

## TIME SERIES MODEL ESTIMATION IN DYNAMIC SYSTEM AND SELECTION OF ARMAX OPTIMAL MODEL ORDER

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**Abstract**— This study examines the process of selecting an appropriate model order for dynamic system modeling using time series techniques. It highlights several estimation algorithms, focusing on their application to the Autoregressive Moving Average with eXogenous input (ARMAX) model. The analysis is conducted on a flexible robotic system where both force and acceleration measurements are available. An optimal model order is proposed based on minimizing the total mean square error variance of the estimated transfer functions. This approach proves effective in addressing noise and modeling uncertainties, enabling the identification of concise, low-order transfer functions that accurately represent system dynamics. The proposed criterion is evaluated alongside established model selection techniques, including the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Noise Order Factor (NOF), showing close agreement and robust performance in facilitating accurate modal analysis of complex mechanical systems.

**Keywords:** *Time series model, ARMAX model, Model order selection.*

### I. INTRODUCTION

The accurate identification of dynamic characteristics in mechanical systems under operational conditions is fundamental to modern control, diagnostics, and structural health monitoring. Transfer functions particularly in the form of Frequency Response Functions (FRFs) and Operational Transfer Functions (OTFs) are indispensable tools for representing system behavior in the frequency domain, enabling the extraction of modal parameters and analysis of vibration phenomena. While non-parametric techniques such as the Empirical Transfer Function Estimate (ETFE) provide intuitive means for estimating system dynamics via frequency-domain ratios of input-output data, they are limited by high variance, sensitivity to noise, and poor

scalability in industrial environments where disturbances are common, and signal quality can be inconsistent [1]. To address these limitations, time series-based parametric models have gained prominence for system identification. Among these, the Autoregressive Moving Average with eXogenous input (ARMAX) model offers a comprehensive structure for capturing both deterministic input-output relationships and stochastic disturbances. Its formulation integrates autoregressive, moving average, and exogenous input terms, making it particularly suited for identifying dynamic systems with measurable external excitations and internal noise [2–4]. However, the effectiveness of parametric modeling is highly dependent on selecting an appropriate model order. An inadequately chosen order may lead to underfitting, overfitting, or numerical instability, ultimately compromising the accuracy of the estimated transfer functions. Model order selection has therefore emerged as a crucial step in the identification process. A range of statistical and information-theoretic criteria has been proposed to guide this decision, including the Akaike Information Criterion (AIC) [5], Bayesian Information Criterion (BIC) [6], and Noise Order Factor (NOF) [7]. These criteria aim to balance model complexity with predictive accuracy by penalizing over-parameterization while ensuring sufficient fidelity to observed dynamics.

This study contributes a focused review and application of model order selection algorithms within the ARMAX modeling framework. The primary objective is to determine an optimal model order that minimizes the total mean square error of the estimated transfer functions, thereby improving robustness against measurement noise and modeling inaccuracies. Special attention is given to low-order models that preserve computational efficiency while retaining dynamic fidelity. To demonstrate the practical applicability of the reviewed methods, experiments are conducted on a flexible robotic manipulator, SCOMPI, undergoing a grinding operation. In this context, the measured grinding force serves as the exogenous input, while the structural response is recorded via acceleration sensors. The

ARMAX model is employed to estimate the transfer functions from this operational data, incorporating the Moving Average component to account for system disturbances. The identified model orders are assessed using standard criteria such as AIC, BIC, and NOF, and are shown to align with theoretical expectations, supporting the model's suitability for modal analysis under real-world conditions.

## II. ARMAX TIME SERIES MODEL

System identification is the process of developing mathematical models of dynamic systems from measured time series data. In this context, the objective is to derive a model that captures the behavior of a system using discrete-time measurements of input  $u(t)$  and output  $y(t)$ , over a finite time horizon  $t=1, \dots, T$ . Various parametric model structures have been proposed for identifying linear systems, notably the Auto-Regressive (AR), Auto-Regressive Moving Average (ARMA), Auto-Regressive with eXogenous input (ARX) [8, 9], and Auto-Regressive Moving Average with eXogenous input (ARMAX) models [10].

