Biot analysis of deformation during variable-head field permeability tests in soft soils

Analyse des déformations durant les essais de perméabilité in situ à niveau variable dans les sols peu rigides à l'aide de la formulation de Biot

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ABSTRACT: Variable-head field permeability tests are conducted by changing rapidly the water level in a monitoring well and by recording its recovery. Interpretation of test data is done using two types of methods. Methods of the first type assume that the pore volume is linearly related to pore pressure, while those of the second type assume a perfectly rigid soil skeleton. In this paper, a COMSOL model based on the Biot consolidation theory is presented for the modelling of variable-head tests in soft soils. The model takes into account changes in stress and well cavity volume. Numerical results indicate that the treatment of deformation for interpretation methods of the first type is incomplete. Pore volume changes and cavity expansion both have an influence on test data, especially for small riser pipe diameters. However, these two deformation types have opposite effects on test data. Because they have the same order of magnitude, they tend to cancel each other. A comparison of modelling and experimental results shows that the model can accurately predict the influence of deformations, and that neglecting deformations gives reliable hydraulic conductivity values for most well geometries.

RÉSUMÉ : L'essai de perméabilité à niveau variable consiste à changer rapidement le niveau d'eau dans un puits d'observation et à suivre le retour du niveau d'eau vers sa position initiale. Deux types de méthodes permettent l'interprétation des résultats. Les méthodes du premier type supposent une relation linéaire entre le volume des pores et la pression interstitielle, alors que celles du deuxième type supposent un sol parfaitement rigide. Dans cet article, un modèle COMSOL basé sur la théorie de Biot est utilisé pour modéliser les essais à niveau variable dans les sols peu rigides. Le modèle tient compte des changements de contrainte et du volume de la cavité à la base du puits. Les résultats numériques indiquent que le traitement des déformations pour les méthodes du premier type est incomplet. Les variations du volume des pores et de la cavité à la base du puits ont une influence sur l'essai. Toutefois, les deux types de déformation ont des effets opposés. Parce que leur impact est du même ordre de grandeur, ils tendent à s'annuler. Une comparaison des résultats expérimentaux et de modélisation montre que le modèle peut prédire l'influence des déformations avec précision, et que des valeurs réalistes de la conductivité hydraulique peuvent être obtenues en négligeant les déformations pour la plupart des géométries de puits.

KEYWORDS: Field permeability tests, variable-head tests, soft soils, COMSOL, Biot analysis

1 INTRODUCTION

Regulations often require the hydraulic conductivity (k) of low-permeability materials to be obtained in the field, for instance using variable-head tests conducted in monitoring wells (e.g., MDDEFP 2012). With variable-head tests, the water level in the riser pipe of a monitoring well is changed rapidly. The k value is calculated from the return of the water level toward is initial elevation. Contrarily to laboratory procedures such as oedometer or triaxial tests, field measurements are known to give k values that better reflect the presence of large scale features that facilitate ground water flow, such as fractures or sand layers (Duhaime et al. 2013; Tavenas et al. 1986).

Two types of methods can be used for the interpretation of test data. The first type assumes that components of the total stress tensor in the soil are constant throughout the test and that soil volume changes are proportional to pore pressure changes. With this family of interpretation methods, the hydraulic head field in the soil is a solution to the following partial differential equation (PDE):

$$\nabla^2 h = \frac{S_s}{k}\frac{\partial h}{\partial t} \tag{1}$$

where h is the hydraulic head field in the soil, t is the time variable and S_s is the specific storage, a compressibility value for the soil skeleton related to the coefficient of volume change used in the Terzaghi consolidation theory ($m_v = S_s/\gamma_w$, where γ_w is the unit weight of water). Type I methods are best exemplified by the Cooper et al. (1967) method and the general approach of Butler (1998).

Type II methods assume negligible volumetric deformations of the soil skeleton. Based on this hypothesis, the water conservation principle for the monitoring well can be written as:

$$ckH = -S_{inj}\frac{dH}{dt} \tag{2}$$

where H is the hydraulic head in the monitoring well with respect to the background hydraulic head in the soil, c is a shape factor that depends on the well geometry and the boundary conditions in the soil, and S_{inj} is the section area of the riser pipe in which the water level varies. Type II methods use different forms of Eq. 2. For example, the velocity graph method replaces the time-derivative of H with a finite difference approximation (Chapuis et al. 1981).

