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A simulation-based optimization approach for an integrated production and quality control model

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Abstract: *This article presents an integrated production and quality control model for an unreliable and imperfect batch manufacturing system. The production is controlled by a hedging point policy, while, the quality control is performed by a lot-by-lot double acceptance sampling plan by attributes. The decision variables of this model are the production lot sizing and the hedging level. The purpose of this work is to develop and validate a simulation model to fairly represent the dynamic and stochastic aspects of the system under study. Then, a simulation optimization approach based on the response surface methodology is used to optimize the decision variables when the failure and repair times and the percentage of nonconforming items produced follow general probability distributions.*

Keywords: *Unreliable manufacturing systems, double acceptance sampling plan, optimal lot sizing, simulation, response surface methodology*

1 Introduction

Manufacturing systems are subject to many stochastic phenomena such as random failures and repairs, imperfect production, quality deterioration, etc. In the literature, batch manufacturing systems are controlled using the economic production quantity (EPQ) models. Research on the EPQ model has been undertaken in different contexts of reliability and/or quality imperfection, especially during the last two decades.

In one of the pioneered papers that addressed the EPQ problem of unreliable batch manufacturing systems, Groenevelt et al. (1992a) investigated the impact of system breakdowns and corrective maintenance on production lot sizing decisions. Under the assumptions of deterministic constant production rate, negligible repair time, exponential failures and no backlogs, the authors determined the optimal lot sizing for two production reorder policies (no-resumption policy and abort/resume policy). Later, Groenevelt et al. (1992b) defined a production control policy to simultaneously determine the optimal lot sizing and the safety stock level that satisfy a prescribed service level. They assumed that, during a production run, a certain fraction of the produced items is instantaneously diverted into the safety stock. Kim et al. (1997) extended the Groenevelt et al. (1992a) model assuming that the times between failures follow general distributions. Chung (1997) determined an approximate formula for the optimal lot sizing of the Groenevelt et al. (1992a) model by calculating its bounds. Giri et al. (2005) focused on the problem of EPQ for an unreliable production system where the production rate is treated as a decision variable. They developed two models; with and without safety stock. Giri and Dohi (2005) extended the Giri et al. (2005) model with safety stock, taking into account the preventive maintenance and assuming that the failure and repair times are general distributions. Bouslah et al. (2011) obtained an integrated optimal lot sizing and feedback production policy, considering a transportation delay of lots produced to the serviceable stock.

All the above cited studies deal with the effect of process reliability on the EPQ model, and do not consider the quality issue. It is assumed that all produced items are perfect quality. On the other hand, many works have considered the quality imperfection problem in the EPQ model, without considering the reliability issue. Porteus (1986) and Rosenblatt and Lee (1986) are among the first researchers who investigated the effect of quality imperfection on the EPQ. In both studies, they assumed that the deterioration of production system is a random process characterized by two states: 'in-control' state when all items produced are conforming of quality and 'out-of-control' state when some percentage of items produced are defectives. Lee and Rosenblatt (1987) considered maintenance by inspection feature to monitor the production process deterioration. If the production process is found to be out of control by inspections, it will be restored to the 'in-control' state. Then, they focused on simultaneously determining the EPQ and optimal inspection schedules. Khouja and Mehrez (1994) have formulated an EPQ model, assuming that production rate is a decision variable and quality of the production process deteriorates with increased production rate. Salameh and Jaber (2000) presented a modified inventory model which extends the traditional EPQ model by accounting for imperfect quality items. Hayek and Salameh (2001) derived an optimal operating policy for an EPQ model under the effect of imperfect quality. They assumed that all of the nonconforming items produced are reworked and added to perfect quality inventory, and shortages are allowed and backordered. Ben-Daya (2002) developed an integrated model for the joint determination of EPQ and preventive maintenance level for an imperfect process having a general deterioration distribution with increasing hazard rate. Chiu (2003) extended the Hayek and Salameh (2001) model, by assuming that not all of the nonconforming items produced are reworked, and that a portion of the imperfect quality items are scrapped and discarded before starting the rework process. Finally, Sana (2010) considered that the percentage of nonconforming items varies linearly with both production rate and production-run time, and the probability distribution of shift time from 'in-control' to 'out-of-control' state depends also on the production rate. Therefore, he focused on determining the optimal lot sizing and the optimal production rate.

In the aforementioned EPQ models, the reliability and quality issues are studied separately. However, these two problems are often observed simultaneously in real-life manufacturing systems. Only few recent EPQ models jointly consider the effects of equipments breakdowns and quality deterioration of production process. Among these works, Chiu et al. (2007) extended the works of (Chung, 1997) and (Chiu, 2003) in order to determine the optimal run time problem of EPQ models with scrap, reworking of nonconforming items, and stochastic machine breakdowns. Liao et al. (2009) integrated maintenance programs (perfect/imperfect preventive maintenance and imperfect repair) with EPQ model for an imperfect and unreliable manufacturing system. Chakraborty et al. (2009) developed integrated production, inventory and maintenance models in order to study the joint effects of process deterioration, machine breakdown and inspections on the optimal lot sizing decisions. Sana and Chaudhuri (2010) extended the Giri and Dohi (2005) model, considering the effect of an imperfect production process subject to random breakdowns and variable safety stocks.

In most existing EPQ models, the effects of using such quality control policy on the production policy parameters (including lot sizing) have not been sufficiently studied. Indeed, the inspection is considered only as a tool to control the quality deterioration of the production process. Also, most models assumed that the inspection delay is negligible. However, inspection is in itself an important part of quality assurance that should be fairly represented in EPQ model. Some authors such as Salameh and Jaber (2000) assume that all lots produced are 100% inspected. Liao et al. (2009) consider complete quality audit using automated inspection. From economic point of view, the cost of 100% inspection is very high, particularly with automation systems which need high technology (Chin and Harlow, 1982).

