ground thermal conductivity, and α_g is its thermal diffusivity. The performance of the boreholes is simulated under a cooling load of 1 Watt (heat delivered to the ground) over durations ranging from a few minutes to many decades for both configurations. These results, as well as the ones reported in [15], are illustrated in Fig. 2.

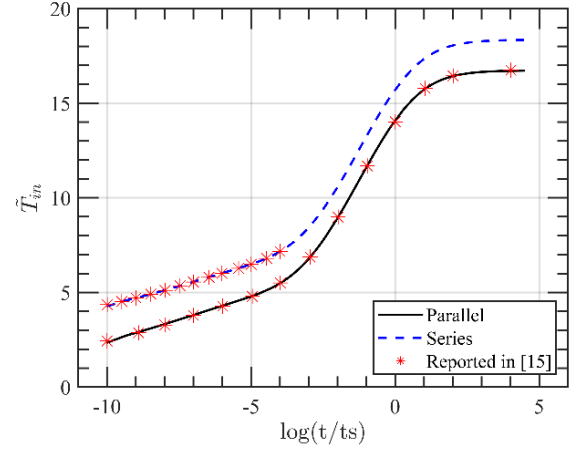


Figure 2: Validation of the test scenario versus results reported in [15]

In Fig. 2, the results obtained for the parallel and series configurations are illustrated by solid and dashed lines, respectively, and those reported in [15] are illustrated by asterisks, which are actually derived from two separate figures: one showing results for short-term and the other for long-term periods. As can be observed, the two sets of results evaluated for the parallel arrangement are in agreement. Similarly, the series comparison shows agreement over the short term. However, for the longer periods, the results are not compared since it is found that the results reported in [15] for the long-term period are not correct as they do not align with the short-term values (possibly due to a misprint or the reporting of an incorrect figure). This test shows that the model is accurate and can be used to simulate the performance of HSPs.

Test 2: Comparison of Modeling Boreholes and Piles

The next test case examines the effects of thermal resistance and height on the thermal performance of the piles and boreholes. In this test, the thermal performance of the 3×3 parallel boreholes simulated earlier is compared with the performance of two sets of 3×3 parallel piles, which have respective heights of 30 and 16 metres. The piles are designed using these parameters: $r_s=0.07$, $r_i=0.06$, $r_{p,o}=0.015$ m, $r_{p,i}=0.01$ m (see Fig. 1). All other parameters are identical to those previously mentioned for parallel boreholes. Thus, the main differences between the piles and boreholes include their sectional geometry (piles are slightly smaller and have different internal designs) and their height (when considering the 16-metre pile), which causes the three configurations to perform

differently. The thermal resistance of the boreholes is assumed to be $R_b=0.12$ (m.K)/W, whereas the thermal resistance of the piles is dependant on the internal geometry and is determined for each simulation.

Figs. 3.a and 3.b show the variation of non-dimensional and dimensional inlet temperatures of the three configurations over time, respectively. It is important to note that the dimensional inlet temperatures are evaluated for only 1 watt of heat exchange rate, which is why their values are small, and would scale up for realistic loads. By analyzing the results, it can be concluded that although \tilde{T}_{in} serves as a useful parameter for comparing boreholes or piles with identical heights, it is not as effective for cases with varying heights. This observation is supported by comparing the \tilde{T}_{in} of piles and boreholes of the same height in Fig. 3.a, which maintain their relationship when comparing their T_{in} in Fig. 3.b. In both figures, the piles outperform boreholes by injecting more heat into the ground and exhibiting lower \tilde{T}_{in} (or T_{in}). On the other hand, the relatively low \tilde{T}_{in} of the piles with a 16-metre height might suggest that they are more efficient than the others. However, their T_{in} indicates that they are, in fact, less efficient as they have a higher temperature, which implies that they were less able to inject heat into the ground.