Type I methods and Eq. 1 do not allow a formal mechanical boundary condition (fixity or total stress value) to be applied at the interface between the tested soil and the sand filter of the well. In practice two types of boundary conditions can be envisioned for the soil/sand filter interface: 1) free displacement with the radial component of total stress at the interface equal to water pressure, and 2) fixed displacements.

The main objective of this paper is to determine the influence of the boundary condition at the interface between the sand filter and the tested soil on variable-head tests. The paper begins with a presentation of the COMSOL model based on the Biot formulation that was developed to model variable-head permeability tests with proper mechanical boundary conditions. The model is then used to simulate permeability tests for realistic clay properties and different riser pipe diameters. The paper ends with a presentation of experimental results obtained in a relatively soft soil for two riser pipe diameters.

2 COMSOL MODEL

The COMSOL model presented in this paper is a generalization of the model presented by Duhaime and Chapuis (2014) for the modelling of pulse tests, a type of field permeability test where a known volume of water is injected below a packer in a monitoring well.

The numerical model is based on the Biot (1941) formulation (*Poroelastic* interface in COMSOL). The Biot formulation is centered on three main equations: a water conservation equation (Eq. 3), an equation that verifies static equilibrium (Eq. 4) and a constitutive model that relates the stress and strain tensors (Eq. 5) (Duhaime and Chapuis 2014):

$$\nabla^2 h = -\frac{1}{k}\frac{\partial \varepsilon_v}{\partial t} \quad (3)$$

$$\nabla \boldsymbol{\sigma} = 0 \quad (4)$$

$$\boldsymbol{\sigma} - p\mathbf{I} = \mathbf{C}\,\boldsymbol{\varepsilon} \quad (5)$$

where ε_v is the volumetric strain, the sum of the radial, vertical and tangential deformation components, $\boldsymbol{\sigma}$ is the total stress tensor, p is the pore pressure, $\mathbf{I}$ is the identity matrix, $\mathbf{C}$ is a constitutive matrix that relates effective stress and deformation, and $\boldsymbol{\varepsilon}$ is the deformation tensor. The simulations presented in this paper are based on a linear-elastic relationship between deformation and effective stress. In this case, $\mathbf{C}$ is a function of the soil Young's modulus (E) and Poisson's ratio (ν)

Equations 3, 4 and 5 were solved for the axisymmetric domain shown in Fig. 1. Numerical experiments conducted by Duhaime (2012) have shown that simulation results can be influenced by the domain size and the element size at the interface between the sand filter and the tested soil. A domain size of 20 m was found to give results that simulate an infinite domain for most well geometries and soil properties. Elements of 1 mm at the interface between the well and the tested soil were found to produce mesh-independent results.

Most of Fig. 1 boundaries impose fixity and a constant hydraulic head. The main distinguishing feature for the model is the boundary condition at the interface between the sand filter and the tested soil. This boundary condition combines the hydrodynamic and mechanical aspects of the model by verifying the following mass conservation equation:

$$\Delta V_{cavity} + \Delta V_{flow} + \Delta V_{pipe} = 0 \quad (6)$$

where ΔV_{cavity} is the change in the water volume stored in the well that is due to the expansion of the cavity containing the sand filter, ΔV_{flow} is the volume of water exchanged between the soil and the well due to the hydraulic gradient and Darcy's law, and ΔV_{pipe} is the change in the water volume stored in the riser pipe. Equation 6 is verified through COMSOL's MATLAB interface and a series of MATLAB functions and scripts. With simulations where $\sigma_r = p$ and displacements at the interface are free, ΔV_{cavity} is calculated from the initial and final displacement vectors for each time step using a MATLAB function presented by Duhaime (2012). For simulations with fixed displacements at the interface, $\Delta V_{cavity} = 0$. The value of ΔV_{flow} is obtained by integrating the hydraulic gradient at the domain boundary. The ΔV_{pipe} component is calculated from the pressure change during each time step and the riser pipe section ($\Delta V_{\mathrm{pipe}} = S_{inj}\,\Delta p / \gamma_w$).

