

EXPERIMENTAL MEASUREMENT OF GAP LEAKAGE AND THE FILL HEIGHT GRADIENT IN ARCHIMEDES SCREW PUMPS

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Abstract—A laboratory-scale Archimedes screw pump was tested in a fully transparent apparatus. Using two cameras, the free surface action of the screw buckets were quantified for the first time during experimental operation. This kind of data has been gathered using computational fluid dynamics, but never in a real Archimedes screw pump. It was observed that rotation speed impacted the amount of leakage experienced during operation, which created a variation in bucket fill height along the length of the operating screw – called the fill height gradient. Further, a sloshing action was observed in the screw buckets, which was suggested to be impacted by rotation speed and flow regime. This study demonstrated new experimental methods that used video footage and open-source annotation software to gather reliable, quantifiable data from free-surface flows. This novel experimental approach can be used to gather reliable, inexpensive data from complex geometries.

Keywords—Archimedes screw pump; experiments; fill height variation; flow visualization; new experimental methods;

I. INTRODUCTION

Since antiquity, Archimedes screws have been used for many hydraulic applications, including irrigation, land reclamation and dewatering, and pumping at wastewater treatment facilities [1]. When used for pumping applications, the device is called an Archimedes screw pump (ASP). In modern applications, a motor supplies torque and rotation speed to the screw's shaft. As the screw rotates, it forms discrete buckets [2] of water (or other media) separated by the blades and translates them from a lower reservoir at the inlet, depositing them to an upper reservoir at the outlet.

The screw is a helical array of blades wrapped around a cylindrical shaft. Similarly to other screws, it may have multiple blade starts; it is most common for Archimedes screw pumps to have between 2 and 4 blades [3]. The screw is fixed between two bearings on an incline and placed within a cylindrical trough.

In most orientations, the trough is fixed, and the screw rotates within it [4]; having a small gap between the screw blade tips and the trough to prevent damage, friction, and wearing. This gap introduces a form of leakage (termed “gap leakage”, Q_g) into the system [1]. In a previous study, it was found that the gap leakage caused additional phenomena to occur within an operating screw [5]. The purpose of the current study was to explore the impacts of gap leakage experimentally and provide further data and insights into the published literature to aid engineers and designers.

Before details of the previous study and phenomena can be discussed, further details about ASPs need to be introduced. Figure 1 presents the general layout of an Archimedes screw pump station. The figure is annotated with the design variables and operating parameters used to describe the system.

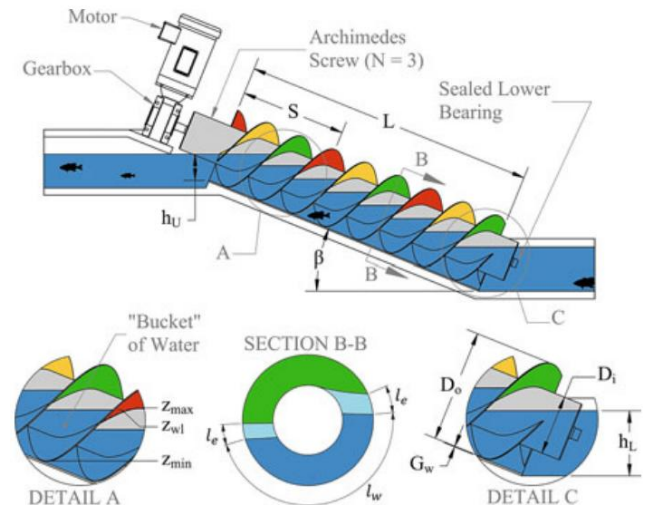


Figure 1. Archimedes screw generator geometry with detailed views showing bucket dimensions (A), wetted blade perimeter (B), and screw inlet (C).

The main geometry of the screw is defined by the outer diameter (D_o), inner diameter (D_i), pitch (S), length (L), and number of blades (N). The screw is set at an incline (β) matching the trough and given a small, intentional gap width (G_w) between the blade tips and trough. It is placed at a position to yield the desired nominal upper (h_U) and lower water levels (h_L).