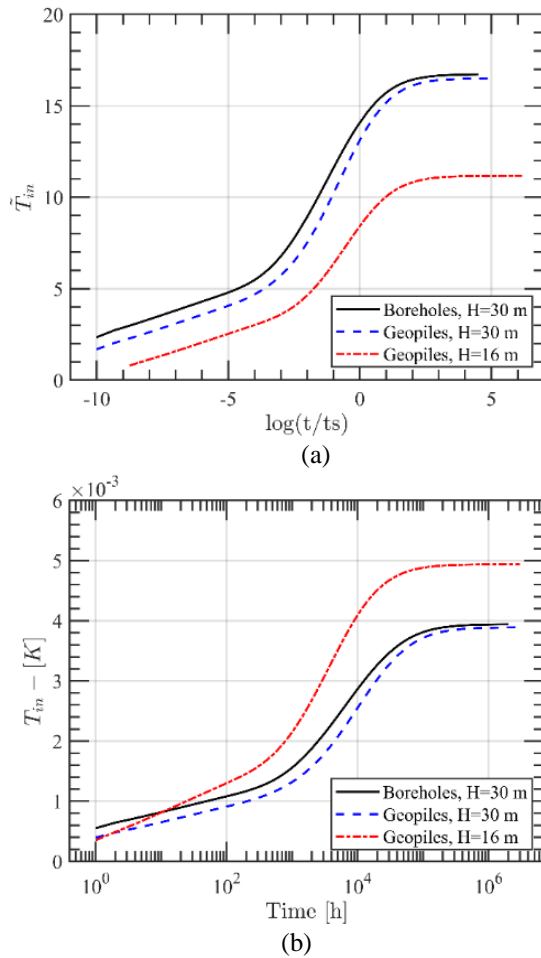


Figure 3: nondimensional (a) and dimensional (b) inlet temperature for the comparison simulations

The length of a pile or borehole impacts many parameters of (7) and (9), such as the effective thermal resistance and matrix D . Its impact on the inlet fluid temperature is a reflection of the overall impact of these effects. The length also affects the characteristic time, ($t_s = H^2/9\alpha_g$) used in evaluation of the logarithmic time scale which is why in Fig. 3.a while it may appear that the inlet fluid temperature of shorter piles increases slower over time, in a real-time their temperature increases much faster, indicating that shorter piles have less thermal capacity. From this analysis, it can be concluded that piles and boreholes should only be compared using their \tilde{T}_{in} if they have the same length. Otherwise, the dimensional T_{in} must be used.

Test 3: Effects of Pile Configuration and Flow Rate

In the final set of test cases, the thermal performance of four different pile configurations is analyzed. Each configuration includes 12 piles, each 16 meters long and spaced 3 meters apart, arranged along the edges of a 9x9 metre square and connected in four distinct configurations: (A) parallel, (B) series, (C) two branches of six series piles, and (D) four branches of three series piles, as depicted in Figs. 4.a and 4.b.

