EFFECTIVE LEARNING OF BUCKLING OF COLUMNS IN ENGINEERING PROGRAMS

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Abstract *— In classic mechanical and civil engineering programs, buckling of columns is learned at end of the basic strength of materials course due to prerequisite subjects on integrals, differential equations, and engineering statics. This classic chronology of subject "buckling of columns" presents only a small advantage, which is to understand mathematical demonstration of buckling formulas, but does not give students opportunity of designing structures during learning engineering statics and strength of materials. By reorganizing few subjects and introducing practical aspects of buckling formulas earlier without all prerequisite subjects, students could practice structural design projects and some simultaneous engineering aspects by team work as early as from midway of the very first engineering course. This new approach has been applied for many years to Mechanical Engineering program of École de technologie supérieure in Montréal, Canada and proven very effective and well appreciated by students.*

Index Terms — Strength of materials, buckling of columns, design and analysis, truss structures, reorganization, team work, simultaneous engineering.

INTRODUCTION

Simultaneous Engineering (SE) or Concurrent Engineering (CE), which denotes simultaneously applying all related activities during design or development of products for reducing turnaround time still improving quality and cost, is more and more applied in industries due to increasing concurrence worldwide, [1]-[3].

Many SE aspects, such as *design methodology, team work including meetings, planning, communication, brain storming, research, preliminary design, engineering economy, project management, quality analysis, calculation, optimization, decision making, reports, etc.,* are only earned by experiences when dealing with real projects, which are usually much bigger than simplified examples presented in under graduate courses. For this reason, some colleges and universities have introduced Project-Based Learning (PBL) approaches since the 1970's for enhancing backgrounds of SE aspects [4].

However, difficulties in evaluating knowledge of every student based on team work reports made PBL approaches just a limited success, so that Engineering Accreditation Boards nowadays still require minimum Accreditation Units for teaching and learning engineering-related subjects, such as computer programming, mathematics, basic sciences, complementary studies, engineering sciences and design, and particularly encourage team work projects whenever possible during engineering curricula [5].

With classic curricula of Engineering Statics and Strength of Materials, such as text books [6]-[7], only simplified and small examples are given and there is no possibility of assigning any structural design project.

It will be shown in the following sections that design projects can be put into practice in two reorganized courses on Statics and Strength of Materials by modifying the chronology and the teaching approach of few subjects including "buckling of columns", specifically.

TRUSS-STRUCTURE DESIGN PROJECTS

A typical project on most basic and common structural design is presented in this section and referred to in subsequent sections for demonstrating how learning of buckling of columns could have a great effect on design and team work practices for engineering students.

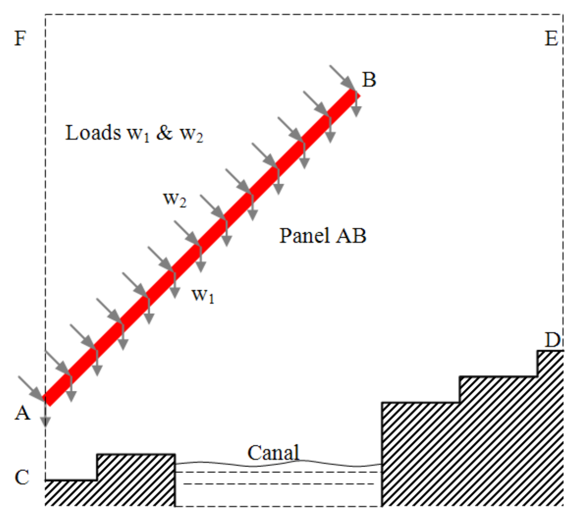

FIGURE 1
DESIGNING A STRUCTURE SUPPORTING PANEL AB IN SPACE CDEF.

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The project consists of designing a fixed structure in a relatively large space (like CDEF in Figure 1) for supporting objects (solar panel AB for example) subjected to loads (w_1 & w_2) with safety, reliability and economy.

Figure 2 and Figure 3 show two truss structures among several possible solutions which satisfy safety and reliability criteria. However, solutions having high quality-cost ratios among those possibilities can only be found after performing several SE tasks which take time, manpower and are usually better achieved by team work than by just one person.