Screw buckets are often described by their dimensionless bucket fill height ratio (f):

$$f = \frac{z_{wl} - z_{min}}{z_{max} - z_{min}} \quad (1)$$

where the parameters in Eqn. 1 represent vertical position for different water levels in the bucket (see Figure 1, Detail A). The bucket water level is z_{wl} , the minimum water level in the bucket is z_{min} , and the maximum water level (z_{max}) is the highest the water level can be within the bucket before it begins to overflow into a lower bucket. This is called “overflow leakage”, and it is generally considered a form of loss in the screw.

The fill height can describe the behavior of water in a bucket. When the fill height is lower than 1, the bucket is considered “underfilled”. When the fill height is equal to 1, the bucket is considered optimally filled. When the fill height is greater than 1 the bucket is considered “overfilled” and will experience overflow leakage.

The lower water level is set to a height corresponding to the desired water level for operation. Since $f = 1$ is the nominal design target of most screw pump installations, a lower water level corresponding to $f = 1$ is called the “optimal lower water level”. The optimal lower water level (h'_L) can be defined geometrically as

$$h'_L = \frac{D_o + D_i}{2} \cos \beta \quad (2)$$

Nagel [3, 6] and Nuernbergk [7, 8] present diagrams and discussions that are consistent with Eqn. 2.

As mentioned, a previous study investigated the impact of the blade-trough gap [5]. In an Archimedes screw installation, the gap is sized so it is large enough to minimise friction and wearing, but small enough to minimize gap leakage (Q_g). Gap leakage is when water flows out of one bucket through the gap into the successive lower bucket. Since the buckets translate up along the screw, water slowly drains out of the bucket as it traverses the length of the screw. When the bucket reaches the outlet of the screw, it has a lower fill height than when it started. This variation of fill height with distance along the screw has been called the “fill height gradient” [5], and it was observed experimentally in the previous study (see Figure 2). Note that the same screw was used in the current study, its blade tips were coloured blue, green, and yellow to distinguish each of the three blades during the analysis.

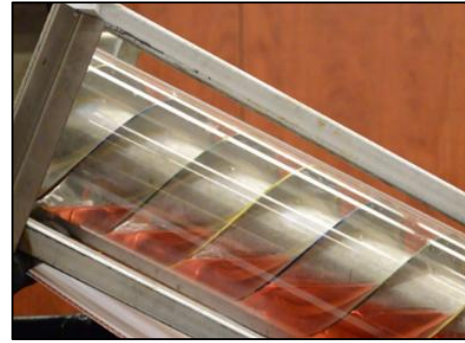


Figure 2. Laboratory apparatus used by Simmons et al. (2024) with observed fill height gradient. From top left, blades tips were coloured blue, green, and yellow.

The fill height gradient was explored in the previous study [5]. The ASP was operated within a transparent trough with dyed water to make the free-surface more visible. A DSLR camera was used to capture video footage of the screw during operation. The video files were post-processed with Kinovea (v0.8.15) motion capture software (www.kinovea.org); the software is often used in sports analysis and kinesiology to track the motion path of athletes and patients. It can trace and quantify the position of elements in video footage.

Kinovea was used in this project to trace the free surface of three successive buckets during operation. The results demonstrated that the fill height of the screw buckets decreased along the length of the screw. The fill height gradient was observed and quantified.

Since the screw was set to a start with a fill height of $f = 1$, the fill height gradient was solely caused by gap leakage in the experimental cases. So, the fill height gradient was plotted with respect to time and integrated to yield the gap leakage with respect to speed.

The study successfully demonstrated that a laboratory-scale Archimedes screw pump could be analysed visually to quantify its performance. However, during experimentation it was noticed that, though the screw was set at its optimal lower water level, the first bucket was $f > 1$. It was postulated that the friction of the screw blades caused water to surge into the screw to a higher-than-expected volume according to the inlet geometry.

That cast further doubt to the experimental results. If the first bucket was impacted by friction to that extent, were the rest of the buckets impacted as well? When viewing the bottom of the screw in Figure 2, the screw blades move from the background to the foreground of the image. It is possible that the blades were causing water to move to the side of the screw visible in the foreground of the image (hereafter, the “front” of the screw).