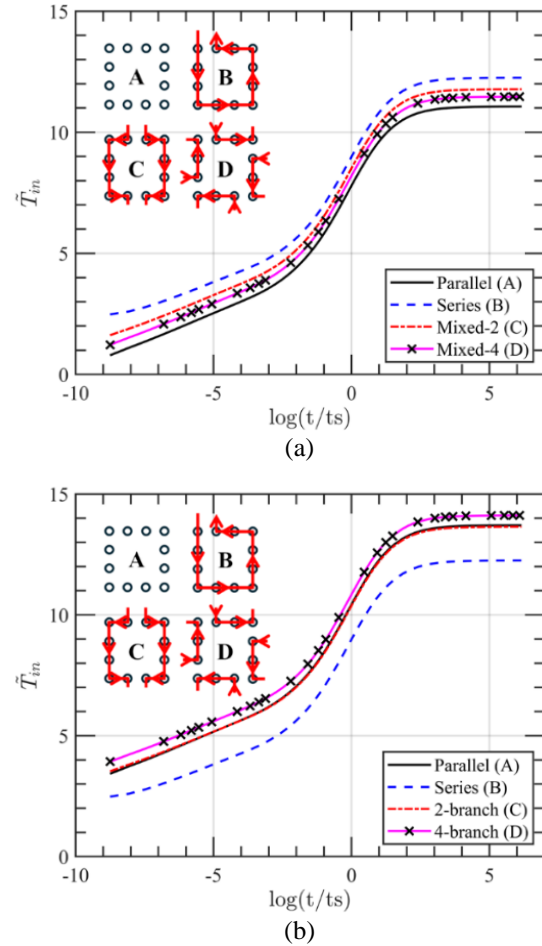


Figure 4: Case study results for (a) constant per pile and (b) constant total mass flow rate

For this test, the study considers two conditions for the fluid flow rate through the piles. Under the first condition, all piles in

each configuration have the same fluid flow rate, consistent with [15]. Thus, the total fluid flow rates for configurations A, B, C, and D are respectively $12\dot{m}$, \dot{m} , $2\dot{m}$, and $4\dot{m}$, where \dot{m} is the flow rate for an individual pile and, thus, the total mass flow rate for a parallel configuration is twelve times that of a series configuration. The second condition standardizes the overall fluid flow rate across all configurations to \dot{m}_{tot} . Accordingly, the flow rates for each pile in configurations A, B, C, and D become $\dot{m}_{tot}/12$, \dot{m}_{tot} , $\dot{m}_{tot}/2$, and $\dot{m}_{tot}/4$, respectively. This design assumes an identical standard heat pump is used for all configurations, which requires a consistent fluid flow rate across different configurations to maintain performance. Using these assumptions, the thermal performance of the four configurations is evaluated and reported respectively in Figs 4.a and 4.b.

By comparing Figs. 4.a and 4.b, it can be seen that the performance of the piles in series is identical, since in both conditions they receive the same fluid flow rate. This permits comparison of the thermal performance of other configurations. From Fig. 4.a, configurations with a greater number of parallel branches were found to perform better than others. In contrast, in Fig. 4.b, the series configuration shows the best performance. Increasing the number of parallel branches may decrease the efficiency of the system, although Configuration A does not follow this pattern. Upon investigating these observations, it is observed that part of this phenomenon is due to the decrease of the fluid flow rates in parallel branches (in the 2nd Condition), which increases the pile's effective thermal resistance so much that it causes the thermal efficiency of the piles to decrease. These observations are in line with findings reported in [19] and [20]. These results show that the thermal behaviour of the array may change substantially when the total mass flow rate changes. Hence, when designing a GSHP system for a specific heat pump, the effects of flow rates should be carefully investigated.

V. CONCLUSION

This study confirms the feasibility of helical steel piles (HSPs) as an efficient alternative to conventional boreholes for geothermal heat exchange and structural support. By employing the finite line source (FLS) model with a modified fast Fourier transform (FFT) approach, an accurate simulation was conducted on the thermal behaviour of HSPs in parallel, series, and mixed configurations. Model validation against data in the literature confirms the approach's accuracy, ensuring reliable long-term performance predictions. The findings indicate that configuration and flow rate significantly influence thermal performance, underscoring the necessity of tailored design considerations. Overall, this study offers a robust analytical framework for designing HSP-based geo-exchange systems, promoting their integration into sustainable and cost-effective geothermal solutions.

