Sensitivity of the convective heat transfer coefficient to the uncertain surface roughness characteristics

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ABSTRACT

Numerically predicting the heat transfer on a rough surface is a challenge. Modelling a rough surface without adding a thermal correction model tends to over predict the heat transfer coefficient in a RANS simulation. Thermal correction models must be added to compute the effect of roughness elements, described by their height, which allows computing the equivalent sand grain roughness, on the thermal boundary layer. The uncertainties on the heat transfer can be critical in in-flight icing studies, where the early stages of icing increase the surface roughness. The objective of the paper is to study the sensitivity of the heat transfer on a rough flat plate to the roughness characteristics. The analysis, using the generation of metamodels and the calculation of sensitivity indexes, will allow determining which characteristics influence the most the heat transfer coefficient variability. After detailing the metamodeling methodology, metamodels predictions obtained and sensitivity indexes calculated will be exposed.

1. INTRODUCTION

Modelling the surface roughness impact on the heat transfer has attracted lots of experimental [1, 2] and numerical [3, 4] efforts. It has been highlighted that the heat transfer is increased on a rough surface compared to a smooth surface. The main usual roughness characteristics are the roughness height, the geometrical shape of the elements and the frontal area of these elements. From a roughness pattern to another, the main heat transfer coefficient characteristics impacted are the maximum value, main value and local values [5].
Numerically, in CFD simulations, the equivalent sand grain roughness is commonly used to characterize a roughness pattern. Dirling [6] proposed a correlation allowing computing the equivalent sand grain roughness, denoted as k_s, from the roughness height, k, the spacing between the roughness elements and their geometrical features. Past studies led to the modification of the turbulence models, such as the Spalart-Allmaras model, to take into account the roughness elements [7]. Nevertheless, these modifications over predict the heat transfer compared to the experimental results and thus requires additional thermal correction models. Aupoix [3] and Morency & Beaugendre [8], implemented thermal correction models aiming at reducing the heat transfer by increasing the turbulent Prandtl number and allowing a positive turbulent viscosity at the wall. One of the applications of the rough heat transfer is aircraft icing. The early stages of ice accretion increase the surface roughness of the aircraft's surface. The roughness features have uncertainties in their pattern resulting from the high variability of atmospheric and flight conditions. In addition, the roughness elements geometry induced by icing is not the cones or the hemispheres commonly used in classical roughness studies [5]. The characterization of the roughness elements in icing conditions has been extensively studied for example by Dukhan [2] who determined the ranges of variation of the roughness height, allowing estimating an empirical probability distribution for the roughness features. Experimental works, for example on airfoils [9] or on flat plates [10] highlighted the strong dependence of the heat transfer coefficient (H_c) to the surface roughness geometry. The ice accretion is influenced by the heat transfer which is itself dependant of the surface roughness. The ice accretion must then present a sensitivity to the uncertain roughness characteristics. A method allowing measuring the uncertainty propagation from the roughness to the ice

accretion is to evaluate the probability distribution function (PDF) of the output of interest [11]. To do so, sample simulations of the full model are run to build a database needed to generate a metamodel. The metamodel is then used to predict the value of the output of interest for a known set of inputs without the need for running additional full-model simulations [12]. Evaluating the sensitivity of the output to the input parameters can then be achieved by computing the Borgonovo sensitivity indexes [13].

The objective of the present paper is to evaluate the sensitivity of the heat transfer coefficient characteristics to the parameters of the thermal correction models. More specifically, the HAX thermal correction model from Aupoix [3] will be compared to the 2PP model [8]. CFD simulations of a horizontal flat plate with both models will be separately run, allowing setting up databases of heat transfer coefficients. The database will then be used to generate polynomial chaos expansions (PCE) metamodels allowing estimating the PDF of the output of interest. The metamodels generated, one for each output of interest, will then allow a complete sensitivity analysis with the calculation of the Borgonovo indexes for each input parameter. This study on the heat transfer coefficient is a step towards the complete study of the ice accretion sensitivity to the roughness parameters in future works.

2. METHODOLOGY

The present section will detail the complete process used to perform the sensitivity analysis. The first step will be the description of the physical problem to be solved, highlighting the geometry and flow conditions used. The next step will be the metamodels generation and finally the method retained for the sensitivity analysis. In the present study, the input parameters are the thermal correction models parameters: roughness height k and equivalent sand grain roughness k_s for both HAX and 2PP models, plus the surface ratio corrected S_{corr} for the HAX model only. The output of interest studied are the following heat transfer coefficient (H_c) features:

- The maximum value of H_c;
- The mean value of H_c;
- The local value of H_c at a specified position x along the flat plate surface.

The graphical representation of the outputs of interest is showed in Fig. 1.