Since video was only capture from the front side of the screw, it was not possible to verify if the water levels were lower on the back side of the screw.

Advancing the previous study, the study presented in this paper sought to capture video footage from both the front and back of the screw to investigate this phenomenon. Additionally, the apparatus used in the previous study did not allow for visualization of the entire screw. The screw was in an opaque tank, and the first few buckets were covered by the tank in the

video footage. So, a new apparatus was designed to allow for video capture of the entire screw on both its front and back sides during operation.

With the new apparatus, it was possible to ascertain if the water levels were indeed different in the front and back of the screw. The results of the experiments were analysed and used to calculate the gap leakage in the system.

II. METHODS

A transparent acrylic tank was constructed to visualize both sides of an operating Archimedes screw entirely, from inlet to outlet. The tank was constructed large enough to fit the whole screw during operation (1220 mm x 330 mm x 735 mm).

The apparatus is shown in Figure 3. The top image of the Figure 3 shows the front side of the screw. This can be understood as the side where the blades are spinning from the bottom to the top; it is the side that may leak via overflow. The side where the blades are spinning from top to bottom is known as the “back” of the screw. Overflow leakage flows into the back of a screw bucket from a corresponding bucket at a higher elevation.

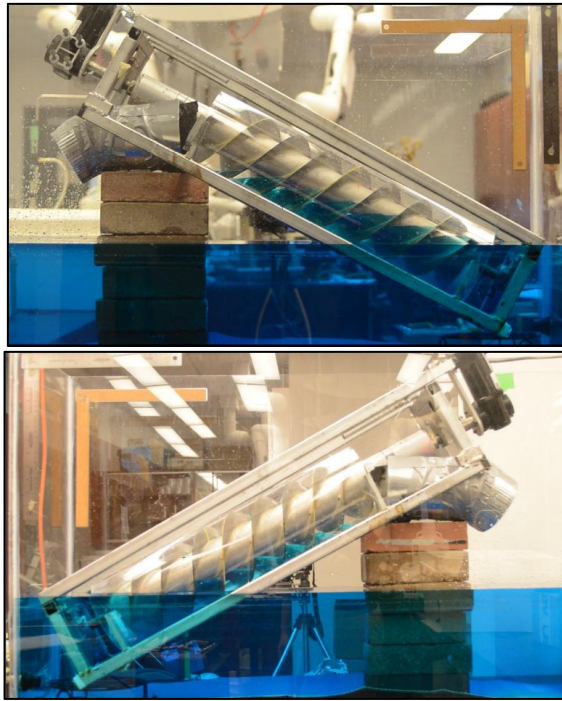


Figure 3. Experimental apparatus viewed from the front (top) and back (bottom).

In this study, a small laboratory screw was used with the dimensions shown in Table 1.

TABLE I. DIMENSIONS OF THE EXPERIMENTAL LABORATORY SCREW

Do (m)	Di (m)	S (m)	L (m)	N (-)	β (°)
0.150	0.078	0.210	0.600	3	29

Two DSLR cameras were used. The cameras were set on both sides of the tank with the frames capturing the entire front

and back side panes of the tank. The cameras were both set to a horizontal angle of 0° using an angle gauge and aligned so that the camera lenses were perpendicular to the side panes of the tank. This was important to reduce ocular distortion.

The screw inlet was set on a wire rack to reduce turbulent behavior at the inlet. The screw outlet was set at a fixed height to maintain a consistent inclination angle during testing. A small outlet duct was placed at the end of the screw to return the flow to the tank and minimize the introduction of secondary flows near the screw inlet.

The tank was filled with water to an arbitrary height and the screw was turned on. The water was dyed a deep shade of blue for easier recognition by the tracing tools used in Kinovea. The screw was set to a desired speed using a variable frequency drive and a tachometer. Table 2 shows the speed values used for the four experimental runs.

TABLE II. EXPERIMENTAL RUN SETTINGS.