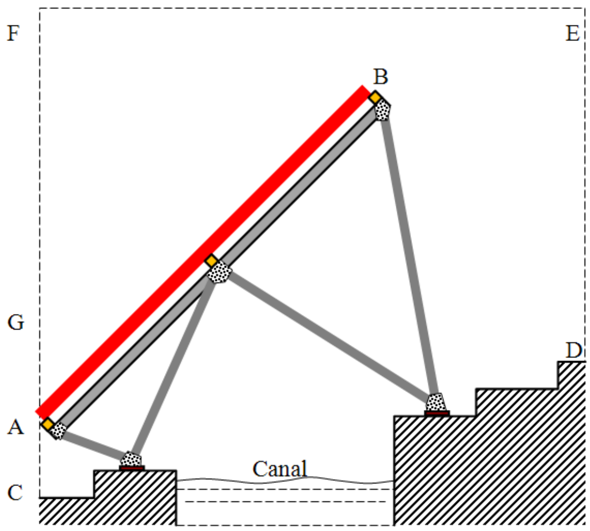

FIGURE 2
POSSIBLE STRUCTURE 1 SUPPORTING PANEL AB IN SPACE CDEF.

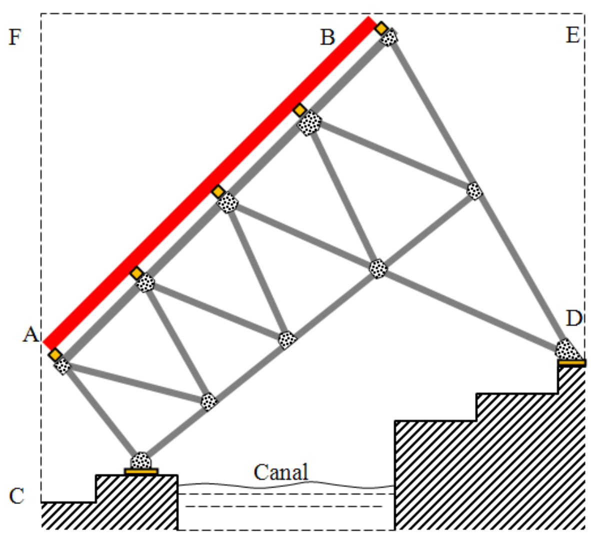

FIGURE 3
POSSIBLE STRUCTURE 2 SUPPORTING PANEL AB IN SPACE CDEF.

One major task in such projects is justifying the safety of truss structures. The safety criteria (1) below is commonly used for straight bars, where F, A and S_a are axial force, cross section area and allowable stress, respectively.

$$\frac{F}{A} \leq S_a \tag{1}$$

Even if criteria (1) and calculation procedure for axial force F and stress F/A are the same for tension and compression bars, there is a significant difference between the allowable stresses S_a for tension and S_a for compression, such as shown by (2) to (4) below.

The allowable axial stress in tension is

$$S_a = \frac{S_Y}{f_s} \tag{2}$$

where f_s is safety factor and S_Y is material yield stress, independent from length and cross section of bars.

The allowable axial stress in compression is

$$S_a = \frac{S_{cr}}{f_s} \tag{3}$$

where S_{cr} is critical compression stress which is not only dependent on material but also on length and cross section properties (A and I), such as seen in Euler's buckling formulas (4) for S_{cr} of long columns, where E is material modulus of elasticity, L is length, I is minimum second moment of cross section area A, and k is coefficient associated with end restrain conditions of column,

$$S_{cr} = \frac{\pi^2 EI}{(kL)^2 A}, \text{ applicable if } S_{cr} \leq \tfrac{1}{2} S_Y \tag{4}$$

or by some empirical formulas for S_{cr} of short columns based on experimental data of different materials [7].

Unfortunately, classic engineering programs do not give much occasion for practicing this task (justifying safety of truss structures) just because the subject "buckling of columns" is introduced too late, such as analyzed in the following section.