In real life manufacturing organisations, it is recommended to use statistical quality control techniques, such as control charts and acceptance sampling plans, when the cost of 100% inspection is higher than the cost of delivering a certain proportion of nonconforming items (Besterfield, 2009). Only few researchers have integrated quality control techniques into EPQ models. Among these, Ben-Daya (1999) presented an integrated model for the joint optimization of production quantity, design of quality control parameters using $\overline{x}$-control chart, and maintenance level. Quality control using lot-by-lot acceptance sampling plans by attributes are not sufficiently studied in the literature. Another critical assumption made in most EPQ models is that, the lot which is currently processed, can instantly meet the demand, and even build a safety stock (the difference between production and demand). This assumption is unrealistic for a wide range of manufacturing systems where certain delays, for lot sampling, inspection, reworking, etc., exist between the production and the final stock that truly serves the demand.

In this paper, we propose an integrated production and quality control model for unreliable manufacturing systems, which has the following three features: the production is controlled by a hedging point policy (HPP), the lot sizing and the hedging level are decision variables, and the quality control is performed by a double acceptance sampling plan by attributes. Our choice of the HPP for the production control is motivated by its flexibility, feedback and optimality properties ((Akella and Kumar, 1986), (Bielecki and Kumar, 1988)). The double sampling plan is widely used in industry, such as military (MIL-STD-105D, 1963), electronic (Sultan, 1994) and construction (Chang and Hsie, 1995) industries. Given that the considered problem is complex and highly stochastic, which time between failures, time to repair and the percentage of nonconforming items produced are general random variables, our objective is to develop and validate a simulation model to fairly represent the system under study. Then, we use a combination of experimental design, simulation and statistical methods to optimize the parameters of the production control policy which minimize the total incurred cost including manufacturing cost, transportation cost, quality control costs, holding and backlog costs.

The remainder of this paper is organized as follows. Section 2 presents the notation. Section 3 describes the problem under study. The optimization problem formulation is presented in section 4. Section 5 explains the resolution approach used to determine the optimal parameters of the control policy and the optimal incurred cost. An illustrative numerical example of the resolution approach with a sensitivity analysis is given in section 6. Finally, section 7 concludes this paper.

2 Notation

The following are the notations used in this paper:

$q(t)$	WIP lot level at time t (units)
x(t)	Inventory level at time t
y(t)	Inventory position at time t
u(.)	Production rate (units/time)
u^i	Production rate of the ith lot (units/time)
u_{max}	Maximum production rate (units/time)
d	Constant demand rate (units/time)
$\sim p$	Proportion of nonconforming items (random variable)
$\overline{p}$	Long-term average proportion of nonconforming items
n_1	Sample size on the first sample
A_1	Acceptance number on the first sample
R_1	Rejection number on the first sample
n_2	Sample size on the second sample
A_2	Acceptance number on the second sample
R_2	Rejection number on the second sample
Q	Lot sizing (units)
θ_i	Production start time of the ith lot
δ_i	Production end time of the ith lot
N_∞	Long-term cumulative total number of lots produced
$\sim TBF$	Time Between Failures (random variable)
$\sim TTR$	Time To Repair (random variable)
τ_{insp}	Inspection delay (time)
τ_{rect}	Rectification delay (time)
c^+	Unit holding cost ($/unit)
c^-	Unit backlog cost ($/unit)
c_p	Unit production cost ($/unit/time)
c_{tr}	Cost of transportation a lot ($/load)
c_{insp}	Unit inspection cost ($/unit)
c_{rect}	Unit rectification cost ($/unit)
c_{rep}	Unit replacement cost ($/unit)

3 Problem description

3.1 Production system

We study an imperfect production system subject to stochastic breakdowns and repairs, and supplying a downstream stock $x(.)$. One single item is manufactured in lots of size Q in order to face a constant and continuous demand. The work-in-process (WIP) lot is stored in a downstream area of the facility until the production lot is completed (Figure 1). The system availability state can be described at each time t by a stochastic process $\{\alpha(t)\}$ taking values $\{0,1\}$. $\alpha(t) = 1$, if the production system is available. $\alpha(t) = 0$, if not. We assume that, when a failure occurs during the production cycle, the production of interrupted lots is always resumed after repair. Let $q(t)$ be a piecewise continuous variable which describes the lot processing progress (WIP level) at time t. Let $0 \leq q(t) \leq Q$ be the capacity constraint of the WIP lot.

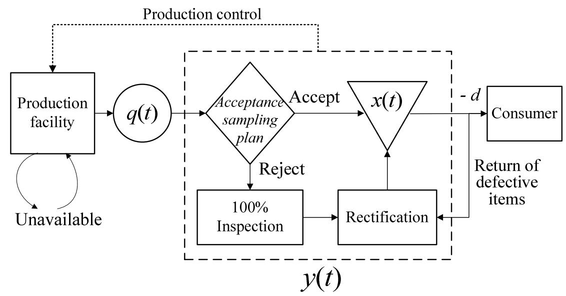

Figure 1: Unreliable and imperfect production system with quality control.

The production system is imperfect where the production of a certain proportion p(.) of nonconforming items. As there is lot-to-lot variation in manufacturing, we assume that the percentage of nonconforming items also varies from lot-to-lot with a known probability distribution. As in Salameh and Jaber (2000), we assume that the number of nonconforming items in each ith lot is equal to p^iQ proportionally to the lot sizing Q, where p^i is the proportion of nonconforming items in the ith lot. Once produced, a quality control is performed on the lot to decide whether it is acceptable or not.

