THROUGHPUT VARIABILITY OF HOMOGENEOUS TRANSFER LINES SUBJECT TO OPERATION-DEPENDANT FAILURES

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ABSTRACT: *Most of the research concerning production systems is so far limited to the assessment of the steady-state throughput despite the impact of the parameter variability on the overall performance of such systems. In this paper, we propose an analytical model to assess the variance of the throughput of homogeneous transfer lines subject to operation-dependant failures. The transfer lines subject of study are composed of a series of manufacturing machines with no intermediate buffers where failure and repair times are exponentially distributed. Based on Markov reward processes, the proposed analytical model determines the limiting variance of the sojourn time in the operational state. A numerical procedure has also been developed to assess the variance of the throughput for transfer lines composed of several machines with different reliability and maintainability characteristics. Numerical examples show that, although the steady-state throughput decreases monotonically, the throughout variance of transfer lines increases and then decreases as the number of manufacturing machines constituting it increases. Compared to past works considering time-dependent failure mode, the results show that in general transfer lines subject to operation-dependant failure mode are superior to those subject to time-dependant failure mode not only in term of the steady-state throughput but also when considering their respective throughput variability.*

KEYWORDS: *Production/transfer lines, expected throughput, throughput variability, Markov reward process.*

1 INTRODUCTION

Most of manufacturing facilities consist of multiple interconnected subsystems such as production lines, transfer mechanisms, material handling, etc. Unfortunately for designers of such facilities, little is known about the interactions among subsystems. In fact, the output of one subsystem is usually the input to one or more others. This paper examines the output process of an automatic production line with no intermediate buffers having deterministic processing times and prone to failures. The output process is analyzed in terms of the transfer line throughput and its variability.

Transfer lines are product oriented automated manufacturing systems employed in industry for mass production. They were composed of a series of machines interconnected by an automatic transfer mechanism. Tasks required to manufacture the final product are assigned to specific workstations along the transfer line. In spite of their lack of flexibility, transfer lines are considered as the most performing systems in terms of throughput, efficiency, work in process, easy managing, and cost effectiveness. Throughput is often considered as the main performance measure of transfer lines which is defined as the overall average long run production rate of the transfer line.

An important work has been done to evaluate the throughput of mono-product, homogeneous transfer lines assuming a perfect work balancing through all the manufacturing workstations (Buzacott and Shanthikumar, 1993; Gershwin, 1994; Papadopoulos and Heavy, 1996; Dhouib *et al*., 2006).

All aforementioned researches have focused on the effect of the randomness associated with failure and repair occurrences on the steady-state, and long term average performance of transfer lines: the steady-state efficiency and the expected throughput. However, failure and repair phenomena have an important impact on the variability of the production. In fact, Gershwin (1993) reports that numerical and simulation experimentation and factory observations indicate that the standard deviation of weekly production can be over 10% of the mean. Furthermore, and based on a focused study over 30 continues process operations in the petrochemical refining, food processing and pharmaceutical industries, Rajaraman and Robotis (2004) report that there was a significant output variability. In fact, they report that the coefficient of variance of the daily output ranged from 10% to 50%, averaging around 25%. This means that it is very likely that customer requirements cannot be met exactly on time due to this variability. Since meeting customer requirements on time and shortening this time are becoming more and more important every day, there is an increasing need for analytical models to study the variability of the output process.

Available results of production variability in manufacturing systems are few, while most of them address only the problem of output variance. Some researches concern the output process variability for mono-processor manufacturing systems. Among those, Lavenburg (1975) evaluates the mean and the variance of queuing time in an M/G/1 queue with a finite buffer. Brouwers (1986) derives the distribution for the fractional loss of production for a single unreliable machine. Jacobs and Meerkov (1995) studied the due-time performance for a single machine system exactly and asymptotically with respect to the length of the shipping period. Kim and Alden (1997) derived an analytical approximation of the density function and variance of the time to produce a fixed lot size on a single workstation with deterministic processing times and random downtimes.

Some authors have studied the impact of variability propagation through a system on the output process of manufacturing systems (Forester, 1961; Hopp and Spearman, 1996). Lee et al. (1997) consider variability propagation effects on supply chain management. Rajaram and Karmarkar (2002) analyzed uncertain yields in the context of process industries. Rajaraman and Robotis (2004) analyze the impact of variability propagation on a continuous flow production process with no intermediate buffers. The authors propose a continuous model to capture the propagation of variability through the system in order to evaluate the mean and the variance of the distribution of the output process.