	Run 1	Run 2	Run 3	Run 4
Rotation Speed (rev/min)	8.82	9.98	14.92	20.00

These four speeds were selected based on the previous study. The speed for Run 1 was found by trial-and-error. It was the highest speed that the screw could operate before delivering flow to the outlet duct; it was termed the “maximum no-flow speed”. The remaining trials were nominally set at 10, 15, and 20 RPM.

The maximum no-flow speed was a very important parameter to determine since it is the point that the screw is least efficient under these conditions. At this speed, the ASP transports water all the way up the trough but drains through leakage just before it can deliver it into the outlet duct. This condition was very important to capture when characterizing gap leakage and screw performance.

Whenever the speed was set, the water level was adjusted so that the first bucket closed at its optimal lower level. Adjusting the water level had an impact on the speed of the screw since a higher water level increased torque requirements and lessened speed. So, the speed was readjusted to the desired value.

Since the tank was a closed system, changing the speed of the screw also effected the water level of the tank. So, the process of adjusting the water level and speed was iterated until the appropriate conditions were met. This process was repeated for the four experimental runs.

Once a speed was set, the cameras simultaneously began to record video. To sync the videos in the analysis stage, it was important to have a key frame that occurred in both videos at the same time. The cameras recorded approximately one minute of operation. The DSLR cameras used in this study captured footage at 1080p 24fps.

After the four experimental runs were conducted and recorded, the video footage was exported to Kinovea. To track the water level using Kinovea, two measurements were taken. The first was the vertical position of the top of the water in a bucket as it translated up the screw, and the second was the

position of the bottom of that screw bucket. Tracking and annotation was carried out on both the front and back side of the screw for all unique consecutive buckets. The tested screw had three blades, which meant there were three unique bucket channels in the screw during operation. So, three consecutive buckets were traced in Kinovea.

Three reference measurements were taken for both the front and back in each video set. One to define the length of the screw trough, another to define the length of the screw pitch, and the last to define the height of a bucket when $f = 1$ (i.e., to define z_{min} and z_{max} from Detail A in Figure 1).

The results from Kinovea were then exported to Excel for further analysis. The data and reference measurements were used to calculate the bucket fill height ratio, gap leakage, and conduct further analysis quantifying the impact of speed on the observed phenomena.

III. RESULTS

The bucket fill height ratio was first compared from the front and back of the bucket for each of the three successive buckets and each speed trial. In Figure 4, the results of the 15 RPM speed trial are shown.

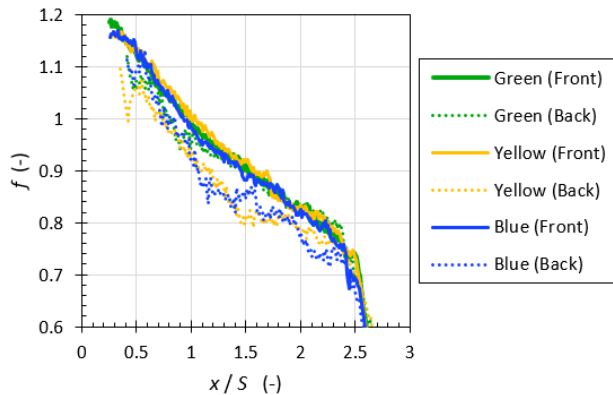


Figure 4. Bucket fill height level throughout duration of travel in the 15 RPM trial. Front and back bucket height for three successive buckets denoted by lower adjacent blade colour (see Figure 2) are compared against their dimensionless position from the screw inlet during travel.

In Figure 5, the fill height was compared against the dimensionless position of the bucket along the screw's length. The distance was made non-dimensional by dividing it by the screw pitch. This made the results more generalized so they can be useful for other researchers. There is also a direct correlation between rotation speed (rotations per time) and pitch (axial distance per screw rotation). Additionally, the length of an Archimedes screw is often described in terms of pitch. For example, the tested screw had a length ratio of $L/S = 2.86$.

Presenting the results against the dimensionless position also allowed for direct comparisons between the different speed trials, since increasing speed decreased the time duration of the buckets in transport.