At the beginning of each simulation, the pore pressure at the interface between the soil and the sand filter is increased by 9.81 kPa (a water column of 1 m) in 1 s. Each simulation is then divided in two hundred time steps. For each time step, Eq. 6 is used to determine the pressure in the well at the end of the time step. The Newton-Raphson method is used to iterate on the pressure in the well cavity at the end of each time step. A detailed description of the algorithm and MATLAB functions is given by Duhaime (2012). Duhaime and Chapuis (2014) also give a similar description, but for the specific case of pulse tests with $\Delta V_{\mathrm{pipe}} = 0$.

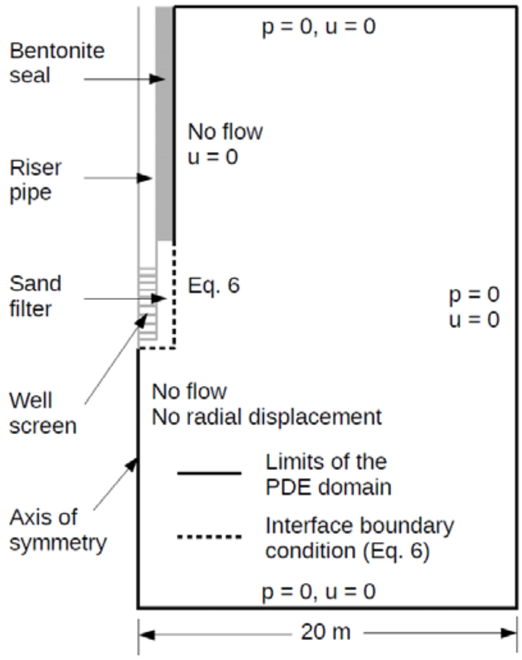

Figure 1. Axisymmetric well geometry, mathematical domain and boundary conditions for the COMSOL model.

3 RESULTS

Using the COMSOL model, a series of field permeability tests were simulated using six riser pipe diameters (d = 100, 50, 25, 12.5, 6 and 3 mm). Soil properties representative of the Champlain clay deposit described by Duhaime et al. (2013) were used ($k = 1{\times}10^{-9}$ m/s, E = 74 MPa, ν = 0.3). The diameter and length of the sand cavity were respectively set to D = 114 mm and L = 1368 mm. For each riser pipe diameter, simulations were conducted for the two hypotheses regarding displacements at the clay-sand filter interface (fixity and free displacements).

With the velocity graph method, k is obtained from the slope of a plot of H_m versus $\Delta H/\Delta t$, where H_m is the mean hydraulic head for the Δt interval (Eq. 2). Figure 2 shows the velocity graphs that were obtained for d = 6 and 25 mm and for the two hypotheses regarding cavity wall displacements. For d = 25 mm (Fig. 2a), both types of boundary condition lead to identical velocity graphs. The first data points (right side of the figure) show a curvature due to the high hydraulic gradients caused by the rapid water level change used to initiate the test. Later data points are aligned on the straight line that should be used to calculate k (e.g., Chapuis et al. 2012; Duhaime et al. 2016). The velocity graph for d = 6 mm (Fig. 2b) shows a more pronounced curvature. The straight part is less evident. Figure 2b also shows that the fixed displacements condition accentuates the velocity graph curvature for smaller d values.

The k value for each velocity graph was calculated from the slope obtained for data points with $H_m < 0.66$ (Chapuis et al. 2012). The following definition was assumed for the shape factor in Eq. 2 (Duhaime et al. 2016):

$$c = \frac{2.22\,\pi\, L}{\ln\left(\frac{L}{D} + \sqrt{1+\left(\frac{L}{D}\right)^2}\right)} \tag{7}$$

Figure 3 shows the k values that were obtained for each velocity graph. All k values are greater than $k = 1\times10^{-9}$ m/s, the hydraulic conductivity used in the numerical model. The apparent k values are also found to increase with decreasing riser pipe diameters. This implies that deformations can lead to overestimated k values. For $d \geq 12.5$ mm, the k error represents less than 30 % of the model k value and both hypotheses on displacements give similar results. With $d < 12.5$ mm, the error on k and the difference between the results obtained for both hypotheses increase. Fixed displacements lead to higher k values than free displacements. For very small riser pipe diameters, the apparent k values show a decrease for the free displacements hypothesis.