The choice of an appropriate model depends on the nature of the available data and the characteristics of the system under investigation. In general, time series models include an AR term to account for dependence on past outputs, an X (exogenous) term to represent the effect of external inputs, and an MA term to model residual noise or unmeasured disturbances.

Among these, the ARMAX model offers a comprehensive structure, capable of modeling systems with both measurable inputs and non-negligible stochastic disturbances. It extends the ARX model by incorporating a Moving Average (MA) component, allowing it to capture a broader class of real-world systems where measurement noise and environmental disturbances are prevalent.

### A. ARMAX Model Representation

A general linear time-invariant (LTI) system in the ARMAX form can be described by the following discrete-time transfer function model:

$$A(q)y(t) = B(q)u(t - n_k) + C(q)e(t) \quad (1)$$

Where  $y(t) \in \mathbb{R}$  is system output at discrete time step  $t$ ,  $u(t) \in \mathbb{R}$  is system input (exogenous signal) at time  $t$ ,  $e(t) \in \mathbb{R}$  denotes zero means white noise or innovation process (unmeasured disturbance),  $q^{-1}$  is the backward shift operator, i.e.,  $q^{-k}y(t) = y(t-k)$ , and  $A(q)$ ,  $B(q)$ , and  $C(q)$  are polynomials in  $q^{-1}$  of degrees  $n_a$ ,  $n_b$ , and  $n_c$ , respectively,  $n_a$ ,  $n_b$ , and  $n_c$  are model order for AR, X, MA terms respectively, and  $n_k$  is the input delay.

The polynomials are defined as:

$$A(q) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a} \quad (2)$$

$$B(q) = b_0 + b_1q^{-1} + \dots + b_{n_b}q^{-n_b} \quad (3)$$

$$C(q) = 1 + c_1q^{-1} + \dots + c_{n_c}q^{-n_c} \quad (4)$$

This model describes a system where the output  $y(t)$  is influenced not only by current and past inputs  $u(t)$ , but also by a

stochastic noise process that is filtered by  $C(q)$ , making it particularly effective in noisy industrial environments.

### B. Prediction and Estimation

Given the model structure, to estimate the parameter vector  $\theta = [a_1, \dots, a_{n_a}, b_1, \dots, b_{n_b}, c_1, \dots, c_{n_c}]^T$ , we defined a one-step-ahead predictor  $\hat{y}(t|t-1)$  based on known past input and outputs, and the prediction error is

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t|t-1) \quad (5)$$

Given assumption that  $e(t) \sim N(0, \sigma^2)$  is uncorrelated with inputs, the predict error method (PEM) estimate  $\theta$  by minimizing the sum of squared errors:

$$\hat{\theta} = \arg \min \sum_{t=1}^T \varepsilon^2(t, \theta) \quad (6)$$

The system can also be represented via its transfer functions:

$$y(t) = G(q)u(t) + H(q)e(t) \quad (7)$$

where:

$$G(q) = \frac{B(q)}{A(q)} q^{-n_k} \text{ is transfer function from input to output}$$

$$H(q) = \frac{C(q)}{A(q)} q \text{ is noise model transfer function}$$

Under the assumption of Gaussian noise, the prediction error method leads to consistent and asymptotically efficient estimates of the parameters. The inclusion of the MA component  $C(q)$  in the ARMAX model helps account for colored noise, which is common in physical measurements and is not properly handled by simpler models like ARX. Unlike the ARX model, where the same denominator polynomial  $A(q)$  models both the deterministic and stochastic parts of the system, the ARMAX model decouples these effects through separate  $A(q)$  and  $C(q)$  polynomials. This distinction enhances the accuracy of the noise model and improves the fidelity of the overall system representation, particularly when disturbances are significant or spectrally colored.