Makino (1964) studied unbuffered production lines and determined the moment generating function of the inter-departure distribution for two and three machine flow lines with exponentially distributed processing times. Hendricks (1992) studied the output process of a serial production line composed of multiple machines with exponentially distributed processing times and finite buffer capacities. The author developed an analytical approach to obtain expressions of the inter-departure distribution and the correlation structure of the output process of these flow lines. However, his approach is numerically intensive, which precludes the analysis of systems with many machines and large buffers. Based on simulation approach, Hendricks and McClain (1993) extend the study by Hendrick (1992) on finite buffered flow lines to include general processing time distributions. The authors examine the effects of flow line length, buffer capacity, and buffer placement on the inter-departure distribution and the correlation structure of the output process of these flow lines. Based on Markovian arrival process, He *et al.* (2007) propose an approximate approach to determine production variability of flow lines composed of multiple machines separated by buffers with finite size and with exponentially distributed processing times. The production variability approximate scheme includes the variance of the number of parts produced in a given time period and the variance of the time to produce a given number of products.

Miltenburg (1987) is the first researcher who presented a general numerical technique for the evaluation of production variance in buffered transfer lines with geometrically distributed failures and repairs. His method uses the results developed for the asymptotic mean and variance of the total state residence time in Markov chains. Due to complexity consideration, Miltenburg considered in a detailed form only the two identical station homogeneous and buffered case. Based on Markov chains, Ou and Gershwin (1989) obtain a closed form expression for the variance of the lead time for a homogeneous two-machine finite buffer transfer lines with geometric and exponential failures and repairs. Duenyas and Hopp (1990) proposed an approximation method to estimate the output variance for a cyclic queuing system (single closed loop) with exponential distributed times. Duenyas *et al.* (1993) extend this result to analyze the variability of the output process from a CONWIP transfer line with deterministic processing times and exponentially distributed repairs and failures. A crucial research was given by Gershwin (1993) whose approach is based on the exact calculation of the production variance for a single machine subject to geometrically distributed failures and repairs. Based on decomposition techniques (Gershwin, 1987; Dallery *et al.*, 1989), Gershwin developed an approximate approach for longer homogeneous transfer lines. Carrascosa (1995) extended Gershwin's results for the one-station system subject to two failure modes, for the two-station transfer lines with no buffers, and provided simulation results on the variability of the output for a two-station transfer line with a finite buffer. The considered transfer lines are homogeneous and subject to operation-dependent failures. Tan (2000) utilized the mean and variance of Markov reward systems to determine the asymptotic variance rate of homogeneous transfer lines with finite buffers and geometrically distributed failures and repairs. Compared to the Miltenburg's method (1987), the proposed method is computationally very efficient and yields a thousand-fold improvement in the number of operations. Li and Meerkov (2000) analyzed the due-date performance for homogeneous transfer lines with intermediate buffers and bernoulli reliability characteristics. The authors consider transfer line variability when the demand is subject to random variations.

All aforementioned researches concerning the variability in transfer lines with intermediate buffers used Markov chain theory to derive approximate formulations and numerical procedures for the variance assessment of the output process. For production lines with no inter-station buffers, Tan (1997) proposed an exact formulation for the asymptotic variance of the amount of items produced per unit time in homogeneous, multi-station continuous-materials-flow transfer lines with time-dependent failures. These results have been extended by Tan (1998) to series-parallel production systems under a more general framework. Tan (1999a) also generalized this method to deal with homogeneous, discrete-materials-flow transfer lines with cycle-dependent failures (discrete version for time-dependent failures). Tan (1999b) also addressed the problem of the variance of the time to produce a given order approximately according to the asymptotic normality of the number of products produced in a specific time interval. The considered transfer line is homogeneous, with geometrically distributed failures and repairs, and subject to time dependent failures. Based on renewal theory, Chen and Yuan (2004) considered the problem of the transient throughput for homogeneous, unbuffered transfer lines. They present a closed form and a sample path method to estimate the mean and the variance of the transient throughput for such transfer lines assuming that each machine is modeled as an alternating renewal process. Chan and Yuan also compare their results to those generated by Tan (1997) for the long run horizon.

Due to mathematical convenience, most of the researches conducted to derive exact formulations of the output process variability of homogeneous transfer lines consider that machines are subject to time-dependent failures. However, and based on practical considerations, many authors agree that operation-dependent failures is the main failure mode for manufacturing systems and specifically for transfer line dynamics (Buzacott, 1968; Gershwin, 1992; Sherwin, 2000; Schneeweiss, 2005; Dhouib *et al.*, 2006). In fact, Operation-dependent failures (ODF) can occur only when stations are in a processing state. But, time-dependent failures (TDF) can also occur when a manufacturing station is at the rest.

This paper proposes exact analytical models to assess the variance of the throughput for homogeneous, unbuffered transfer lines subject to operation-dependant failures. The next section describes the characteristics of the studied transfer lines, and presents the assumptions and notations used in this work. Based on Markov reward processes, section 3 developed an analytical model to assess the asymptotic variance of the throughput for homogeneous, unbuffered, transfer lines with exponentially distributed failures and repairs. The transfer line machines are subject to operation-dependent failures and have identical machine repair distributions. A general approach and a numerical procedure is proposed in section 4 to assess the exact value of the throughput variance for homogeneous, unbuffered transfer lines subject to operation-dependent failures. Numerical results and a comparison study between transfer lines subject to operation-dependent failures and those subject to time-dependent failures are given in section 5. Finally, section 6 contains a summary of the paper and some concluding remarks.