BUCKLING OF COLUMNS IN CLASSIC PROGRAMS

In classic engineering programs, buckling of columns is mathematically introduced from development through solution of differential equations: differential equation (5) for deflection of column, obtained from relationships between curvature and bending moment,

$$\frac{d^2\upsilon}{dx^2} = \frac{M}{EI} \tag{5}$$

where υ is deflection and M is bending moment. Bending moment M is not an explicit function but implicitly defined by differential equation (6) obtained from $\sum M = 0$ for forces and moments applied on element dx shown in Figure 4, where P is axial compression force and V is shear force. Shear force V is constant throughout the length L due to $\sum F=0$ in υ direction for forces applied on element dx.

$$dM = V\,dx - P\,d\upsilon \qquad (6)$$

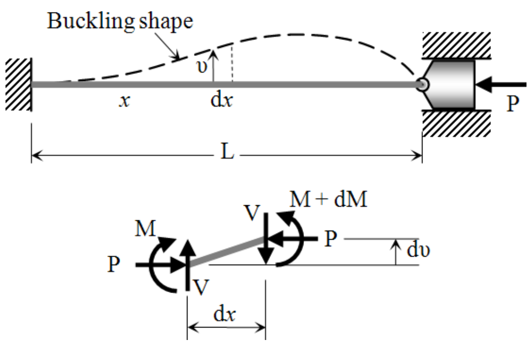

FIGURE 4
FREE BODY DIAGRAM OF A DIFFERENTIAL ELEMENT OF A COLUMN

The system of differential equations (5) and (6) with particular end restrain conditions is solved after some mathematical manipulations, not always easy to understand, in order to give a non trivial solution called Euler's buckling load written as formula (7), where k is dimensionless coefficient associated with end conditions.

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \qquad (7)$$

The mathematical demonstration of Euler's formula (7) can only be done at or near the end of basic Strength of Materials course because it requires a long list of prerequisite subjects including analytical solutions of ordinary differential equations, equilibrium equations, centroid and second moments of areas, stress, strain, mechanical properties of materials, shear force and bending moment in beams, and deflection of beams.

The approximate order of subjects and Accreditation Units (AU) for classic Engineering Statics course (classic course one) and basic Strength of Materials course (classic course two) are summarized in Tables I and II, each course having about 45±2 AU, each AU being equivalent to 50 minute in-class teaching such as recognized by Canadian Engineering Accreditation Board (CEAB), [5].

Let us discuss about four subjects in these two courses that are simultaneously needed for evaluating safety of truss structures such as those of Figures 2 and 3.

- The subject of item *d* (Table I) just teaches how to calculate forces in truss structures with no possibility of showing how to design tension and compression bars in trusses because stress and strain have not been seen yet.

TABLE I
SUBJECTS AND AU FOR ENGINEERING STATICS

Subjects	AU
a. Basic notions, force and moment	4
b. Equilibrium of bodies in 2D space	7
c. Force, moment and equilibrium of bodies in 3D space	7
d. Statics of truss structures	5
e. Statics of frames and mechanisms	5
f. Shear force and bending moment diagrams in beams	4
g. Friction	4
h. Centroid and second moments of plane areas (G, I)	7

TABLE II
SUBJECTS AND AU FOR BASIC STRENGTH OF MATERIALS

Subjects	AU
i. Stress, strain and mechanical properties of materials (E, S_Y) . . .	3
j. Axial stress and deformation of straight bars	4
k. Torsion - Circular bars and of hollow thin-walled bars	7
l. Bending stress and shear stress in beams	7
m. Stresses due to combined loads including pressure vessels . . .	6
n. Transformation of stress, strain and Mohr's circle	7
o. Deflection of beams	7
p. Buckling of columns	6

- Second moments of plane areas in item *h*, such as the one defined by $I = \int_A y^2\, dA$, appear purely mathematics because they have no immediate applications, so that students usually ask "what are second moments of areas for" and are not quite satisfied with the answer "they will be applied later in other courses".
- In item *i* and *j* (Table II), only simplified exercises on axially loaded bars and on some small application of allowable stress for tension bars could be given because buckling of compression bars has not been learned yet.
- After learning buckling of columns too late in Strength of Materials course (item *p* in Table II), students hardly retain all related mathematics and just have few time left in classic course two for applying buckling formulas in simplified problems with compression bars only.

In brief, four essential subjects in these two courses for evaluating safety of truss structures, namely statics of truss structures in item *d*, centroid and second moments of plane areas in item *h*, allowable stress for tension bars in item *j*, and allowable stress for compression bars included in item *p*, are separately learned too far away from each other, so that students may not easily see their simultaneous application in design of truss structures.