3.2 Quality control

The quality control policy consists on a lot-by-lot double acceptance sampling plan with parameters n_1, A_1, R_1, n_2, A_2 and R_2. A sample of size n_1 is drawn randomly from the lot, and inspected item-by-item by attributes. The duration of sample inspection is equal to $n_1 \times \tau_{insp}$. If the number of nonconforming items d_1 in the first sample does not exceed the acceptance number A_1, the lot is accepted and the d_1 nonconforming items are replaced from a stock of known good items, before the transport of the entire lot to the final stock area. If the nonconforming items d_1 found in the first sample exceeds the rejection number R_1 on the first sample, the lot is rejected. Otherwise ($A_1 < d_1 < R_1$), no decision is made and a second sample of size n_2 is taken. The duration of the second sample inspection is equal to $n_2 \times \tau_{insp}$. If the number of nonconforming items in both samples ($d_1 + d_2$) does not exceed the acceptance number A_2, the lot is accepted. Otherwise ($d_1 + d_2 \geq R_2$), the lot is rejected. Note that the rejection number of the second sample is equal to the acceptance number A_2 plus 1. Figure 2 illustrates the procedure for double sampling by attributes.

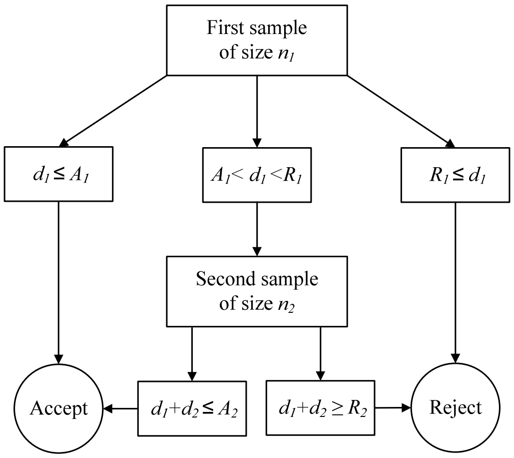

Figure 2: Decision process for double acceptance sampling by attributes.

When a lot is rejected, a 100% inspection is performed and all nonconforming items are sorted by inspection personnel. The duration of this operation is equal to $(Q-n_1)\times\tau_{insp}$ if the lot is rejected in the first sample, and equal to $(Q-n_1-n_2)\times\tau_{insp}$ if the lot is rejected in the second sample. Then, the nonconforming items are rectified. The delay of rectification of the nonconforming items discovered in the ith lot is equal to $p^i\times Q\times\tau_{rect}$. After that, the entire lot is transported to the serviceable stock. Let ξ_i be the arrival time of the ith lot to the on-hand serviceable inventory $x(.)$. Then, $\xi_i = \delta_i + n_1\times\tau_{insp}$, if the ith lot is accepted after the first sample. $\xi_i = \delta_i + (n_1+n_2)\times\tau_{insp}$, if it is accepted after a second sample. $\xi_i = \delta_i + Q\times\tau_{insp}$, if the lot is rejected. We assume, in our study, that $\xi_i \leq \delta_{i+1}$ (i=1..N), which means that the quality control operations of the ith lot is finished before the end of production of the next $i+1$th lot.

The probability P_{a1} of accepting an ith lot in the first sample can be calculated using the Poisson probability distribution $F(.|.)$ (Besterfield, 2009), as follows:

$$P_{a1}\left(p^i\right) = F\left(d_1 \leq A_1 \middle| n_1 p^i\right) \tag{1}$$

where, $d_1 \leq A_1$ is the condition that determining the number of occurrence and $n_1\times p^i$ is the mean of the Poisson function.

The probability P_{a2} of accepting an ith lot in the second sample can be calculated as follows:

$$P_{a2}\left(p^i\right) = \sum_{k=A_1+1}^{R_1-1} F\left(d_1 = k \middle| n_1 p^i\right).F\left(d_2 \leq A_2 - k \middle| n_2 p^i\right) \tag{2}$$

The probability P_a of accepting an ith lot containing p^i proportion of nonconforming items after quality control is obtained by combining the two equations (1) and (2):

$$P_a\left(p^i\right) = P_{a1}\left(p^i\right) + P_{a2}\left(p^i\right) \tag{3}$$

Similarly, the probability of rejection P_r of an ith lot p^i proportion of nonconforming items after quality control is obtained by combining the probabilities of rejection P_{r1} and P_{r2} respectively in the first and the second samples, as follows:

$$P_r\left(p^i\right) = P_{r1}\left(p^i\right) + P_{r2}\left(p^i\right) \tag{4}$$

where, $P_{r1}\left(p^i\right) = F\left(d_1 \geq R_1 \middle| n_1 p^i\right) = 1 - F\left(d_1 \leq R_1 - 1 \middle| n_1 p^i\right)$ (5)

and,

$$P_{r2}\left(p^i\right) = \sum_{k=A_1+1}^{R_1-1} F\left(d_1 = k \middle| n_1 p^i\right).F\left(d_2 \geq R_2 - k \middle| n_2 p^i\right) \tag{6}$$

As the accepted lots do not receive 100% inspection, the nonconforming items existing in these lots will be transmitted to the final stock and therefore to the consumer. The long-term average proportion of nonconforming items that contains the final stock, also named the Average Outgoing Quality *AOQ*, can be calculated using the following formulae (Schilling and Neubauer, 2009):

$$AOQ = \bar{p}P_{a1}\left(\frac{Q-n_1}{Q}\right) + \bar{p}P_{a2}\left(\frac{Q-n_1-n_2}{Q}\right) \tag{7}$$

We assume that, in the producer-consumer relationship, all nonconforming items are returned to the producer and replaced by good ones. While the demand/backlog is filled, the returned quantity at each time t is considered proportional to the demand rate d, and is replaced by good items immediately. Then, the long-term real demand rate during times of serving the demand becomes equal to $d/(1-AOQ)$.