Furthermore, the ARMAX structure provides a more flexible and realistic representation of industrial systems, such as robotic manipulators operating in the presence of machining noise or environmental vibrations. For instance, in grinding operations, force signals may be used as exogenous inputs while structural accelerations serve as outputs, and the ARMAX model offers the ability to robustly estimate transfer functions under such conditions.

In practice, when identifying a model from data, the total error in estimating the true frequency response function  $G_0(j\omega)$  can be decomposed as:

$$\begin{aligned} E \left[ |G_0(j\omega) - \hat{G}(j\omega)|^2 \right] &= \\ &= |G_0(j\omega) - G_\theta(j\omega)|^2 + E \left[ |G_\theta(j\omega) - \hat{G}(j\omega)|^2 \right] \end{aligned} \quad (8)$$

where  $G_0(j\omega)$  is the true system,  $G_\theta(j\omega)$  is the best model within the model class (bias), and  $\hat{G}(j\omega)$  is estimated model from data (variance).

Here, the bias error arises from model structure or order mismatches, while the variance error stems from measurement noise and estimation uncertainty. Selecting an appropriate model order is thus essential to minimize total estimation error. Overly simplistic models may result in high bias, while overly complex models can amplify variance and lead to overfitting. The model order must therefore balance fidelity and generalization, which will further be addressed in the next section through a review of model order selection criteria.

### III. MODEL ORDER ESTIMATION

#### A. Model Order Estimation Criteria for ARMAX Model

In parametric time series modeling using ARX or ARMAX structures, the accuracy and generalization capability of the identified model are critically influenced by the selected model order. Determining an appropriate model order is a longstanding challenge in system identification. A model that is overly complex may overfit noise and increase computational cost, while a model of insufficient order risks failing to capture essential dynamic features of the system. In practical applications, especially in operational modal analysis of mechanical structures operating in low-frequency regimes, reliable identification of AR, MA, and exogenous (X) polynomial orders is essential. Model order selection not only improves estimation accuracy but also contributes to reducing bias and variance in predicted transfer functions.

To address this, several order selection criteria have been developed in the literature. Among the earliest is the Final Prediction Error (FPE) criterion proposed by Akaike [11] for AR models, later extended to ARMA models by [12]. The FPE balances model complexity against prediction performance by penalizing overfitting. A more robust alternative is based on the eigenvalues of a modified covariance matrix, as proposed in [13, 14], which remains effective in the presence of noise. Information-theoretical criteria are widely adopted due to their solid theoretical foundation. These include: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Minimum Description Length (MDL) [15]. These approaches penalize model complexity through a likelihood-based cost function, attempting to strike a balance between model fit and the number of parameters.

Let  $N$  be the number of observations, and  $z$  the total number of parameters in the model. The AIC is given by:

$$\text{AIC}(z) = N \ln(\det |\hat{\Sigma}|) + 2(z) \quad (9)$$

where  $N$  denotes the number of data points,  $(z)$  is a dimension associated with the vector of unknown parameters to be estimated and

$$\hat{\Sigma} = \frac{1}{N} \sum_{t=1}^N \hat{\mathbf{w}}[t] \cdot \hat{\mathbf{w}}^T[t] \quad (10)$$

$\hat{\Sigma}$  is the covariance matrix of the innovation sequence associated with the estimated coefficients; the vector  $\mathbf{w}[t]$  denotes the innovation sequence at time  $t$ .

For an ARMAX model with orders  $(n_a, n_b, n_c, n_k)$ , with  $n_a, n_b, n_c$  and  $n_k$  orders for its AR, MA, eXogenous components and time delay, respectively, while  $p$  represents the number of orthonormal functions by which each of these components is multiplied. Here, the total number of estimated parameters is:

$$z = (n_a + n_b + n_c + n_k) \cdot (p) \quad (11)$$

The AIC for ARMAX becomes:

$$\begin{aligned} \text{AIC}(n_a, n_b, n_c, n_k)(p) &= \\ &= N \ln(\det |\hat{\Sigma}|) + 2(n_a + n_b + n_c + n_k)(p) \end{aligned} \quad (12)$$

where  $z$  is the number of scalar parameters in the ARMAX model.