EFFECTIVE LEARNING OF BUCKLING OF COLUMNS

Trusses are very common and effective structures for supporting heavy loads and constitute good examples for practicing simultaneous application of four subjects discussed above, including buckling of columns.

For allowing undergraduate students to do so, it is necessary to reorganize the order and the teaching approach of some subjects of two classic courses outlined in Tables I and II. A satisfactory reorganization of these subjects suggests two alternative courses called "Statics and Strength of Materials I" or course one, and "Statics and Strength of Materials II" or course two, such as outlined in Tables III and IV. Advantages and disadvantages of this reorganization are discussed below.

- The subject "Centroid and second moments of plane areas" is moved from end to first half of course one (from *h* of Table I to *C* of Table III). **Advantages**: 1)-This subject is more easily presented as an immediate application and a good continuation of resultant force and moment learned in item *B* with cases of uniformly and linearly distributed pressures or stresses on plane areas; 2)-The notion "Second moments of plane areas" learned in item *C* is immediately applied in next item *D* when learning "Strength and deformation of axially loaded bars" including buckling formulas for compression bars; and 3)-The question "what are second moments of areas for" is no more relevant thanks to good sequence of subjects *B*, *C* and *D* in Table III.

TABLE III
SUBJECTS AND AU FOR STATICS & STRENGTH OF MATERIALS I

Subjects	AU
A. Basic notions, force and moment	4
B. Equilibrium of bodies in 2D space	7
C. Centroid and second moments of plane areas (G, I)	7
D. Strength and deformation of axially loaded bars	7
E. Design and analysis of truss structures	7
F. Shear force and bending moment diagrams in beams	4
G. Stresses due to bending moment and design of beams	7
H. Friction	4

TABLE IV
SUBJECTS AND AU FOR STATICS & STRENGTH OF MATERIALS II

Subjects	AU
I. Analysis of frames and mechanisms	7
J. Force, moment and equilibrium of bodies in 3D space	5
K. Torsion - Circular bars and of hollow thin-walled members	7
L. Stresses due to shear force in beams	4
M. Stresses due to combined loads including pressure vessels	6
N. Transformation of stress, strain and Mohr's circle	7
O. Deflection of beams	7
P. Buckling of columns	4

- The subject "Strength and deformation of axially loaded bars" in item *D* of Table III combines item *i* for stress strain notions and mechanical properties of materials, item *j* for design and analysis of tension bars, and part of item *p* for design and analysis of compression bars in which buckling phenomena and formulas are introduced in class using simple buckling of slender sticks or rulers and are supported by laboratory tests on long and short columns, with no need of mathematical demonstration. **Advantages**: 1)- The acceptance of Euler's and empirical buckling formulas for designing compression bars is quite equivalent to the acceptance of yield strength based on experimental stress strain diagrams for designing tension bars. 2)-Within seven Accreditation Units, item *D* allows to show the resemblance in calculation of axial force and stress in tension and compression bars for applying safety criteria (1) and to emphasize at the same time the difference in allowable stresses between tension and compression bars such as shown by (2), (3) and (4).
- The classic "Statics of truss structures" in item *d* is renamed as "Design and analysis of truss structures" in item *E* for fully extending the subject to design of tension and compression bars, not just limited to calculation of forces only. **Advantage**: All ingredients for design and analysis of tension and compression bars being learned within the first half of course one (items *A* to *D*), a project such as presented in Figures 1 to 3 could be assigned and students still have enough time to practice design aspects and some elementary SE aspects for finalizing a truss design and submitting team work reports by the end of this course.
- The subject "Bending stress in beams" (first half of item *l* in Table II) is moved from course two to course one (item *G* in Table III). **Advantages**: 1)-This subject allows selection of standard beams or shapes for supporting loads on panels or platforms, like solar panels AB in previously described project; 2)-The second half of item *l*, including general formula of shear stress in beams such as $\tau = \frac{V\,Q}{I\,t}$ and various forms derived from this formula for particular cross sections [7], is advantageously left in course two (item *L* of Table IV) because it is not important for truss structures, more difficult to understand, and balances the numbers of Accreditation Units of two courses.
- The subject "Statics of frames and mechanisms" presented near midway of classic course one (item *e*) is slightly extended and renamed as "Analysis of frames and mechanisms" at the beginning of course two (item *I*). **Advantages**: 1)-Axial force, shear force and bending moment being learned in course one (items *D* to *G*), their simultaneous existence is beneficially introduced using frames and mechanisms, not just limited to calculation of reaction forces like item *e* of classic course; 2)-Being familiar once with truss design project in course one, a second project, similar to but more advanced than project one, could be assigned almost at the beginning of course two for designing a frame structure or a mechanism, such as shown in Figure 5. All subjects learned in items *I* to *O* are, turn by turn, applicable to the second project.
- Buckling formulas being learned and applied in truss design project, the last item *P* in Table IV becomes a recall of buckling phenomena with mathematical demonstration and focuses more on columns with eccentric loading. This is neither an advantage nor a disadvantage.
- **Disadvantages**: The two reorganized courses, "Statics and Strength of Materials I" and "Statics and Strength of