3.3 Production control policy

In production systems management, one of the main useful strategies for responding to uncertainty is to build a surplus inventory, or safety stock, to hedge against periods in which the production capacity cannot satisfy the demand (Hu et al., 2004). For continuous-flow unreliable manufacturing systems, the optimal production policy is of a *hedging point policy* (HPP) type ((Akella and Kumar, 1986), (Bielecki and Kumar, 1988)). For unreliable batch manufacturing systems, some authors, e.g. (Giri and Dohi, 2005) and (Sana and Chaudhuri, 2010), used an optimal safety stock in inventory to protect against possible stock-out during system repair and to enhance customer service level. For batch manufacturing systems with delays which cannot be considered as continuous-flow systems, (Bouslah et al., 2011) showed that the optimal feedback control policy can be closely approximated by a base-stock policy expressed by a modified HPP. Considering a transportation delay for lots produced to the serviceable stock, the authors assumed that the feedback inventory control is based on the concept of the inventory position which includes the on-hand inventory in the final stock and the total pending quantities in transportation, as in Mourani et al. (2008) and Li et al. (2009).

In our study, we define the inventory position $y(t)$ at each time t as the sum of the stock (inventory/backlog) level $x(t)$ and the total amount of lots-under-sampling, 100% inspection and rectification. Then, the production control policy based on the concept of HPP and taking into account the effect of quality imperfection on the real demand rate can be described by the following equation:

$$u^{i}\left(\underset{i=1,2,..,\infty}{t\in\left]\theta_{i},\theta_{i+1}\right]},\alpha\right)=\begin{cases}\alpha(t)\,u_{\max} & \text{if}\left(y(\theta_{i}^{+})<S\right)\\ \dfrac{\alpha(t)\,d}{1-AOQ} & \text{if}\left(y(\theta_{i}^{+})=S\right)\\ 0 & \text{if}\left(y(\delta_{i}^{+})>S\right)\end{cases}\tag{8}$$

In fact, the production rate $u^i(.)$ of the ith lot can take three possible levels depending on the inventory position evolution and the instantaneous system availability, as follows:

1. If the inventory position at the beginning of the ith production cycle ($t=\theta_i$) is strictly below the threshold level S, and while the production system is available ($\alpha(t)=1$), the corresponding ith lot is manufactured at the maximum production rate u_{max}. Such a case happens when the production is restarting just after a corrective maintenance.

2. If the inventory position at the beginning of the ith production cycle is exactly equal to the threshold level S, and while the production system is available ($\alpha(t)=1$), the production rate of the corresponding ith lot is set to the demand rate $d/(1-AOQ)$ in order to maintain the on-hand inventory position.

3. If the inventory position at a time t ϵ $]\theta_i, \theta_{i+1}]$ becomes strictly greater than the threshold level S or the production system becomes unavailable, the manufacturing is stopped ($u(.)=0$) until the inventory position falls to the threshold S by the effect of the demand.

4 Optimization problem formulation

The dynamics of production $q(.)$, inventory position $y(.)$ and final inventory level $x(.)$ can be characterized by the following difference and differential equations:

$$\begin{aligned}
&\frac{dq(t)}{dt}=u(t,\alpha),\ q(0)=q,\ \forall t\in\left]\theta_i,\delta_i\right[,\\
&q(\delta_i^+)=q(\delta_i^-)-Q,\\
&\frac{dy(t)}{dt}=\begin{cases}-d & \text{if }(x(t)<0)\&(\alpha(t)=0)\\ \dfrac{-d}{1-AOQ} & \text{otherwise}\end{cases},\ \begin{array}{l}y(0)=y,\\ \forall t\in\left]\delta_i,\delta_{i+1}\right[,\end{array}\\
&y(\delta_i^+)=y(\delta_i^-)+Q,\\
&\frac{dx(t)}{dt}=\begin{cases}-d & \text{if }(x(t)<0)\&(\alpha(t)=0)\\ \dfrac{-d}{1-AOQ} & \text{otherwise}\end{cases},\ \begin{array}{l}x(0)=x,\\ \forall t\in\left]\xi_i,\xi_{i+1}\right[,\end{array}\\
&x(\xi_i^+)=x(\xi_i^-)+Q,\\
&\forall\, i=1,..,N
\end{aligned}\tag{9}$$

where, q, x and y denote respectively the WIP level, the inventory position and the finished product inventory level at initial time. δ_i^- and δ_i^+ denote the left and right boundaries of the ith production run end time δ_i, and ξ_i^- and ξ_i^+ denote the left and right boundaries of the arrival time ξ_i of the ith lot to the final stock $x(.)$.

Figure 3 depicts graphically the dynamic of production (WIP lot level), and the evolution of the serviceable inventory level as function of instantaneous system availability, production cycle length, and acceptance or not of lots produced.

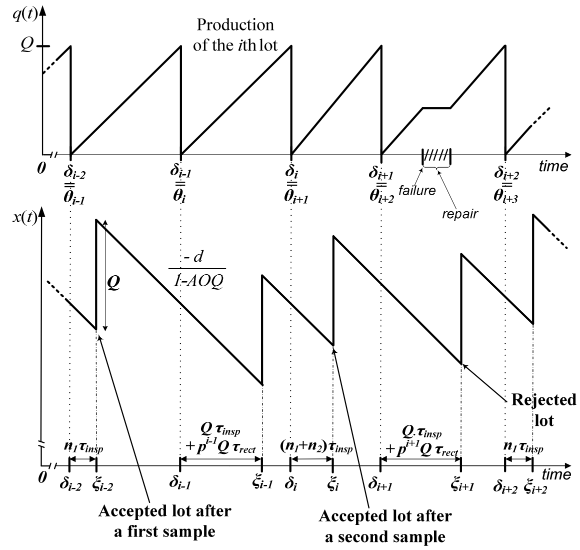

Figure 3: Production and inventory level dynamics.