AIC tends to favor more complex models. To mitigate over parameterization, the Bayesian Information Criterion (BIC) introduces a stronger penalty term:

$$\text{BIC}(z) = \ln(\det |\hat{\Sigma}|) + \frac{z \ln(N)}{N} \quad (13)$$

And for the ARMAX model:

$$\begin{aligned} \text{BIC}(n_a, n_b, n_c, n_k)(p) &= \\ &= \ln(\det |\hat{\Sigma}|) + \frac{(n_a + n_b + n_c + n_k)(p) \ln(N)}{N} \end{aligned} \quad (14)$$

These criteria are consistent under certain assumptions and tend to select the correct model order as the sample size increases. However, their performance can degrade in small-sample settings or in systems with low signal-to-noise ratios.

#### B. Minimization of Transfer Function Estimation Error

To address the limitations of traditional criteria, this paper proposes a novel model order selection method that minimizes the total mean square error (MSE) between the true transfer function  $\mathbf{G}(q)$  and the estimated one  $\mathbf{G}(q, \hat{\theta}_N)$ . The focus is on frequency domain error between estimated and actual system dynamics.

We define the error metric:

$$\varepsilon \left( \left\| \mathbf{G}(q) - \mathbf{G}(q, \hat{\theta}_N) \right\|^2 \right) \quad (15)$$

This error can be decomposed into a bias term, which decreases with increasing model order, and a variance term, which increases with model order. Thus, the optimal model order  $P_{\text{optimal}}$  must balance both components. The criterion is:

$$P_{\text{optimal}} = \arg \min_{p_o=1,2,\dots} \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{\mathbf{E}}_p(\omega) \mathbf{D}_u(\omega) d\omega \right) \quad (16)$$

where  $\hat{\mathbf{E}}_p(\omega)$  is the estimated mean square error between true and identified transfer functions at frequency  $\omega$ , and  $\mathbf{D}_u(\omega)$  denotes power spectral density (PSD) of the input signal  $u(t)$ .

Assuming the input is quasi-stationary and has zero mean, the estimated transfer function can be decomposed as:

$$\mathbf{G}_T(q) = \mathbf{G}_T(q, \theta_0) + \mathbf{G}_\Delta(q) \quad (17)$$

where  $\mathbf{G}_T(q, \theta_0)$  is mean of the estimated transfer function, and  $\mathbf{G}_\Delta(q)$  is a zero-mean residual error component

$$\varepsilon\{\mathbf{G}_\Delta(q)\} = 0 \quad (18)$$

Given ARMAX model structure:

$$\mathbf{G}(q, \hat{\theta}_N) = \frac{\mathbf{B}(q, \theta)}{\mathbf{A}(q, \theta)} = \frac{b_0 + b_1 q^{-1} + b_2 q^{-2} \dots + b_{n_b} q^{-n_b}}{I + a_1 q^{-1} + a_2 q^{-2} \dots + a_{n_a} q^{-n_a}} \quad (19)$$

The true transfer function can be estimated directly from data as:

$$\mathbf{G}_T(q) = \frac{\mathbf{Y}(q)}{\mathbf{U}(q)} = \frac{\mathbf{y}(t)}{\mathbf{u}(t)} \quad (20)$$

The means square error at each frequency  $\omega$  becomes:

$$\hat{\mathbf{E}}_{p_o}(\omega) = \varepsilon\left\{\left|\mathbf{G}(q, \hat{\theta}_N) - \mathbf{G}_T(q)\right|^2\right\} = \text{Trace}\{\mathbf{Q}_{\tilde{g}}\} \quad (21)$$

where:

$$\mathbf{Q}_{\tilde{g}} \triangleq \varepsilon\left\{\tilde{g}(q) \tilde{g}(q)^T\right\} \quad (22)$$

$$\tilde{g}(q) \triangleq \begin{bmatrix} \text{Re}\{\mathbf{G}_T(q) - \mathbf{G}(q, \hat{\theta}_N)\} \\ \text{Im}\{\mathbf{G}_T(q) - \mathbf{G}(q, \hat{\theta}_N)\} \end{bmatrix} \quad (23)$$

This methodology reflects the complete error dynamics by incorporating both real and imaginary components of the deviation in the frequency domain.