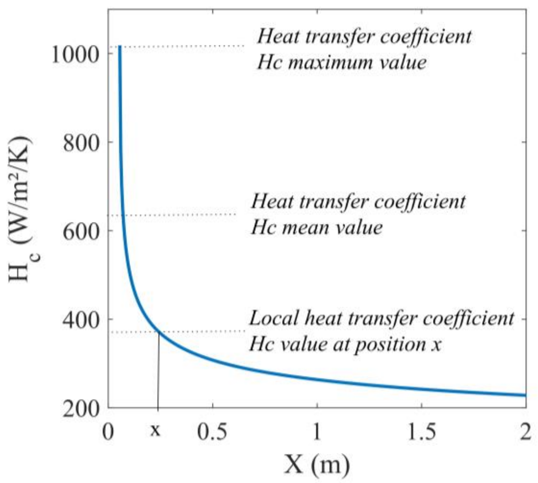

Figure 1. Outputs of interest in the present study

2.1. The problem definition

The present study is based on the assumption that a flat plate is a good approximation of the "unrolled" upper part of an airfoil, as in [14]. The conceptual view of the method is illustrated on Fig. 2. Fig. 2a shows the original NACA0012 airfoil which upper part is represented as a flat plate. Fig. 2b shows the uncertainty on the surface roughness pattern, which implies uncertainty on the heat transfer coefficient on Fig. 2c. Finally, Fig. 2d shows the future extensions of the project: studying the uncertainty on the ice accretion geometry.

The correspondence between the airfoil flow and the flat plate flow is done by adding some conditions on the flat plate surface. The beginning of the studied rough zone of the flat plate must have an established boundary layer. The first 5.2 cm of the flat plate are unheated and smooth, as in [2], allowing having a boundary layer thickness of about 1.8 mm at the beginning of the rough zone in the present case. The rough zone of interest begins then at the end of the starting length of 5.2 cm.

To allow the generation of a heat transfer coefficient database, CFD simulations will be run on a horizontal flat plate geometry. The structured mesh of 137 by 97 cells, with 114 points on the flat plate surface including the starting length, is obtained from the SU2 test case collection [15]. The visual representation of the domain, highlighting the main dimensions and boundary conditions, is showed on Fig. 3.