Our objective is to determine the optimal lot sizing Q and the optimal hedging level S wich minimize the long-term expected total cost ETC per unit time including; the average total holding cost including the storage of batch in processing, the batches under inspection, 100% inspection and rectification and the final inventory stock, the average backlog cost, the average cost of sampling (including first and second samples), the average cost of 100% inspection and rectification of the rejected lots, the average cost of transportation of lots to the serviceable stock, and the average cost of replacement of nonconforming loaded to the consumer.

Any admissible solution (Q, S) must satisfy the following two constraints:

- $0 < Q \leq \min\{Q_{max}^{wip}, Q_{max}^{insp}\}$ (10)

where, Q_{max}^{wip} is the maximum WIP storage capacity, and Q_{max}^{insp} is the maximum inspection area capacity.

- $0 < S \leq S_{\max}$ (11)

where, $S_{\max}$ is the maximum storage capacity of the inventory position.

Therefore, the optimization model associated to the problem under study can be described as follows :

$$\begin{cases}\underset{Q,S}{Min} & ETC=c^{+}\left(E[q]+E[y^{+}]\right)+c^{-}E[x^{-}]\\ & +c_{insp}n_1E[N_\infty]+c_{insp}n_2P_{a2}\left(\bar{p}\right)E[N_\infty]\\ & +P_{r1}\left(\bar{p}\right)\left(c_{insp}\left(Q-n_1\right)+c_{rect}\,\bar{p}Q\right)E[N_\infty]\\ & +P_{r2}\left(\bar{p}\right)\left(c_{insp}\left(Q-n_1-n_2\right)+c_{rect}\,\bar{p}Q\right)E[N_\infty]\\ & +c_{rep}E[\beta]\,AOQ\,d\\ & +c_{tr}\,E[N_\infty]\\ S.C & \text{Equations (2)-(5)-(6)-(7)-(8)-(9)}\\ & \text{Constraints (10)-(11)}\end{cases}\tag{12}$$

where, $y^+(t)=\max(0,\ y(t))$, $x^-(t)=\max(0,-x(t))$, $E[N_\infty]$ is the expected number of lots produced per unit time.

$\beta(t)$ is the instantaneous satisfaction level of the demand/backlog; $\beta(t)=1$ if $x(t)>0$ or $\alpha(t)=1$. $\beta(t)=0$, otherwise.

Due to the complexity of the system dynamic described by Eqs. (9), and given the high stochastic nature of the problem under study, it is almost impossible to derive an analytical solution for (12). Thus, we advocate a simulation-based optimization approach to determine the optimal values of the decision variables (Q, S).

5 Resolution approach

5.1 Simulation-based optimization approach

To optimize the expected total cost with respect to the design factors (Q^*, S^*), we adopt a simulation optimization approach which combines simulation model with design of experiments, statistical analysis and response surface methodology ((Kleijen, 1999), (Fu, 2002)). This approach has been used to control diverse problems in manufacturing (Gharbi and Kenné, 2000). It can be applied in our study through the following four steps:

1. *Mathematical problem formulation:* The objective of this step is to formulate analytically the optimization problem, as shown in section 4. This allows to understand the dynamic of the system as function of its states, and to calculate the expected long-run average cost.

2. *Simulation model*: The simulation model describes the dynamic of the system and evaluates its performances (i.e., cost) for given factors (Q, S) using the mathematical problem formulation. These factors are considered as input of such a model, and the related incurred cost is defined as its output.

3. *Design of experiments*: The experimental design defines how the control factors (Q, S) should be varied in order to determine the effects of the mains factors and their interactions (i.e. analysis of variance ANOVA) on the incurred total cost.

4. *ANOVA, Regression analysis and Response surface methodology*: A multi-factor statistical analysis (ANOVA) of the simulated data is carried out to provide the effects of the design factors (Q, S), their interaction and their quadratic effects on the response variable (i.e. the cost). Then, the main significant factors and their interactions are considered as input of a regression analysis which is used in conjunction with the response surface methodology, to fit the relationship between the cost and the input factors. Response surface methodology is a collection of mathematical and statistical techniques that are useful for modelling and analysing problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Montgomery, 2008).

5.2 Simulation model

A combined discrete-continuous model was developed using the SIMAN simulation language with C++ subroutines (Pegden et al., 1995), and then executed through the ARENA simulation software. The advantage of using a combined discrete-continuous model is to reduce the execution time (Lavoie et al., 2007), and to model accurately the impulse-continuous nature of the production-inventory dynamic.

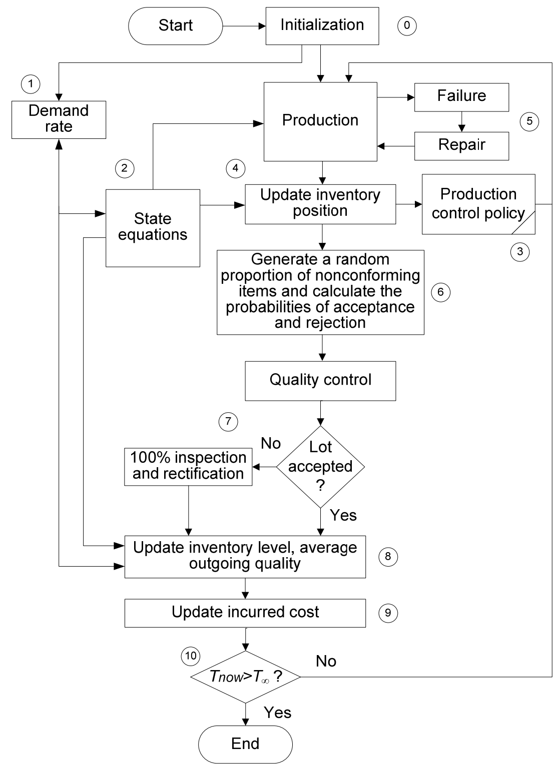

Figure 4: Simulation block diagram.