#### IV. APPLICATION FOR SCOMPI ROBOT

Flexible manipulators are increasingly utilized in the maintenance of large-scale hydroelectric equipment due to their portability and adaptability to constrained work environments [16]. However, the control of lightweight, flexible robotic manipulators remains a complex challenge. The inherent structural flexibility of these systems introduces undesired vibrations that degrade trajectory tracking performance and may lead to instability during operation.

In this study, the proposed ARMAX-based identification methodology is applied to a light and portable track-mounted robotic system known as SCOMPI (Super-COMPact Ireq) [17, 18]. SCOMPI is a multi-process manipulator specifically designed for in-situ repair operations. Due to the dynamic nature of the grinding process, it is critical to accurately identify the system's transfer functions and modal parameters to ensure precise vibration control and system stability.

To evaluate the system's dynamic response, the operational Frequency Response Functions (FRFs) and associated modal parameters are estimated using time domain input-output data collected during a real grinding process. In this setup, measured grinding forces serve as the input excitation, while the structural response of the manipulator is captured via acceleration measurements. The grinding test is performed at a spindle speed of 3225 rpm, using an axial depth of cut of approximately 0.08 mm. The workpiece is composed of EN31-64HNC hardened steel with dimensions of 150 mm  $\times$  7 mm  $\times$  48 mm. The cutting forces are captured using a Kistler 3-axis dynamometer (model CH8408), which is rigidly mounted beneath the workpiece. The

dynamometer records forces in three orthogonal directions:  $F_x$  - normal force,  $F_y$  - Tangential force,  $F_z$  - Axial force. Simultaneously, structural vibrations at the robot's end-effector are measured using three PCB-352C34 piezoelectric accelerometers, each with a sensitivity of 5.29 mV/g. Three sensors capture acceleration data at the end effector, along the corresponding axes to represent dynamic responses in three spatial directions. Data acquisition is performed using an LMS Test Lab system, which records signals over a 10-second duration at a sampling frequency of 512 Hz. The same platform is employed to calculate average FRFs from experimental data. These FRFs serve as a benchmark for validating the performance of the proposed ARMAX identification methodology.

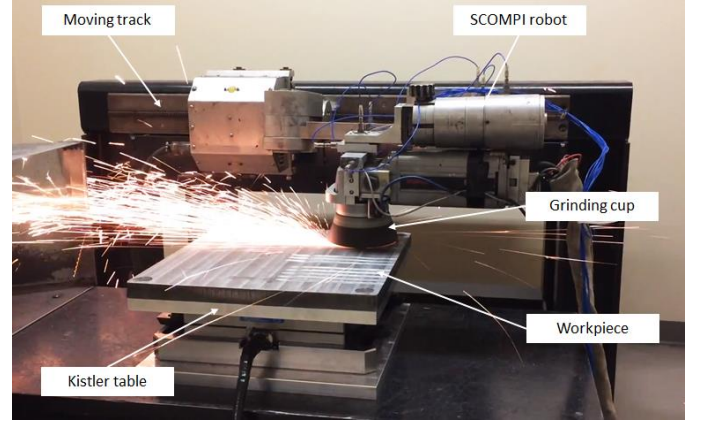


Figure 1. Experimental setup for SCOMPI robot during grinding operation

Figure 1 presents the actual experimental configuration, showing the SCOMPI manipulator in operation during the grinding test. Importantly, the excitation of the system arises directly from the grinding process, ensuring that the data reflects all relevant operational phenomena, including spindle rotation dynamics, cutting interactions, and structural coupling effects.