It was observed that the front and back side of the screw had differences in fill height. This was observed in all trials but was

easiest to visualize in the plot for the 15 RPM trial (shown in Figure 15). As expected, the water level decreased from the start of the screw to the end. The screw started with a fill height of $f \approx 1.2$ at the inlet ($x/S = 0$) and reached a fill height of $f \approx 0.6$ near the outlet (at $x/S \approx 2.6$).

A. Average Bucket Fill Height and Oscillations

This analysis was conducted for each of the speed trials. For each trial, the average fill height was found for the front and back of each of the three buckets. The results for all speed trials are shown in Figure 5.

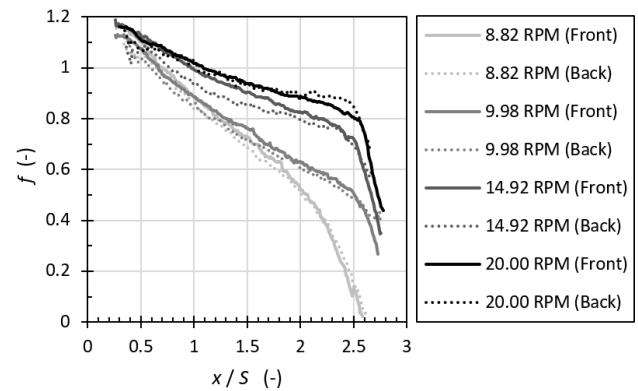


Figure 5. The average bucket fill height of the the front and abck of the three successive buckets in each trial were compared to their dimensionless distance from the screw inlet.

The general trend of a decreasing fill height along the length of the screw was present. So, the fill height gradient was present for all screw rotation speeds tested. The slope of the gradient was heavily influenced by rotation speed. An increase in rotation speed directly correlated to a decrease in slope (an increased last-bucket fill height). When rotation speed increased, the bucket spent less time within the screw leaking (via gap leakage), and therefore maintained a higher fill height than a slower moving screw bucket.

The front and back of the bucket seemed to consistently differ in magnitude, suggesting that the bucket was experiencing dynamic action like the sloshing noticed in a previous CFD study [9]. To investigate this action further, the average between the front and back bucket fill height was found. That value was then subtracted from the front bucket fill height to find the amplitude of oscillation in the bucket. Note that the back bucket fill height oscillation is just the inverse of the front (i.e., when the front is negative, the back is positive). This oscillation is shown against the dimensionless position along the screw in Figure 6.

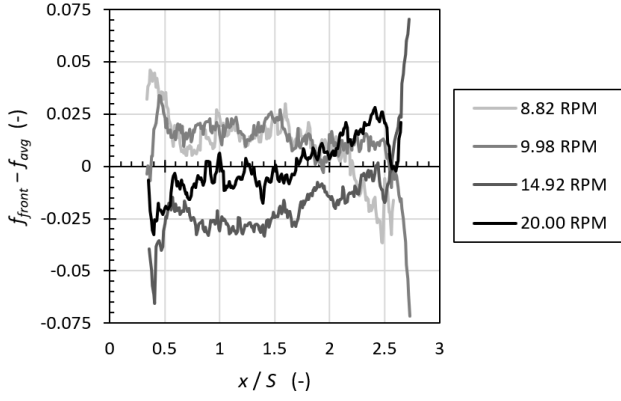


Figure 6. Buck fill height fluctuation plotted against the dimensionless length along the screw.

The difference in fill height from the average fill height indicated that the bucket was undergoing a sloshing action during transport through the screw. The fluctuations were very different as rotation speed changed. In the case of the maximum no-flow speed, the front side of the screw seemed to be higher than the back throughout travel – until the last pitch length, while the screw was open to the outlet and emptying. This action could be reasonably expected since the screw rotated through the bucket from back to front. As was postulated in the previous study [5], perhaps wall shear stress was causing a front-bias to bucket fill height.