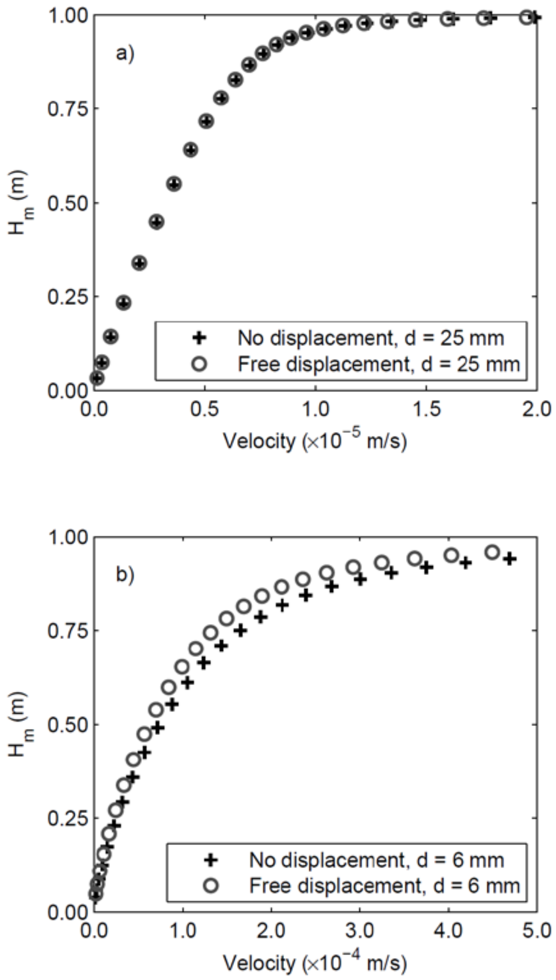

Figure 2. Velocity graphs for the two displacement boundary conditions and for a) d = 25 mm and b) d = 6 mm.

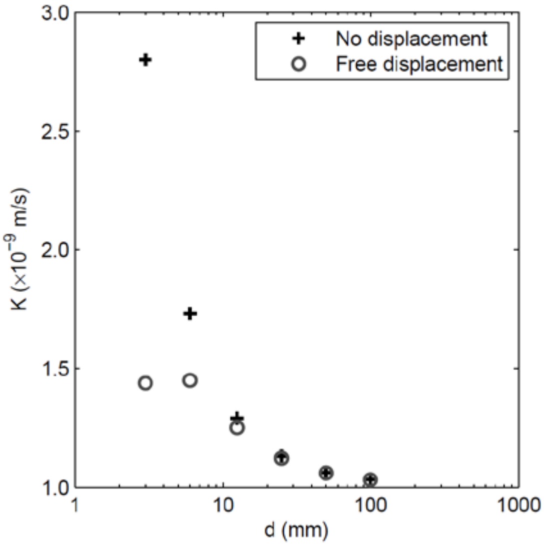

Figure 3. Apparent hydraulic conductivity as a function of riser pipe diameter and displacement hypothesis.

3 DISCUSSION

The difference between the apparent k values for the two displacement hypotheses can be explained by the influence of cavity expansion on the volume of water stored in the riser pipe. Cavity expansion effectively increases the riser pipe section. For example, with a falling-head test, the radial component of total stress decreases as the water level moves downward. This decrease in total stress produces a decrease of the volume enclosed by the interface (Chapuis 2009). The water volume displaced by the change in cavity volume must be stored back in the riser pipe, thus decreasing the rate of change of the water level.

As demonstrated by Duhaime (2012), type I methods produce numerical velocity graphs that correspond to those obtained with the Biot formulation for the no-displacement hypothesis. As shown on Figure 3, type I methods can thus be considered as an upper bound for the influence of deformation on k values. Considering cavity volume changes leads to a decrease in the apparent k value and the influence of deformations.

Depending on the test set-up, both types of displacement boundary conditions can be considered realistic. Free displacements are probably more realistic for variable-head tests where the cavity is filled with loose sand that is less likely to oppose interface displacements. On the other hand, pushed-in piezometers with rigid porous elements (e.g., Tavenas et al. 1986) are less likely to allow a decrease of the cavity volume during a falling-head test. Intermediate cases where displacements are neither free nor completely restrained are also likely. As a consequence, when possible, it appears preferable to limit the influence of deformations on a test instead of trying to take deformations into account.

The α parameter introduced by Cooper et al. (1967), and expressed here by Eq. 8, can be used to appraise the influence of deformations for a given test setting:

$$\alpha = \frac{\pi\, m_v\, \gamma_w\, L\, D^2}{4\, S_{inj}} \tag{8}$$

As demonstrated by Duhaime et al. (2016), tests with $\alpha \leq 10^{-2}$ tend to be less influenced by deformations. On Figures 3 and 4, this threshold corresponds to d = 13 mm. The influence of deformation can be neglected for tests involving a larger riser pipe.