2 SYSTEM DESCRIPTION, ASSUMPTIONS, AND NOTATIONS

2.1 Manufacturing system description

Transfer lines being studied in this paper are sets of *m* machines (M_1, M_2, …, M_m) arranged in a serial structure without intermediated buffers and dedicated to manufacturing a specific product type. Hence, parts flow from outside the production line to the first machine (M_1), then to the second one (M_2), and so forth until it reaches the last machine (M_m), after which they leave the production line (Fig.1).

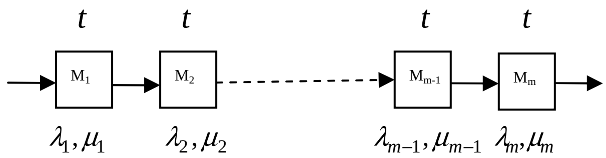

Figure 1: non-homogeneous, unbuffered transfer line with *m* machines

Machines are linked by an automatic transfer mechanism and are subject to random operation-dependant failures so that a failure can only occur while the machine is working (Dallery and Gershwin, 1992; Buzacott and Shanthikumar, 1993; Papadopoulos and Heavy, 1996; Sherwin, 2000; Dhouib *et al.*, 2006). Each workstation is assigned the task of manufacturing, assembling or inspecting parts and each part is processed on each workstation during a fixed amount of time which is said processing time.

2.2 Notations

The following notations are considered during the next sections:

m : number of machines composing a transfer line.

i : index identifying the machine number i (M_i); (i = 1, 2, ..., m).

t : Processing time of one item on any machine of the homogeneous transfer line.

λ_i : Failure rate of machine i.

μ_i : Repair rate of machine i.

$MTTF_i$: Mean time to failure of machine i ($MTTF_i = 1/\lambda_i$).

$MTTR_i$: Mean time to repair of machine i ($MTTR_i = 1/\mu_i$).

UTR_i : Steady-state availability of machine i.

$UpT(T)$: random variable identifying the transfer line UpTime explaining the total operational time during a processing period of length T.

$P(T)$: State transition probabilities of Markov process.

$P_{ij}(T)$: Transient probability that a Markov process will be in state *j* at time *T* given that it has been initially at state *i*.

$P_{00}(T)$: Instantaneous Availability of the transfer line.

A : State-space transition rate matrix for a homogeneous Markov process.

ξ_k : eigenvalues associated with matrix *A*.

l_k : eigenvectors associated with matrix *A*.

UTR_{ODF} : Steady-state availability of the entire transfer line subject to operation-dependent failures.

UTR_{TDF} : Steady-state availability of the entire transfer line subject to time-dependent failures.

Th_{ODF} : Throughput of the entire transfer line subject to operation-dependent failures.

Th_{TDF} : Throughput of the entire transfer line subject to time-dependent failures.

$E(Th)$: Expected throughput of the transfer line.

$Var(Th)$: Throughput variance of the transfer line.

2.3 Working assumptions

It is assumed that the homogeneous transfer line satisfies the following conditions:

1- Failure times and repair times are exponentially distributed random variables.

2- Repair actions are done perfectly so the machines are restored to the 'as good as new' state.

3- On failure, parts remain at machines and processing resumes after repair completion.

4- The line operates under saturation: the first station is never starved and the last one is never blocked.

5- Transfer times between machines are considered negligible.

6- No parts are scrapped.

3 THROUGHPUT VARAIABILITY OF HOMOGENEOUS TRANSFER LINES WITH IDENTICAL REPAIR DISTRIBUTIONS

3.1 State Space Model

For homogeneous, unbuffered *m*-machine transfer lines subject to operation-dependent failures where time to failure and time to repair are exponentially distributed, the transfer line dynamics can be modeled as a continuous time, discrete state space, homogeneous Markov process with (*m*+1) states (Fig. 2) where only state 0 is the operational state during which the transfer line is manufacturing parts. Since all the model states constitute a single communicating class, the Markov process is ergodic and consequently the steady-state probabilities exist (Çinlar, 1975).

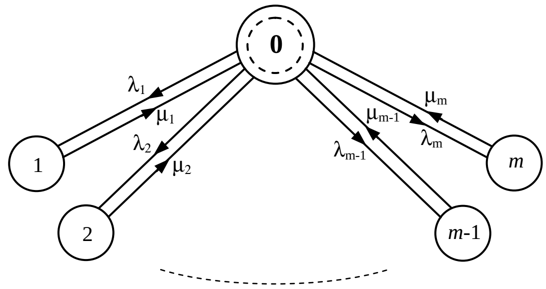

Figure 2: State-space Markov process for an *m*-machine transfer line

In the case where all repair time distributions are identically distributed with parameter $\mu_i = \mu$ (i = 1,2, …, m), the non-operational states (1, 2, …, *m*) can be aggregated into one state, and the transfer line dynamics will be modeled by a two-state homogeneous Markov process (Fig. 3), where 0 is the operational state and 1 is the down state (Dhouib *et al*. (2006)).