The simulation model can be described following the sequence of numbers appearing in Figure 4, as follows:

0 INITIALIZATION: setting the values of the parameters (u_{max}, d, n_1, c_1, n_2, c_2, τ_{insp}, τ_{rect}), the simulation runtime T_∞, the decision variables (Q, S), the unit partial costs (c^+, c^-, c_{insp}, c_{rect}, c_{rep}), the initial states (q, x) and the probability distributions of the proportion of defective items ($\sim p$), time between failures ($\sim TBF$), time to repair ($\sim TTR$) and the simulation run-time T_∞. Note that the model is developed to accept any probability distribution for the $\sim p$, $\sim TBF$ and $\sim TTR$.

1 The DEMAND RATE is used as an input of the state equations. In order to represent the real system operation, we define the instantaneous real demand rate as $d/(1-AOQ(t))$, where, $AOQ(t)$ is the instantaneous average outgoing quality. This can be calculated using the following formulae:

$$AOQ\left(p(t)\right)=\frac{\sum\limits_{i=0/a^i=1}^{N(t)} p^i\left(Q^i-n_1\right)+\sum\limits_{i=0/a^i=2}^{N(t)} p^i\left(Q^i-n_1-n_2\right)}{\sum\limits_{i=0}^{N(t)} Q^i} \tag{13}$$

where, $a^i=1$, if the lot is accepted after the first sample, and $a^i=2$, if the lot is accepted after the second sample. Q^i means the ith lot produced of size Q. $N(t)$ is the cumulative number of lots produced at time each t.

The AOQ(t) and the real demand rate are update each time a lot enters to the serviceable stock x(.).

② The STATE EQUATIONS are described by the differential equations of (9) and are modeled with a C++ language insert. When a lot is released at the end of production cycle or the lot arrives to the final stock x(.) a signal is send to the C++ routines to update the values of the variables $q(t)$, $y(t)$ and $x(t)$ using the difference equations of (9).

③ The PRODUCTION CONTROL POLICY is implemented using equation (8). At the end of each production cycle, the control policy is triggered to determine the production rate of the next production cycle depending on the current position inventory and the system availability.

④ The PRODUCTION block models the processing delay which is calculated by dividing the lot sizing Q by the corresponding production rate u^i(.). When the lot production is completed, the original entity is sent back to the PRODUCTION CONTROL POLICY block and a duplicated entity is created and sent to an ASSIGN block where the WIP lot level is impulsively annulled and the lot size is added to the inventory position.

⑤ This models the failure and repair events as a close loop following the ~*TBF* and ~*TTR* distributions.

⑥ A random proportion of nonconforming items is attributed to each lot produced following the ~p probability distribution, and the associated probabilities of acceptance and rejection are calculated, using equations (1)-(2) and (5)-(6).

⑦ Then, the entity (lot produced) holds in a DELAY block for the first sample during $n_1 \times \tau_{insp}$. Lots which need a second sample holds in another DELAY block during $n_2 \times \tau_{insp}$. The decision to accept or reject the lot is modeled by a *probabilistic* BRANCH block of SIMAN using the probabilities of acceptance and rejection attributed to each lot. Rejected lots hold in an additional DELAY block for 100% inspection and rectification.

⑧ When a lot arrives in the serviceable final stock, the corresponding entity impulsively updates the inventory level as in (9). The average outgoing quantity AOQ(.) is also updated using Eq. (13).

⑨ This block updates instantly the incurred cost according to the instantaneous values of the different variables and the unit costs.

⑧ Simulation run-time control: if the current time *Tnow* exceeds the predefined simulation run-time T_∞, the simulation run is stopped.

5.3 Validation of the simulation model

To validate the simulation model we should verify the accuracy of the quality control (double acceptance sampling plan) and production control policy (hedging point policy) modeling.

A set of experiments are conducted to compare the observed characteristics of a given double sampling plans with the theoretical characteristics. Operating characteristic (OC) curve is an important technique to evaluate a given sampling plan which determines the acceptance probability P_a(.) of a lot with respect to percent nonconforming items that exist in the lot. Figure 5 shows the observed and theoretical operating characteristic (OC) curves with respect to the average proportion of nonconforming items. The observed OC curve is determined by calculating the long-term observed acceptance probability which is calculated at the end of simulation run by dividing the cumulative total number of accepted lots by the cumulative total number of lots inspected. While, the theoretical OC curve is designed using equation (3). We can see clearly that the observed OC curve coincides with the theoretical OC curve.

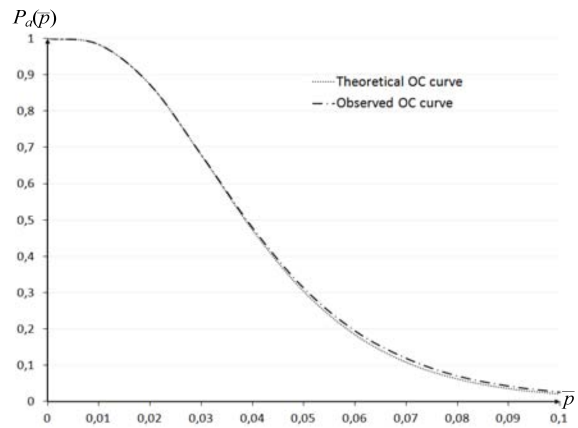

Figure 5: Comparison of the observed and theoretical OC curves.