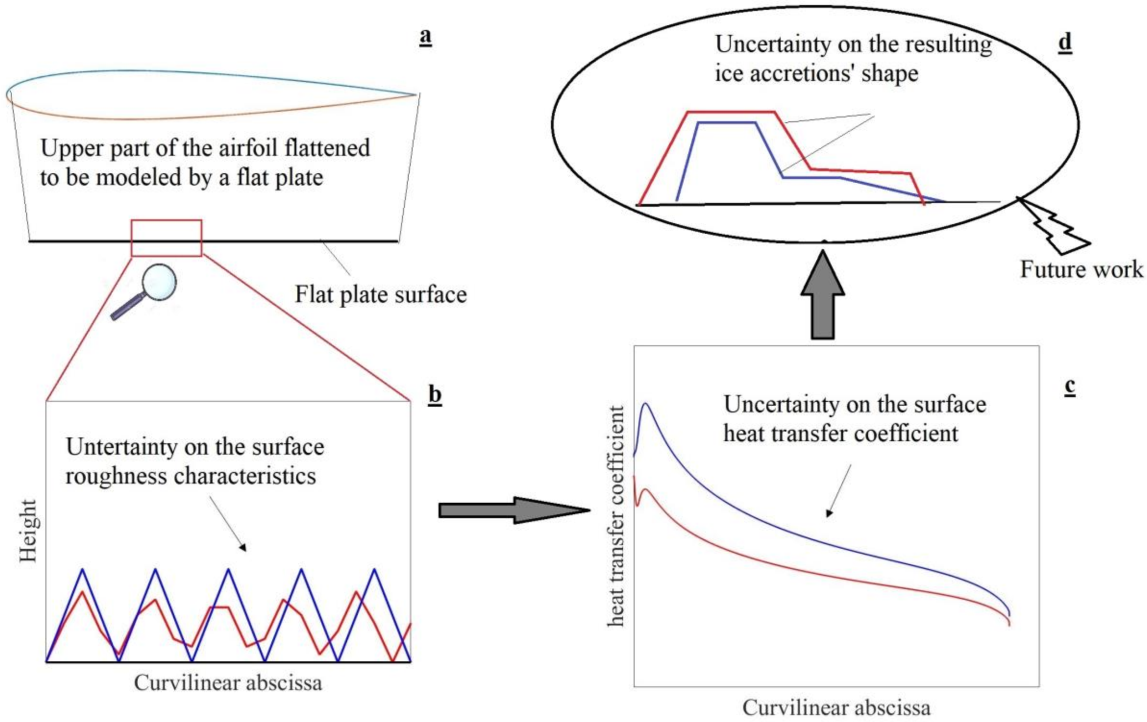

Figure 2. Conceptual view of the problem

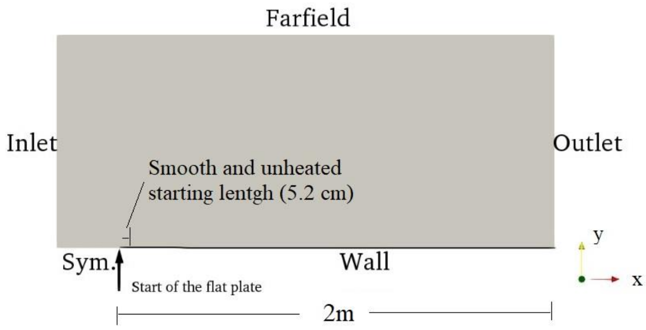

Figure 3. Geometry used for the CFD simulations

The flow field is in the *x* direction at Mach 0.2 at a temperature of 262.04 K. The starting length of the flat plate is unheated while the remaining part is an isothermal rough wall at 273.15 K. Between the simulations, and to build a heat transfer database, the rough parameters (i.e., the roughness height, the equivalent sand grain roughness for both thermal correction models and S_{corr} for the HAX model) will be changed. The CFD solver uses a compressible Navier-Stokes formulation with a rough modification of the Spalart-Allmaras turbulence model [7]. Two sets of simulations will be carried out, one with each thermal correction model.

2.2. The metamodeling process

Building metamodels requires a sample set of “true model response” obtained by the CFD simulations. The metamodel is then built based on the sample set using the PCE technique. For the HAX thermal correction model, the sample set is constituted of 190 simulations and the 2PP model has a sample set of 120. These sample sizes were chosen in accordance with [12] who took a sample of 115 simulations for a four parameters study. The input parameters k, k_s, and S_{corr} have ranges of variations obtained from the literature. The roughness height has a range of variation between 0.41 mm and 4.32 mm [2] and the equivalent sand grain roughness ranges between 0.309 mm and 1.247 mm [16]. As input parameters for the present study, the roughness height k and the ratio k_s/k will be used. The values for k and k_s previously detailed give a range from 0.288 to 0.753 for the ratio k_s/k. Finally, as given by [3], S_{corr} varies between 1 and 2.5. The distributions used for the present study are given in Tab. 1. The assumption of uniform probability distribution is made for every input parameters. Note that the values obtained in the literature are not the only ones possible in practice. These values are taken as a starting point for the present study.

Table 1. Distributions for the input parameters

Parameter	Minimum	Maximum	Distribution
k (mm)	0.41	4.32	Uniform
Ratio k_s/k	0.288	0.753	Uniform
S_{corr}	1	2.5	Uniform

Following the distributions given in Tab. 1, input samples for each thermal correction models are obtained by Latin hypercube sampling [17]. For the HAX model, 190 triplets (k, k_s/k, S_{corr}) are generated and 120 doublets (k, k_s/k) are obtained for the 2PP model. The run of all the CFD simulations then gives databases of 190 (HAX) and 120 (2PP) heat transfer coefficient distributions at the flat plate surface.
The software used to generate the metamodels, and later perform the sensitivity analysis, is the UQLab suite [18]. A metamodel is a function, denoted as M_i, that allows computing an output of interest Y_i from a given input vector X. The components of X are denoted as X_j. The advantage of the metamodel is its ability to compute thousands of Y_i predictions in seconds, avoiding the need for running time and resource-consuming CFD simulations instead. In the framework of the present study, three metamodels for each thermal correction model will be generated. Their inputs and output are summed up in Table 2.

Table 2. The metamodels generated

Metamodel	Input components X_j		Output of interest Y_i
	HAX	2PP	
M_1	k, k_s/k, S_{corr}	k, k_s/k	H_c maximum value
M_2	k, k_s/k, S_{corr}	k, k_s/k	H_c mean value
M_3	k, k_s/k, S_{corr}, x	k, k_s/k, x	H_c value at position x

The M_i's are PCE metamodels whose general formulation is given by Eq. 1.

$$Y_i = M_i(X) = \sum_{\alpha} y_\alpha \times \psi_\alpha(X) \tag{1}$$

In Eq. 1 ψ_α and y_α are the multivariate polynomials and the corresponding coefficients, respectively, of the PCE decomposition. The evaluation of the precision of the metamodel is computed through the regression coefficient R^2, based on the ratio of the mean square error between the true CFD output Y_{CFD} and the metamodel predicted output Y_{PCE} for each sample point and the variance of the CFD output, as in Eq. 2. $\bar{Y}_{CFD}$ is the mean value on all the sample points of the CFD-calculated output.

$$R^2 = 1 - \frac{\sum(Y_{CFD} - Y_{PCE})^2}{\sum(Y_{CFD} - \bar{Y}_{CFD})^2} \tag{2}$$

R^2 close to 1 means a metamodel with a good precision, capable of predicting the output values close to what the CFD computation would have evaluated.