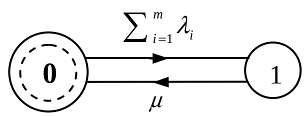

Figure 3: State-space Markov process for an *m*-machine transfer line with identical repair time distributions

3.2 Expected throughput and throughput variance

The transfer line throughput is given by the number of parts produced during one time unit in the long run horizon. Consequently, it can be explained in term of the total operational time during which the transfer line is producing ($UpT(T)$) as *T* tends to infinity (Eq. 1); the ratio $UpT(T)/T$ is the proportion of time during which the transfer line is in an upstate and producing items (i.e. interval availability).

$$Th = \lim_{T\to\infty} \frac{UpT(T)}{T} \cdot \frac{1}{t} \tag{1}$$

Since the total operational time ($UpT(T)$) is a continuous random variable depending on the random character of failure and repair times, the transfer line expected throughput and its throughput variance are given by equations (2) and (3), respectively (Tan, 1997).

$$E(Th) = \frac{1}{t} \cdot \lim_{T\to\infty} \frac{E(UpT(T))}{T} \tag{2}$$

$$Var(Th) = \frac{1}{t^2} \cdot \lim_{T\to\infty} \frac{Var(UpT(T))}{T} \tag{3}$$

For homogeneous Markov processes, the limiting expectation and the limiting variance of the total residence time in a specific state i can be written as a function of the terms of the transient probabilities ($P_{ii}(T)$) of the Markov process (Eq. 4).

$$P_{ii}(T) = \frac{1}{B} \cdot \left(a_0 + \sum_{k=1}^{K} a_k \cdot e^{-\xi_k \cdot t} \right) \quad (4)$$

where ξ_k are the non-zero eigenvalues of the transition rate matrix of the Markov process (k = 1, 2, …, K), a_k is the weight of ξ_k and a_0 is the weight corresponding to the zero eigenvalue (Çinlar, 1975).

Given that the total operational time during which the transfer line is producing is equal to the total residence time in the operational state 0, then one can derive the expected throughput (Eq. 5) and the throughput variance (Eq. 6) by determining the terms B, a_0, a_k, and ξ_k of the transient probability $P_{00}(T)$. In fact, $P_{00}(T)$ is the probability that the transfer line will be operational at a specific instant T given that it was initially in an upstate; $P_{00}(T)$ defines the instantaneous availability of the transfer line. A similar proof of the explicit equations of the troughput expectation and its variance (Eqs. 5 and 6) can be found in Matis *et al*. (1983) and Tan (1997).

$$E(Th) = \frac{1}{t} \cdot \frac{a_0}{B} \quad (5)$$

$$Var(Th) = \frac{2}{t^2} \cdot \frac{a_0}{B^2} \cdot \sum_{k=1}^{K} \frac{a_k}{\xi_k} \quad (6)$$

Consequently, in order to assess the expected throughput and the throughput variance of the transfer line, we should determine the respective expressions of the terms B, a_0, a_k, and ξ_k. These parameters can be evaluated by deriving the general form of the state transition probabilities associated with the Markov process. For homogeneous Markov processes, the state transition probabilities are given by equation (7), where A is the state-space transition rate matrix (Cinlar, 1975).

$$P(T) = \left(e^{A \cdot T} \right) \quad (7)$$

By considering the two-state, homogeneous Markov process of figure 2, the state-space transition rate matrix A is given by equation (8).

$$A = \begin{pmatrix} -\sum_{i=1}^{m} \lambda_i & \sum_{i=1}^{m} \lambda_i \\ \mu & -\mu \end{pmatrix} \quad (8)$$

Toward determining the expressions of the state transition probabilities (Eq. 7), we first evaluate the eigenvalues and the eigenvectors associated with the matrix A, and then we proceed by the diagonal form of the matrix A. By resolving the characteristic polynomial of matrix A, two eigenvalues denoted ξ_0, ξ_1 (Eqs. 9, 10) and two eigenvectors denoted l_0, l_1 (Eqs. 11, 12) can be found.

$$\xi_0 = 0 \quad (9)$$

$$\xi_1 = \mu + \sum_{i=1}^{m} \lambda_i \quad (10)$$

$$l_0 = (1,1) \quad (11)$$

$$l_1 = \left(-\sum_{i=1}^{m} \lambda_i / \mu, 1 \right) \quad (12)$$

Since all eigenvalues of the state transition rate matrix A are distinct, then matrix A is diagonalizable and it can be expressed by equation (13), where U is the matrix enclosing all eigenvectors, U^{-1} is its inverse, and D is the diagonal matrix whose diagonal entries are ξ_0 and ξ_1 (Çinlar, 1975).