It was also postulated that an increase in rotation speed would increase the front-bias; increase the front fill height and decrease the back, causing a positive fluctuation in Figure 6. However, that was only the case with the maximum no-flow speed (8.82 RPM) and the nominally 10 RPM (9.98 RPM) case. The magnitude of the 9.98 RPM fluctuations were slightly higher than the no-flow speed, which neither definitely supported nor ruled out a changing impact of wall shear/friction.

The 14.92 RPM case was observed to be biased towards the back of the screw during operation (i.e., a negative magnitude in Figure 6). While it is possible that this was due to measurement error, the same techniques were used for all trials, and the same data analyst conducted the analysis of all trials. The bias seemed to shift from front to back about halfway along the screw. That indicated the screw bucket might have sloshed from back to front after an initial surge from front to back during filling at the inlet.

An almost identical profile was found with the 20.00 RPM case, however it had a higher overall magnitude. The 20.00 RPM case did not cross the threshold from back to front bias like the 14.92 RPM case; though, perhaps this was because the bucket translated faster and did not have the opportunity to reach the same point of sloshing action as the 14.92 RPM case.

The authors suggest that it is still possible that the increase of speed corresponded to an increase in friction, and the oscillations in the screw. This may have been conditionally observed in the experiments. As the speed increased from 8.82 RPM to 9.98 RPM, the fill height oscillations increased in magnitude. Again, from 14.92 RPM to 20.00 RPM, the fill

height oscillations increased in magnitude. However, there was a drastic difference in oscillation magnitude and form between 9.98 RPM and 14.92 RPM.

The authors noticed that the slower speed trials had very calm free surfaces during transport when compared to the higher speeds. It is possible that flow in the buckets switched from a near laminar to fully turbulent flow during these speed changes. A flow regime change would explain the drastic change in oscillation, since a turbulent boundary layer flow experiences much less friction/wall shear than a laminar system.

The Archimedes screw is a complex geometry, so estimating its Reynolds number to determine the flow regime was difficult. If the screw and trough were assumed to be analogous to pipe flow through an annulus, one could approximate the Reynolds number as

$$Re = \frac{\rho v_T D_H}{\mu} = \frac{\rho \omega S (D_o - D_i)}{\mu \cdot 60} \quad (2)$$

where the characteristic flow velocity could be approximated as the transport velocity (v_T) of the bucket – the axial velocity of the bucket as it travels along the trough inlet to outlet. The transport velocity is equal to the rotation speed (ω , in RPM), times the screw pitch (S), divided by 60 to get it in terms of meters per second. The hydraulic diameter (D_H) of an annulus was used as the length scale to approximate the Reynolds number. The density (ρ) and dynamic viscosity (μ) of water were used.

For pipe flow, Reynolds numbers under 2300 indicate a laminar flow and about 3500 indicates fully turbulent flow. In-between values correspond to more complex transitional flow regimes. Reynolds number estimates are shown in Table 3.

With this estimate, the 8.82 and 9.98 RPM trials were under a transitional flow regime, while the 14.92 and 20.00 RPM trials were experiencing fully turbulent flow. It is possible that there was a flow regime shift that was responsible for the change in oscillation response to rotation speed. Observations made during experimentation would support this hypothesis. This would be a very interesting avenue for further research.

TABLE III. REYNOLDS NUMBER AND FLOW REGIME ESTIMATES.

ω (rev/min)	v_T (m/s)	Re (-)	Possible Flow Regime
8.82	0.03087	2492	Transitional
9.98	0.03493	2820	Transitional
14.92	0.05222	4216	Turbulent
20.00	0.07	5651	Turbulent

B. Overfilling of First Bucket

Revisiting the dynamic “sloshing” observed in the screw, the authors noticed a consistent inrush of water at the screw’s inlet during operation. Once the first bucket was fully enclosed, the first-bucket fill height was always higher than expected.

The screw was always carefully set to its optimal lower water level (h_L'), which should correspond to a first-bucket fill height of $f = 1$ once the bucket becomes fully enclosed at the inlet. This was not observed in the experiments. Figure 7 shows the closing of a screw bucket in four different snapshots of time.