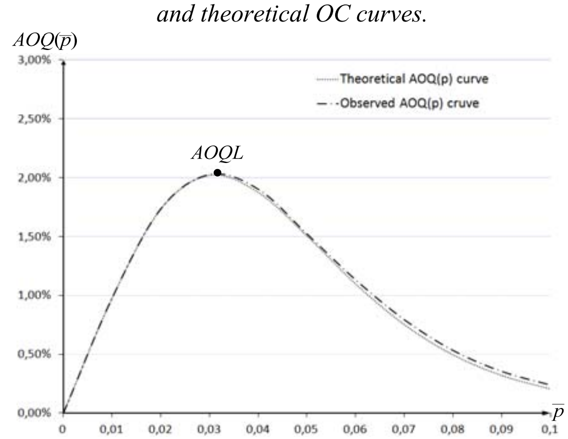

Figure 6: Comparison of the observed and theoretical AOQ curves .

Figure 6 is a graphic comparison of the observed and theoretical AOQ curves. The observed AOQ curve is determined by calculating the AOQ at the end of each simulation run using equation (13), while the theoretical AOQ curve is designed using equation (7). The two curves are confused until the AOQ reaches the $AOQL$ (average outgoing quality limit) with respect to the quality deterioration. When the AOQ declines, the difference between the two curves becomes apparent but not significant (< 5%).

Also, we graphically examine the trajectory of the inventory position during the simulation run. Figure 7 shows that, the model performs correctly as expected and intended and represents adequately the control policy. In addition, we verified that the production rate value changes instantaneously in response to changes in the inventory position and the system availability state as described in Eq. (8).

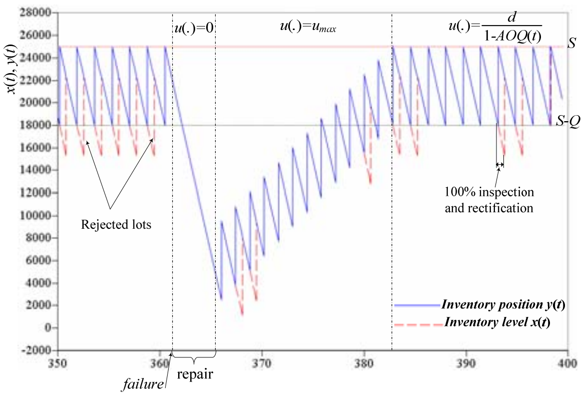

Figure 7: Inventory position evolution during simulation run.

6 Numerical example

In this section, we present a numerical example to concretize the simulation-based optimization approach. The following parameters are considered in appropriate units: u_{max}=3000, d=2000, p~*Uniform*(0.01,0.03), *TBF*~*LogNormal*(50,5), *TTF*~*Gamma*(0.5,10), $\tau_{rect}=10^{-4}$, $\tau_{insp}=5\times10^{-5}$, n_1=40, A_1=0, R_1=4, n_2=60, A_2=3, R_2=4, c^+=0.1, c^-=1.5, c_{tr}=1500, c_{insp}=0.5, c_{rect}=5, c_{rep}=7.5.

Simulation runs are conducted according to a complete 3^2 experiments plan with five replications for each combination of factors (Q, S). The selection of the levels of the experimental design plan parameters is an important factor in the precision of the response surface. It should be precise enough so that the response surface estimates the total expected cost function accurately, but large enough so that the effect of the parameters is not hidden by the inherent variability of the response. In order to select these levels correctly, we repeat the design of experiments, simulation and optimization using Response surface methodology sequence, narrowing the domain of (Q, S) around the last found solution until it is centered about the optimum design point. Through this sequential procedure, the admissible experimentation region is explored and therefore the obtained solution will be a global optimum.

In order to ensure that the steady-state is reached, the duration of simulation run is set in a way to observe 10,000 failures in each replication, i.e. T_∞ = 500,000 units of time. The simulated data is carried out using statistical software (STATISTICA) to seek a regression model fitting the response variable (total expected cost). We assume here that a continuous function Ψ(.) of Q and S exists, fitting a second-order regression model and relating the response variable to the design factors. The function Ψ(.) is called the response surface and takes the following equation:

$$\psi(Q,S)=\beta_0+\beta_1 Q+\beta_2 S+\beta_{12}QS+\beta_{11}Q^2+\beta_{22}S^2+\varepsilon \tag{14}$$

where, β_0, β_i (i = 1, 2), β_{12}, β_{ii} (i = 1, 2) are unknown parameters to be estimated from the collected simulation data, and ε is a random error. It should be noted that the idea of approximating the function cost by quadratic model has been widely used in the literature (Gershwin (1994), Gharbi and Kenné (2000)).

The significant effects are provided through a multifactor analysis of variance (ANOVA), and the regression model is then determined using the response surface methodology.

Factor	SS	d.f.	MS	F-Ratio	P-value
Q (Linear + quadratic)	96019.7	2	48009.8	108.24	0.00000
S (Linear + quadratic)	485657.7	2	242828.9	547.50	0.00000
$Q.S$	392732.6	1	392732.6	885.49	0.00000
Error	17297.2	39	443.5		
Total SS	991707.2	44			

Table 1: ANOVA table for the total cost.