$$A = U \cdot D \cdot U^{-1} \quad (13)$$

Consequently, the state transition probabilities are given by equation (14).

$$P(T) = U \cdot e^{D} \cdot U^{-1} \quad (14)$$

Replacing U, D, U^{-1} by their corresponding values in equation (14), the state transition probabilities of an m-machine, homogeneous transfer line with identical repair time distributions are given by:

$$P(T) = \frac{1}{\sum_{i=1}^{m} \lambda_i + \mu} \cdot \begin{pmatrix} 1 & -\sum_{i=1}^{m} \lambda_i / \mu \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{-\left(\mu + \sum_{i=1}^{m} \lambda_i\right) \cdot T} \end{pmatrix} \cdot \begin{pmatrix} \mu & \sum_{i=1}^{m} \lambda_i \\ -\mu & \mu \end{pmatrix} \quad (15)$$

As a result, the transient probability that the transfer line will be in an operational state (state 0) at time T given that it has been initially at an operational state (state 0) is given by equation (16), and the analytical expressions of parameters B, a_0 and a_k will be respectively deduced to the forms given by equations (17), (18), and (19).

$$P_{00}(T) = \frac{1}{\mu + \sum_{i=1}^{m} \lambda_i} \left(\mu + \left(\sum_{i=1}^{m} \lambda_i \right) \cdot e^{-\left(\mu + \sum_{i=1}^{m} \lambda_i\right) \cdot T} \right) \quad (16)$$

$$a_0 = \mu \quad (17)$$

$$a_1 = \sum_{i=1}^{m} \lambda_i \quad (18)$$

$$B = \mu + \sum_{i=1}^{m} \lambda_i \quad (19)$$

Replacing B, a_0, a_1, ξ_0, and ξ_1 by their respective values, the homogeneous transfer line expected throughput and its throughput variance will be reduced to equations (20) and (21), respectively. The same expression of the expected throughput of an *m*-machine, unbuffered, homogeneous transfer line has been also derived by Buzacott (1968), Papadopoulos and Heavy (1996), Sherwin (2000), Schneiweiss (2005), and Dhouib *et al*. (2006).

$$E\left(Th_{ODF}\right)=\frac{1}{t}\cdot\frac{\mu}{\mu+\sum_{i=1}^{m}\lambda_i} \quad (20)$$

$$Var\left(Th_{ODF}\right)=\frac{2}{t^2}\cdot\frac{\mu\cdot\sum_{i=1}^{m}\lambda_i}{\left(\mu+\sum_{i=1}^{m}\lambda_i\right)^3} \quad (21)$$

4 THROUGHPUT VARAIABILITY OF GENERAL HOMOGENEOUS TRANSFER LINES

4.1 Expected throughput of the transfer line

Many authors working in the reliability domain have derived an analytical formula of the steady-state availability of systems having a serial structure and are subject to operation-dependent failures. The same formula has also been proposed to model homogeneous, unbuffered transfer lines subject to operation-dependent failures, by authors working in the production line field and which is called line efficiency (Dhouib *et al*., 2006). The expected throughput of these transfer lines (Eq. 22) is simply obtained by multiplying the line availability by its production rate.

$$E\left(Th_{ODF}\right)=\frac{1}{t}\cdot\frac{1}{1+\sum_{i=1}^{m}\lambda_i/\mu_i} \quad (22)$$

4.2 Throughput variance of the transfer line

In the general case where the homogeneous transfer line machines have different repair time distributions and are subject to time-dependent failure mode, Tan (1997) consider the property associated with the reliability structure of series systems and which explains the instantaneous availability of the entire transfer line as the product of individual instantaneous availabilities of its workstations. In fact, the failure and repair mechanisms of each workstation are time-dependent and are independent of other stations.

However, in the case where the transfer line is subject to operation-dependent failures, this multiplicative property does not apply due to coupling phenomena that exist between the workstation dynamics (Fig. 2). Accordingly, it's extremely complex to resolve analytically the characteristic polynomial of the transition matrix A (Eq. 23) in order to find the explicit expressions of the matrix eigenvalues and eigenvectors, and then derive the transfer line instantaneous availability ($P_{00}(T)$) to deduce the respective terms (B, a_0, a_k, and ξ_k for k = 1, 2, …, K), and finally evaluate the throughput variance of such lines.

$$A=\begin{pmatrix} -\sum_{i=1}^{m}\lambda_i & \lambda_1 & \cdots & \lambda_{m-1} & \lambda_m \\ \mu_1 & -\mu_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mu_{m-1} & 0 & & -\mu_{m-1} & 0 \\ \mu_m & 0 & \cdots & 0 & -\mu_m \end{pmatrix} \quad (23)$$