##### A. Model Order Estimation

The parametric identification of the structural dynamics is carried out using time-domain input-output data acquired over a sampling window of 10 seconds. The input consists of externally measured excitation forces, while the output corresponds to the system's structural response, recorded via acceleration sensors.

The modeling framework is based on the ARMAX structure. The identification strategy involves the iterative estimation of ARMAX models over a range of candidate orders. For each model order  $p \in [1, 60]$ , models are fitted to the measured force-response data, and their performance is evaluated using two classical information-theoretic criteria: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

Figure 2 presents the evolution of AIC and BIC values as functions of the model order  $p$ , computed directly from ARMAX models fitted to the experimental datasets. These criteria provide quantitative guidance for selecting the optimal

model complexity, balancing fidelity and parsimony in representing the underlying dynamics.

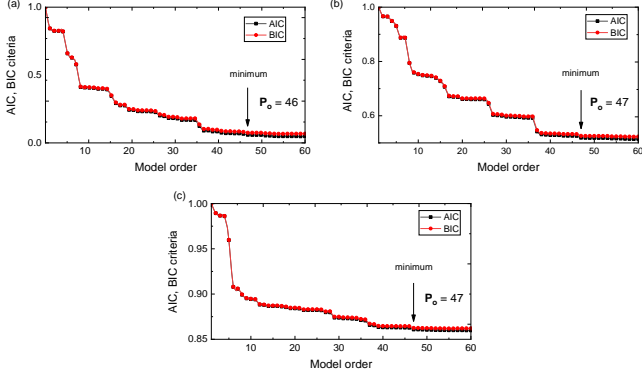


Figure 2. Model order selection based on AIC and BIC criteria for different sets of data: (a) X data, (b) Y data, (c) Z data

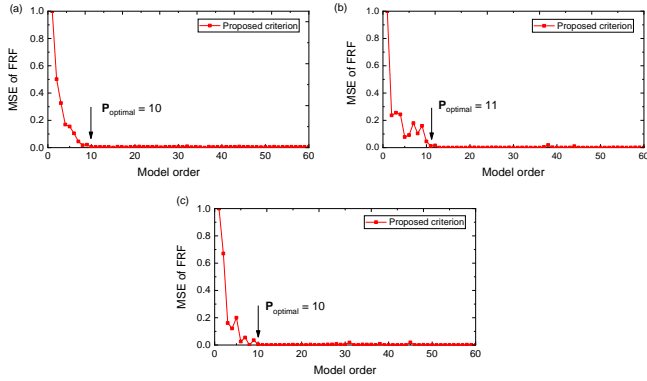


Figure 3. Model orders selection based on the proposed approach for different sets of data: (a) X data, (b) Y data, (c) Z data

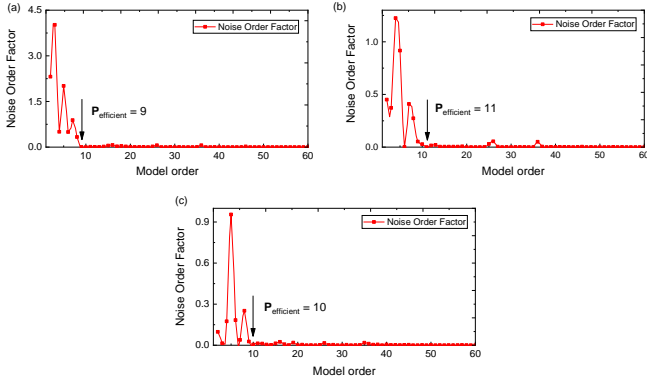


Figure 4. NOF evolution and efficient model orders selection for different sets of data: (a) X data, (b) Y data, (c) Z data

Figure 2 presents the evolution of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values as functions of the model order. Both criteria exhibit a decreasing trend with increasing model order, with a minimum observed around  $p = 47$ . However, a key limitation of these techniques is their tendency to yield differing model order suggestions, without guaranteeing the selection of a truly optimal structure. Therefore, the resulting estimates should be interpreted with caution and supplemented by additional validation or domain knowledge.