Table 1 summarizes the ANOVA of the collected data. For each design factor (including the linear and the quadratic effects) and their interaction, the table presents the Sum of Squares (SS), the degree of freedom (d.f.), the Mean Square (MS), an F-ratio, computed using the residual mean square, and the significance level of P-value. We can see that the linear and quadratic effects of the two factors (Q, S) and their interaction $Q.S$ are significant for the dependent variable at a 0.05 level of significance. The R-squared adjusted value of 0.9803 presented in Table 1, states that 98.03 % of the observed variability in the total expected cost is explained by the model (Montgomery, 2008). A residual analysis was also used to verify the adequacy of the model. Therefore, it confirmed that the expected total cost *ETC* can be fitted by a second-order model. From STATISTICA, the corresponding quadratic function is given by:

$$\begin{aligned}\psi(Q,S)=&\,8560.58+64.52\times10^{-3}Q-259.52\times10^{-3}S\\&-9.34\times10^{-6}QS+10.55\times10^{-6}Q^2+6.18\times10^{-6}S^2\end{aligned} \tag{15}$$

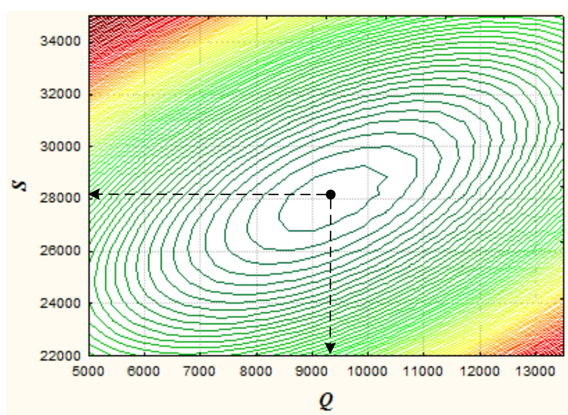

Figure 8: Contour plot of the cost function surface.

Figure 8 shows the contours of the cost function surface. The minimum total expected cost $\Psi^*(.)$=5223.03 is located at Q^*=9354 and S^*=28046 as shown in Figure 7.

A sensitivity analysis of costs parameters is conducted to prove the efficiency and the robustness of the resolution approach. For twelve cases of costs changes derived from the basic case of the numerical example, the optimal design factors and incurred cost variations (i.e., respectively ΔQ^*, ΔS^* and ΔC^*) are explored and discussed. The results are summarized in Table 2.

Sets	Costs	Changes	ΔQ^*	ΔS^*	ΔC^*
Set I	c^+	-50%	+2584	+5805	-1262.8
		+50%	-2559	-5607	+994.9
Set II	c^-	-50%	+188	-7055	-400.5
		+50%	-247	+2373	+172.0
Set III	c_{insp}	-50%	-48	-34	-136.6
		+50%	+91	+64	+273.1
Set IV	c_{rect}	-50%	+2	+2	-26.0
		+50%	-3	-3	+52.0
Set V	c_t	-50%	-5147	-3883	-426.03
		+50%	+1823	+1375	+293.6

Table 2: Sensitivity analysis for cost parameters.

- *Variation of the inventory cost* (Set I): When the inventory cost increases, the optimal hedging threshold S^* decreases in order to avoid further inventory costs. In addition, the optimal lot sizing Q^* decreases to ensure a better supply to the final stock against the risk of shortages becoming higher. The decrease in inventory cost produces the opposite effects.
- *Variation of backlog cost* (Set II): When the backlog cost increases, more safety stock should be held in order to provide better protection to the system against shortages, which explains the increase in the optimal hedging threshold S^*. The optimal lot sizing decreases in order to reduce the production delay and 100% inspection and rectification delays for rejected lots, and therefore ensure better supply to the final stock. The decrease in backlog cost produces the opposite effects.
- *Variation of quality costs* (Sets III and IV): We notice that quality costs do not have significant effects on the optimal parameters (Q^*, S^*). Indeed, the expected total costs of inspection and rectification (model (12)) depend on the quantity $Q\times E[N_\infty]$ which is the expected constant amount of items to be produced per unit time to face the continuous demand. Thus, it can be understood that the change in quality costs does not have a significant impact on the optimal lot sizing. Moreover, from (12), it is clear that the total quality costs of inspection and rectification do not depend on the inventory state, which explains why there is no change in the threshold S^* when varying the unit quality costs.
- *Variation of transportation cost* (Set V): When the transportation cost c_t is higher, the system reacts by reducing the frequency of lots transportation in order to minimize the total transportation cost. Consequently, the optimal lot sizing Q^* increases, and leads to a systematic increase in the optimal hedging level S^* in order to protect the system from backlogs. When the transportation cost decreases, the optimal lot sizing and the optimal threshold decrease very significantly.

7 Conclusion

In the literature, most existing EPQ models for imperfect processes do not consider statistical quality control methods such as acceptance sampling plans, although, these methods are widely employed in industry. In this paper, we have studied an integrated model considering simultaneously a hedging policy for production control and a lot-by-lot double acceptance sampling plan by attributes for quality control. A combined discrete-continuous simulation model has been developed and validated in order to represent the real and complex dynamic and the stochastic behavior of the manufacturing system. Then, we used design of experiments and response surface methodology in conjunction with the simulation model to determine the optimal lot sizing and the optimal hedging level. The proposed simulation optimization approach provides an efficient way to surmount the difficulties of the analytical/numerical resolution of such non-linear, complex and high stochastic problem. Moreover, the applicability of this approach in industrial context is guaranteed because the optimal solution is always obtained whatever the probability distribution functions of the time between failures, the time to repair and the proportion of defective items. Future research can be undertaken to investigate the joint optimization of the production control policy and sampling plan parameters. Another direction of further research is to consider jointly the preventive maintenance, the production-inventory control and the economic sampling plan design problem in deteriorating system in order to joint determine the optimal system reliability and the product quality levels.

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