On the other hand, analytical expressions of terms B and a_0 of the transfer line instantaneous availability ($P_{00}(T)$) can be deduced by considering the two expressions of the expected throughput of the transfer line (Eqs. 5 and 22). In fact, the expected throughput expression (Eq. 22) can be rewritten in the form of equation (24). As a result, the terms B and a_0 will be expressed by equations (25) and (26), respectively. In the case where the transfer line machines have identical repair time distributions ($\mu_i = \mu$ for i = 1,2, …, m), equations (25) and (26) reduce to the respective values given by equations (17) and (19).

$$E\left(Th_{ODF}\right)=\frac{1}{t_{bot}}\cdot\frac{\sum_{i=1}^{m}\frac{\mu_i}{m}}{\sum_{i=1}^{m}\frac{\mu_i}{m}+\sum_{i=1}^{m}\frac{\lambda_i^e}{m}\cdot\sum_{j=1}^{m}\frac{\mu_j}{\mu_i}} \quad (24)$$

$$a_0=\sum_{i=1}^{m}\frac{\mu_i}{m} \quad (25)$$

$$B=\sum_{i=1}^{m}\frac{\mu_i}{m}+\sum_{i=1}^{m}\frac{\lambda_i^e}{m}\cdot\sum_{j=1}^{m}\frac{\mu_j}{\mu_i} \quad (26)$$

The remaining terms required to evaluate the throughput variance of the transfer line, the non-null eigenvalues ξ_k and their corresponding weights a_k for k = 1, 2, …, K, can be evaluated numerically using the following algorithm:

STEP 1. Initialization

$\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$;
(Transfer line parameters)

STEP 2. Expected throughput

Evaluate $E(Th_{ODF})$ (Eq. 22)

STEP 3. Instant availability term calculation

Set transition Matrix A (Eq. 23)
Evaluate: a_0 (Eq. 25) and B (Eq. 26)
Determine numerically non-null eigenvalues (ξ_k) and eigenvectors (l_k) by solving the characteristic polynomial of A.

Determine the explicit form of the homogeneous transfer line instantaneous availability ($P_{00}(T)$).
for k = 1 to K (m-1)
Evaluate the weight a_k corresponding to the eigenvalue ξ_k by considering the value of the term B (Eq. 26).

STEP 4. Throughput variance assessment

Calculate the throughput variance of the transfer line (Eq. 6) by replacing the terms B, a_0, a_k, and ξ_k for k = 1, 2, …, K by their corresponding values.

5 NUMERICAL RESULTS ANALYSIS

In order to analyze the variability of the output process of homogeneous, unbuffered transfer lines subject to operation-dependent failures, and specifically to analyze the impact of the transfer line parameters (m, λ_i, μ_i) and to compare throughput variances of transfer lines subject to operation-dependent failures to those subject to time-dependent failures we consider homogeneous transfer lines with identical machine reliability and maintainability characteristics.

In this study we will implement the same data considered by Tan (1997) and Chen and Yuan (2004); where $t = 1$, $\lambda_i = \lambda = 0.1$, and $\mu_i = \mu = 2$. Figures 4 and 5 depict, respectively, the expected throughput and the throughput variance of transfer lines subject to operation-dependent failures as the number of workstations (m) increases. It is observed that although the transfer line expected throughput decreases as the number of workstations increases, the dependence of the throughput variance on the number of workstations is not monotonic and affected by the workstation parameters. In fact, figure 5 shows that the throughput variance increases and then decreases as the number of the transfer line workstations increases.

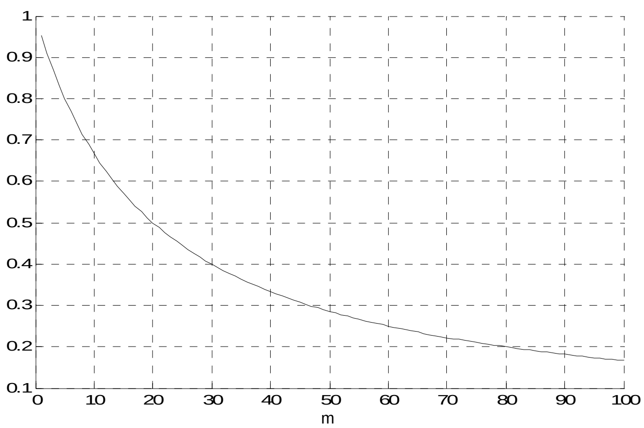

Figure 4: Expected throughput of m-identical machines, homogeneous transfer lines subject to operation dependent failures ($\lambda = 0.1$, $\mu = 2$)

By analyzing the variance throughput expression (Eq. 21) with respect to the number of the transfer line machines (m), we conclude that the variance throughput starts with an initial value equal to ($2\cdot(\mu\cdot\lambda)/(\mu+1)^3$) for $m = 1$, increases with m until reaching a maximum value of ($8/(27\cdot\mu)$) for $m^* = \mu/(2\cdot\lambda)$ and then decreases to reach the asymptotic value 0 as m tends to infinity.