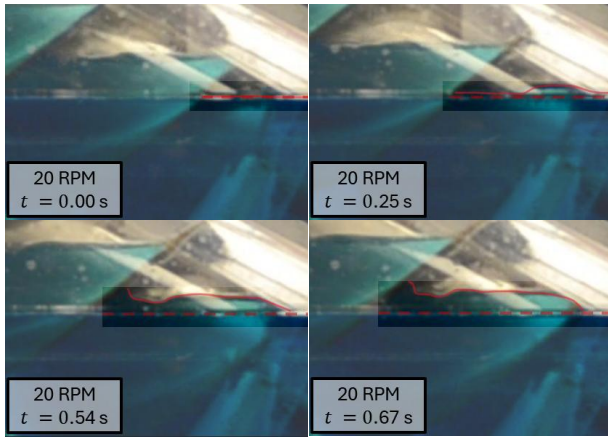


Figure 7. Overfilling of first bucket and surging at the inlet. Dashed line indicates lower water level and, by extension, expected bucket water level. Solid lines indicate actual water levels.

It is possible that the increased fill height of the first bucket was caused by a surge of water entering the screw at the inlet, and aided by a churning action caused by blade friction. In that case, it would be reasonable to expect the back side of the screw bucket to be under-filled; however, it was also overfilled as observed in Figure 8.



Figure 8. Overfilling of first bucket and surging at the inlet at the back side of the screw bucket. Dashed line indicates lower water level and, by extension, expected bucket water level. Solid lines indicate actual water levels.

Interestingly, the back side seemed to overfill to a lesser degree than the front side, so there still was a front-side bias as could be reasonably hypothesized.

The first-bucket fill height (f_0) was found for the front and back side at each speed trial. The expected first-bucket fill height ($f_{0, \text{expected}} = 1$) was subtracted from the measured numbers, and is presented as the overfill amplitude in Figure 9.

It was observed that the front side of the first bucket always experienced higher overfilling than the back side. Both buckets experience overfilling for all tested speeds. This suggests that the optimal fill height does not correspond to an initial fill height of $f_0 = 1$ in the case of this laboratory-scale screw. It is noted that the laboratory-scale screw should proportionally be more impacted by friction than a full-scale Archimedes screw pump, so this observation might not hold true for much larger screws.

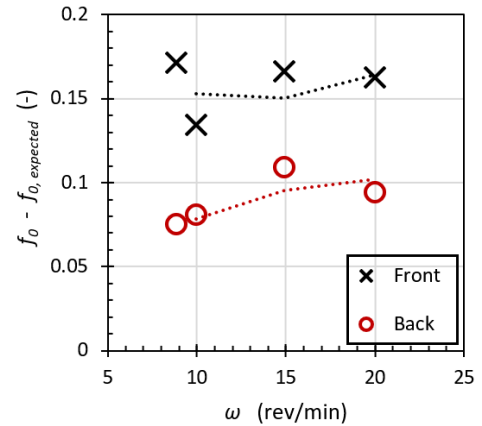


Figure 9. Overfill amplitude ($f_0 - f_{0, \text{expected}}$) of screw first-bucket at the inlet – front and back side of the bucket.

C. Gap Leakage

One of the goals of the previous study [5] and this study was to quantify gap leakage experimentally in an operating Archimedes screw. Before the previous study, gap leakage had never before been quantified experimentally across individual blade tips due to the complex geometry of Archimedes screws. By implementing a second camera, and an apparatus that allowed for the full screw length to be visualized, this current study has improved the reliability of this quantification.

To quantify gap leakage, a numerical model [10] was used to calculate bucket volume (V_b) based on the measured fill height. The average fill height of the front and back side of the screw shown in Figure 5 was used to determine the bucket volume. Results are shown in Figure 10.