To address this, the proposed frequency-domain method is applied to determine an optimal order based on the stability of estimated transfer functions. The key idea is to identify the lowest order beyond which the estimated dynamics converge and do not significantly change with further increases in model complexity. As shown in Figure 3, convergence is observed around order  $p = 10$  across all three motion directions. This value is therefore selected as the optimal model order, as it offers a parsimonious yet dynamically sufficient representation.

For comparative analysis, the approach is benchmarked against the method introduced by Vu et al. [7], which estimates an efficient model order  $p_{\text{eff}}$  based on the Noise-to-Signal Ratio (NSR). The NSR evaluates the relative contribution of residual noise to the deterministic component of the system. The estimate is computed using the trace norm of the innovation covariance matrix:

$$\text{NSR}(p) = \frac{\text{Trace}(\mathbf{M}(p))}{\text{Trace}(\mathbf{K}(p))}(\%) \quad (24)$$

A Noise-ratio Order Factor (NOF) is calculated as a variation of the NSR between two successive orders:

$$\text{NOF}(p) = |\text{NSR}(p) - \text{NSR}(p+1)| \quad (25)$$

NOF provides a measure of convergence, exhibiting significant changes at low model orders and diminishing as the model order increases. The value of NOF approaching zero indicates that further increases in order yield diminishing returns. As illustrated in Figure 4, convergence of NOF occurs around  $p=10$ , reinforcing the result obtained by the proposed method and validating it as an efficient and reliable model order.

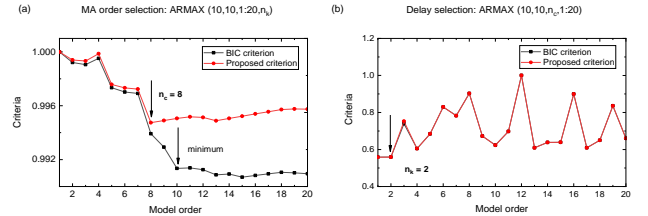


Figure 5. Moving average order (a) and time delay order (b) selection

While modeling a complex system like the SCOMPI manipulator theoretically requires a high-order model, criteria such as AIC and BIC suggest an optimal order near 47. However, both the proposed method and the Noise-to-Signal Ratio (NSR) indicate that a significantly lower order around 10 can sufficiently capture the system dynamics.

For ARMAX models, which include three distinct orders  $n_a$ ,  $n_b$ ,  $n_c$  and an input delay  $n_k$ , order selection is more involved. Following established guidelines [19, 20], and given that acceleration is the primary response measurement, an initial configuration of  $n_a = n_b = 10$  is adopted. The MA order  $n_c$  is set equal to  $n_a$  but prior studies suggest it may be lower in low-noise environments [21].

Since noise characteristics and delay are unknown,  $n_c$  and  $n_k$  are varied within  $\{1, \dots, 20\}$ , while keeping  $n_a = n_b = 10$ . Based on the proposed criterion, the combination (10,10,8,2) was

found to best balance model accuracy and complexity, as shown in Figure 5, and is thus selected for modal analysis.

## V. CONCLUSION

This paper reviewed and evaluated several model order selection methods for time series-based system identification, with a focus on their application to ARMAX models. A new approach was proposed, based on minimizing the total mean square error of the estimated transfer functions, which allows for the identification of low-order models that effectively capture system dynamics while accounting for noise and modeling uncertainties. The method was validated experimentally on a flexible robotic manipulator under grinding operation, where both force and acceleration data were used. The results were compared against established criteria, including AIC, BIC, and NOF, and showed strong agreement in identifying suitable model orders. Compared to ARX-based modeling, the ARMAX model provided more accurate and compact representations of the system. The proposed approach offers a practical solution to estimate the ARMAX model order for application to the operational modal analysis.

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