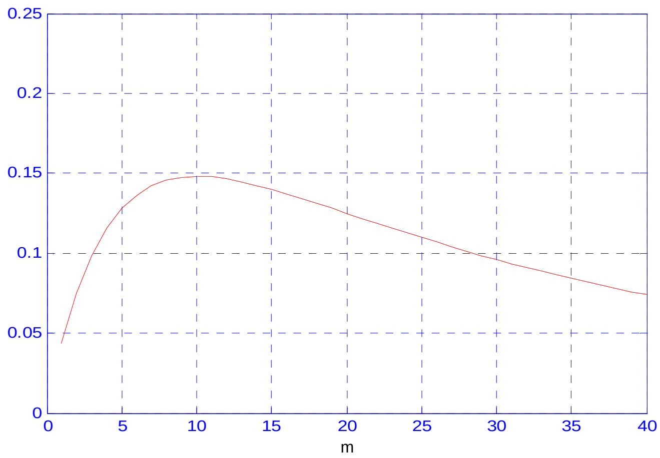

Figure 5: Throughput variance of m-identical machines, homogeneous transfer lines subject to operation dependent failures ($\lambda = 0.1$, $\mu = 2$)

One can note that the maximum observed value of the throughput variance is independent of failure rate values and only depends on repair rate values of the transfer line machines. In fact, the maximum observed value of the throughput variance of a transfer line is maintained with the machine repair rate value but the value of m^* changes depending on the machine failure rates. Indeed, if the machine failure rate increases, the corresponding value of m^* decreases (Fig. 6).

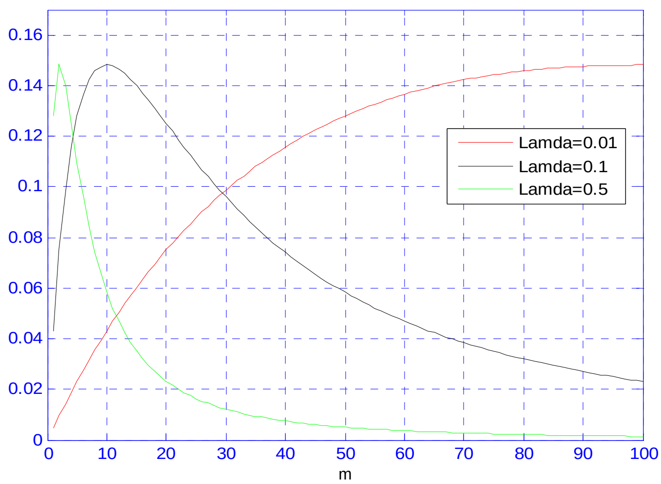

Figure 6: Throughput variance of m-identical machines, homogeneous transfer lines subject to operation dependent failures for different λ values ($\mu = 2$)

For transfer lines having the same failure rate value (λ), an increase in the machine repair rate values (μ) allows a decrease in the maximum throughput variance value and an increase in the corresponding number of the transfer line machines (m^*) (Fig. 7).

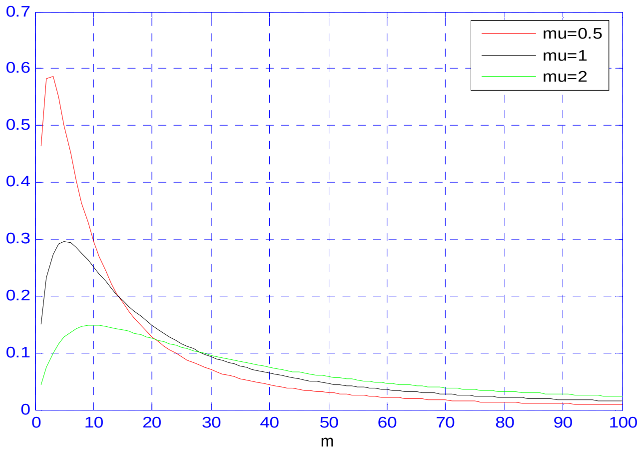

Figure 7: Throughput variance of *m*-identical machines, homogeneous transfer lines subject to operation dependent failures for different μ values ($\lambda = 0.1$)

Also, one can show that if the failure and repair rates of an *m*-identical machine transfer line increase with a factor *n*, then, the throughput variance of the transfer line decreases *n* times (Eq. 27).

$$Var(n\lambda, n\mu) = \frac{1}{n}\cdot Var(\lambda,\mu) \qquad (27)$$

Finally, comparing *m*-identical machine transfer lines subject to operation-dependent failure mode to those subject to time-dependent failure mode (Tan, 1997), we conclude first that the expected throughput of transfer lines subject to operation-dependent failures (Eq. 20) is superior to that of transfer lines subject to time-dependent failures (Eq. 28) (Fig. 7) (Dhouib *et al*. (2006)).