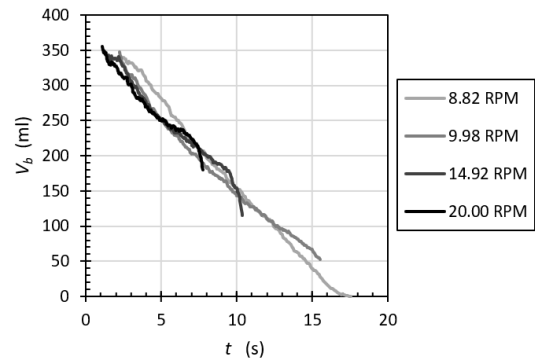


Figure 10. Bucket volume plotted against time the bucket has been transported within the screw.

The speed trial curves of Figure 10 were then used to find the gap leakage. Since only gap leakage could cause the decrease in fill height when the fill height was $f \leq 1$, the integral of the curve when $f \leq 1$ should yield the gap leakage experienced by the screw.

Interestingly, the slope of each speed trial curve seemed visually similar. That indicated gap leakage was a similar magnitude for each speed trial.

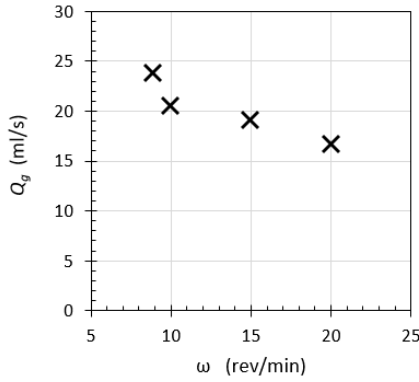


Figure 11. Gap leakage flow rate for varying screw pump rotation

The authors note that the last pitch-length of the screw always has a rapid drop-off in fill height since water exits to the open outlet duct. So, the integral was taken for all cases of $f \leq 1$ when $x/S < 2.86 - 1 = 1.86$. Results of this integration are shown in Figure 11.

It was observed that gap leakage flow rate slightly decreased as rotation speed increased. In the literature, gap leakage is predicted using a derivation of the energy equation, usually assuming a negligible impact from dynamic effects (i.e., driven by potential energy) [7, 10]. The results presented in Figure 11 suggest that rotation speed has a non-negligible impact on gap leakage flow rate; therefore, to improve model accuracy, dynamic effects cannot be neglected.

Finally, it should be noted that the potential uncertainty of these methods is relatively high. With these methods, it is also difficult to estimate reliable uncertainty ranges of variables and propagate that uncertainty through the analysis. Keeping uncertainty to feasible levels means it is important to minimize image distortion by placing cameras as far back as reasonable from the screw being imaged, and ensuring high quality video is obtained. The method was found to be practically limited to cases without substantial splashing of water onto the transparent surfaces or disturbance of the free surface in a bucket. In practice, this meant only the lower speed range of the screw could be studied. The significant needs for optical access to the operating screw also limit the practicality of this method in field studies.

IV. CONCLUSION

This experiment provided a novel view of both the front and back of a turning Archimedes screw. It leveraged image processing tools to optically measure the fill height gradient of the front and back of an operating screw. Fill height was observed to vary between the front and back of the screw, with varying levels of oscillations. It appeared that the oscillations may be impacted by flow regime (i.e., laminar, turbulent) caused by variation of the screw rotation speed.

This phenomenon is critically important in understanding gap leakage and in calculating volume optically in Archimedes screws. Further experimentation should be conducted to better

understand the relationship between rotation speed these dynamic phenomena – especially as they may impact ASP performance.

The variation in water level from the front and back side of the screw bucket has never before been quantified experimentally. This phenomenon was observed in a previous study qualitatively and quantified through computational fluid dynamics simulations [9]. However, for the first time, this phenomenon has been captured, recorded, and quantified experimentally. Its documentation has many implications on modelling Archimedes screw pumps.

It was also observed that gap leakage was non-negligibly impacted by rotation speed, suggesting that rotation speed should be included when predicting gap leakage flow rate in performance models.

To a much greater extent, this study demonstrated that two-phase immiscible flows (i.e., water-air free surface flow) can be reliably traced and quantified experimentally using video annotation software such as Kinovea. This has huge implications on this field of fluid mechanics since it allows researchers to use video footage and open-source annotation software to reliably, and inexpensively measure free surface flows (wave action, open channel hydraulics, tank drainage, etc.).

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