Materials II" outlined in Tables III and IV presents just two negative effects: 1)-Mathematical demonstration could not be done when introducing buckling formulas in item *D* because deflection of beams has not been learned yet. However, this is just a small disadvantage compared to the numerous advantages mentioned above; 2)-There is no short term reference text books that present subjects and adequate exercises for these new courses. Although text books entitled "Statics and Strength of Materials" such as [8] exist, their content is practically the same as two classic text books [6] and [7] together.

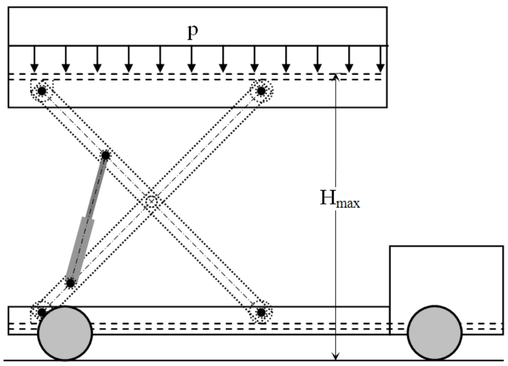

FIGURE 5
TYPICAL PROJECT FOR DESIGNING FRAMES OR MECHANISMS.

RESULTS

- **Teaching evaluations**: By teaching classic Engineering Statics and Strength of Materials courses for several years, it has been noticed that, independently from ability of teachers, negative comments such as "boring subjects, nothing new, etc.", are often showed up by students having mechanical backgrounds from Colleges of Technology. Since 1994 however, alternative courses shown in Tables III and IV being applied to mechanical engineering department of ÉTS in Montreal using course notes developed by the present authors, [9]-[10], such negative comments disappeared and new comments about advantages mentioned above are often recorded by written evaluations from students and more often by direct contacts between students and professors.
- **Team work**: Design projects such as shown in Figures 2, 3 and 5 are assigned as team work projects even if they could be handled alone by each well-organized student. Each one of such assignment produced about 10% to 20% good to excellent 2-to-4-team reports, which would be hardly achieved by just one person.
- **Maturity**: It could be furthermore noticed that student maturity grows faster in courses with team work projects than without one. This fact has been recorded by noticing better ability and better how-to-deal knowledge of students with second project in course two even if it is more advanced than the project in course one.

CONCLUSIONS

Since classic engineering programs introduce the subject "buckling of columns" after all prerequisite subjects for mathematical demonstration of Euler's formula, design aspects of truss structures, frames and mechanisms could not be enhanced in Statics and Strength of Materials courses.

Besides buckling of columns, other essential subjects needed for justifying safety of truss designs, namely statics of trusses, second moments of plane areas, mechanical properties of materials, allowable axial stress in tension, and allowable axial stress in compression, are taught too sparsely from each other in classic courses so that students do not easily realize their simultaneous importance.

By replacing tedious mathematical demonstration by simple proofs of buckling of slender sticks or rulers in class, buckling formulas can easily be introduced much earlier for enhancing the simultaneous use of allowable stresses for tension and compression bars.

Reorganizing subjects into alternative courses "Statics and Strength of Materials I" and "Statics and Strength of Materials II" with objective of enhancing structural design projects and some Simultaneous Engineering aspects has been made possible and successfully applied at École de Technologie Supérieure (ÉTS), [11].

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