Duhaime (2012) presented 43 velocity graphs for field permeability tests with d = 12.6 and 52.5 mm that were conducted in the Champlain clay deposit in Lachenaie, Quebec. The tests were conducted in a series of 17 monitoring wells with D values between 76 and 95 mm, and L values between 660 and 1372 mm. Figure 4 combines the 65 velocity graphs. To facilitate their comparison, the velocity graphs were normalized with $H(t=0)$, the initial hydraulic head at the beginning of each test, and $(\Delta H/\Delta t)_{0.25}$, the rate of change of hydraulic head at $0.25H(t=0)$ (e.g., Duhaime and Chapuis 2014).

Two numerical type curves are shown on Fig. 4. The first one was obtained using L = 1372 mm, D = 95 mm, $m_v = 2.83\times10^{-5}$ kPa^{-1} and the fixity condition. The second type curve was obtained for L = 660 mm, D = 76 mm, $m_v = 1.36$ kPa^{-1} and the free displacement condition at the soil-sand filter interface. The two m_v values that were used to calculate the type curves correspond to upper and lower bound values from cavity expansion tests that were conducted in the same wells (Duhaime and Chapuis 2014). The two type curves correspond respectively to an upper bound and a lower bound for the impact deformation. The lower bound corresponds approximately to a straight line, while the upper bound shows a significant curvature. Figure 4 clearly shows that the experimental velocity graphs are consistent with the range of deformation impact deduced from the type curves obtained with the numerical model based on the Biot formulation.

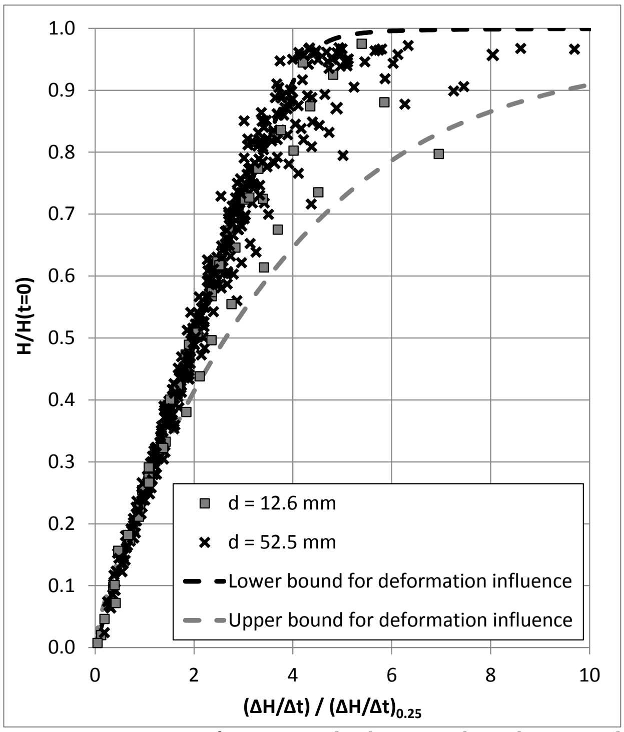

Figure 4. Comparison of experimental velocity graphs with numerical type curves obtained with the Biot formulation.

4 CONCLUSION

This paper introduced a COMSOL numerical model based on the Biot formulation that allows variable-head permeability tests conducted in the field, in soft soils, to be simulated. Using the model, two hypotheses were compared with regards to the mechanical boundary condition at the interface between the sand filter and the tested soil: fixity and free displacements. Model results show that the influence of deformation is more important for the case of fixed interface. Even if classical interpretation methods (types I and II) do not explicitly account for displacements, interpretation of test data based on type I methods is consistent with the fixity hypothesis. Type I methods thus give an upper bound for the influence of deformation on k.

Depending on the type of test, both fixity and free displacements could be realistic boundary conditions for the model. As the real boundary condition in the field is unknown, the authors suggest designing variable-head permeability tests to minimize the influence of deformation. The results presented in this paper and by Duhaime and Chapuis (2014) and Duhaime et al. (2016) show that this can be accomplished by making sure that $\alpha \leq 10^{-2}$.

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