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REFERENCES

- [1] S. R. Nicholson, "Characterization of a novel in-ground heat exchanger for applications in sustainable building energy and maintaining permafrost," Ryerson University, 2020.
- [2] S. R. Nicholson, L. R. Kober, P. Atefrad, A. Mwesigye, and S. B. Dworkin, "The influence of geometry on the performance of a helical steel pile as a geo-exchange system," *Renew. Energy*, vol. 172, pp. 714–727, 2021.
- [3] C. Beragama Jathunge, A. Darbandi, S. B. Dworkin, and A. Mwesigye, "Numerical investigation of the long-term thermal performance of a novel thermo-active foundation pile coupled with a ground source heat pump in a cold-climate," *Energy*, vol. 292, 2024.
- [4] C. Beragama Jathunge, P. Adebayo, S. B. Dworkin, and A. Mwesigye, "Numerical investigation of an energy pile-based solar-assisted ground source heat pump system for space heating and cooling," in *IGSHPA Research Conference*, Montréal, 2024, pp. 1–14.
- [5] C. Beragama Jathunge, S. B. Dworkin, C. B. Wemhöner, and A. Mwesigye, "Performance investigation of a solar-assisted ground source heat pump system coupled with novel offset pipe energy piles and solar PVT collectors for cold climate applications," *Appl. Energy*, vol. 265, 2025.
- [6] K. Henry-Mathieu, S. Antoun, and S. Dworkin, "The characterization of helical steel pile performance under varying soil conditions," in *IGSHPA Research Track*, Las Vegas, 2022, pp. 216–227.
- [7] E. Attard, S. Antoun, P. Hatefraad, and S. B. Dworkin, "Field-scale experimental analysis of helical steel piles as in-ground heat exchangers for ground source heat pumps," *Proc. Can. Soc. Mech. Eng. Int. Congr.* 2022, pp. 1–6, 2022.
- [8] M. Cimmino and M. Bernier, "A semi-analytical method to generate g-functions for geothermal bore fields," *Int. J. Heat Mass Transf.*, vol. 70, pp. 641–650, 2014.
- [9] M. Ahmadvard and M. Bernier, "A review of vertical ground heat exchanger sizing tools including an inter-model comparison," *Renew. Sustain. Energy Rev.*, vol. 110, pp. 247–265, 2019.
- [10] M. A. Bernier, P. Pinel, R. Labib, and R. Paillot, "A multiple load aggregation algorithm for annual hourly simulations of GCHP systems," *HVAC R Res.*, vol. 10, no. 4, pp. 471–487, 2004.
- [11] D. Marcotte and P. Pasquier, "Fast fluid and ground temperature computation for geothermal ground-loop heat exchanger systems," *Geothermics*, vol. 37, no. 6, pp. 651–665, 2008.
- [12] L. Lamarche, "A fast algorithm for the hourly simulations of ground-source heat pumps using arbitrary response factors," *Renew. Energy*, vol. 34, no. 10, pp. 2252–2258, 2009.
- [13] P. Pasquier and D. Marcotte, "Efficient computation of heat flux signals to ensure the reproduction of prescribed temperatures at several interacting heat sources," *Appl. Therm. Eng.*, vol. 59, no. 1–2, pp. 515–526, 2013.
- [14] A. Lazzarotto, "A network-based methodology for the simulation of borehole heat storage systems," *Renew. Energy*, vol. 62, pp. 265–275, 2014.
- [15] D. Marcotte and P. Pasquier, "Unit-response function for ground heat exchanger with parallel, series or mixed borehole arrangement," *Renew. Energy*, vol. 68, pp. 14–24, 2014.
- [16] P. Pasquier and D. Marcotte, "Short-term simulation of ground heat exchanger with an improved TRCM," *Renew. Energy*, vol. 46, pp. 92–99, 2012.
- [17] V. Godefroy and M. Bernier, "A simple model to account for thermal capacity in boreholes," *11th IEA Heat Pump Conf.*, no. May, p. 12, 2014.
- [18] M. Cimmino, "g-Functions for fields of series- and parallel-connected boreholes with variable fluid mass flow rate and reversible flow direction," *Renew. Energy*, vol. 228, 2024.
- [19] G. Hellström, "Ground heat storage: Thermal analyses of duct storage systems," *Lund Univ.*, p. 310, 1991.
- [20] M. Cimmino, "Semi-analytical method for g-function calculation of bore fields with series- and parallel-connected boreholes," *Sci. Technol. Built Environ.*, vol. 25, no. 8, pp. 1007–1022, 2019.