$$E(Th_{TDF}) = \frac{1}{t}\cdot\frac{\mu^{m}}{(\lambda+\mu)^{m}} \qquad (28)$$

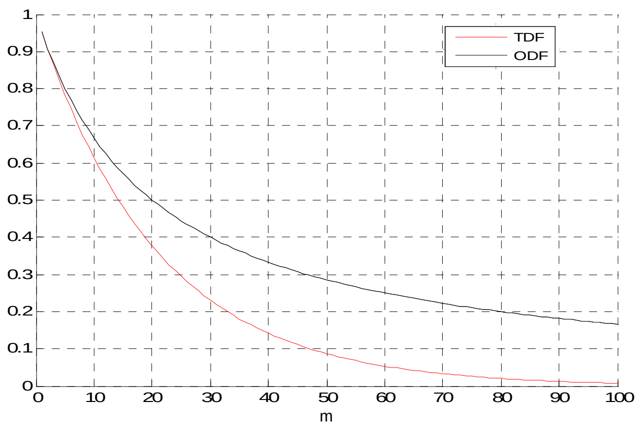

Figure 7: Expected throughput of *m*-identical machines, homogeneous transfer lines ($\lambda = 0.1$, $\mu = 2$)

Furthermore, Figure 8 shows that the throughput variance of transfer lines subject to operation-dependent failures (Eq. 21) is inferior to that of transfer lines subject to time-dependent failures (Eq. 29) (Tan, 1997) if the transfer line has a number of workstations inferior to a specific number ($m \le m_{max}$). For longer transfer lines ($m \ge m_{max}$), those subject to time-dependent failures have smaller variance. For transfer lines with parameters $t = 1$, $\lambda_i = \lambda = 0.1$, and $\mu_i = \mu = 2$, the specific maximum number of workstations (m_{max}) equals 37 workstations. The results also show that the value of m_{max} increases as the repair rate increases and as the failure rate decreases and remains constant for the same ratios μ/λ.

$$Var(Th_{ODF}) = \frac{2}{t^{2}}\cdot\frac{\mu^{2m}}{(\lambda+\mu)^{2m+1}}\sum_{i=1}^{m}\binom{m}{i}\frac{(\lambda/\mu)^{i}}{i} \qquad (29)$$

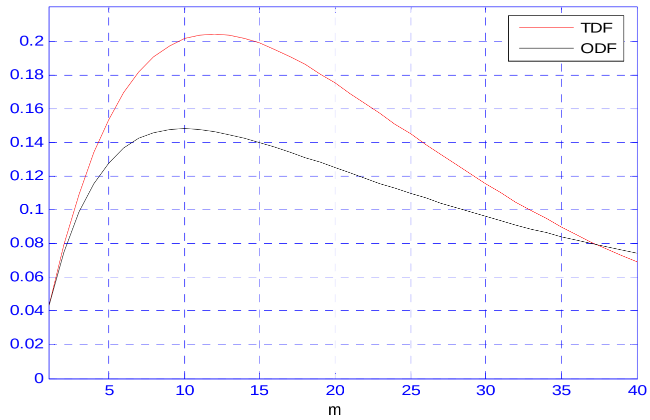

Figure 8: Throughput variance of *m*-identical machines, homogeneous transfer lines ($\lambda = 0.1$, $\mu = 2$)

6 CONCLUSIONS

In this paper the expected throughput and the throughput variance of homogeneous, unbuffered transfer lines were studied. The transfer line is composed of several workstations with exponentially distributed failure and repair times, and subject to operation-dependent failures.

Based on Markov reward processes, explicit analytical formulations of the expected throughput and the throughput variance for homogeneous transfer lines having identical repair time distributions were proposed. For the case where the transfer line have repair time distributions, a numerical procedure has been proposed to determine all parameters required to evaluate the throughput variance of these transfer lines.

Numerical experiments have been carried out to analyze the impact of operational, reliability, and maintainability parameters of workstations on the behavior of the throughput variance of transfer lines. The results show first that unlike the expected throughput which decreases as the number of transfer line machine increases, the throughput variance increases until reaching a maximum value and then decreases to a zero asymptotic value when the number of machines tends to infinity. The results show that the maximum throughput variance does not depend on the failure rate of the transfer line machines. However, the number of machines at which the maximum variance value is reached increases (decreases) as the machine failure (repair) rates decreases.

Compared to homogeneous transfer lines subject to time-dependent failures (Tan, 1997; Chen and Yuan, 2004), transfer lines subject to operation-dependent failures present a higher value of the expected throughput. However, the throughput variance of transfer lines subject to operation-dependent failures is maintained lower if the number of workstations composing the transfer line does not exceed a specific limiting number. The limiting number of workstation increases as the machine repair (failure) rates increases (decreases), but it remains constant for the same ration of repair rate to failure rate.

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