2.3. The sensitivity analysis

Once the metamodels are generated, the sensitivity analysis, using the PCE formulation to calculate sensitivity indexes for each input parameters, can be performed. The Borgonovo indexes, specific to an input parameter, measure the predicted variation in the PDF of the output Y_i when an input parameter (i.e., k, k_s/k or S_{corr}) is fixed at a constant value [13]. By denoting F_Y the PDF of the output Y, F_{Xj} the PDF of the input parameter X_j and $F_{Y|Xj}$ the PDF of Y when X_j is fixed, the Borgonovo index δ_j for the j^{th} input parameter is:

$$\delta_j = \frac{1}{2} \int F_{Xj}(x_j) \int \left| F_Y(y) - F_{Y|Xj}(y|X_j) \right| dy dx_j \tag{3}$$

The higher is the Borgonovo index value, the more sensitive is the output Y to the corresponding input parameter. These indexes are less used than their Sobol indexes counterpart, which are based on a variance decomposition. However, the Borgonovo indexes characterize the PDF of the outputs, which is a feature of interest in the present study.

2.4. The thermal correction models

Finally, prior to expose the results of the analysis, the expressions of the increment ΔPr_t in the turbulent Prandtl number as computed by the HAX model and the 2PP model will be given. This increase in the turbulent Prandtl number is linked to the temperature shift in the thermal boundary layer induced by the roughness elements, as qualitatively illustrated in Fig. 4. On Fig. 4, the Y-axis represents the non-dimensional wall temperature T^+ as a function of the natural logarithm of y^+. The temperature shift between the smooth and the rough surface is denoted as ΔT^+.

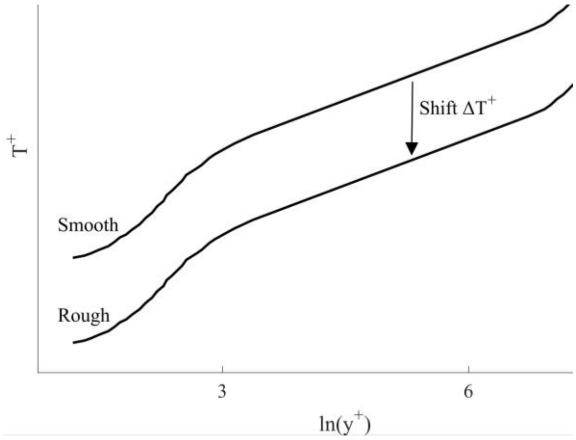

Figure 4. Temperature shift on a rough surface

The mathematical expressions of ΔPr_t will allow highlighting how the input parameters of the analysis k, k_s and S_{corr} are involved in the CFD model. The HAX model computes ΔPr_t as a function of the three input parameters previously mentioned, the distance d to the wall and the velocity shift in the boundary layer induced by the roughness elements [1], as in Eq. 4.

$$\Delta Pr_t = (A(\Delta u^+)^2 + B\Delta u^+)\exp(-d/k) \quad (4)$$

Where:

$$A = (0.0155 - 0.0035S_{corr})(1 - e^{-12(S_{corr}-1)}) \quad (5)$$

$$B = -0.08 + 0.25e^{-10(S_{corr}-1)} \quad (6)$$

$$\Delta u^+ = \frac{1}{\kappa \cdot \log\left(1 + \frac{k_s \cdot u_\tau}{\nu \cdot \exp(1.3325)}\right)} \quad (7)$$

In Eq.7, ν and u_τ are the kinematic viscosity of air and the shear velocity, respectively.

On the other hand, the 2PP formulation relies only on two input parameters (k and k_s) and the expression for ΔPr_t is:

$$\Delta Pr_t = g \times 0.07083 \times {Re_s}^{0.45} \times Pr^{0.8} \times exp\left(-\frac{d}{k}\right) \quad (8)$$

In Eq. (8), the roughness Reynolds number Re_s is defined as $u_\tau k_s/\nu$. Finally, g is subject to the following expressions:

$$g = 1 \text{ if } Re_s \geq 70$$

$$g = \frac{\ln(Re_s) - \ln(5)}{\ln(70) - \ln(5)} \text{ if } 5 < Re_s < 70 \quad (9)$$

$$g = 0 \text{ if } Re_s \leq 5$$

The goal of this section of detailing the methodology used for the metamodeling procedure and the sensitivity analysis has been reached. The physical problem of interest has been depicted as well as the input variables involved in the procedure. Furthermore, the PCE metamodeling process and output of interest of the study have been highlighted. The next section will expose the results of PCE predictions and Borgonovo sensitivity indexes obtained by using this methodology to the present problem.

3. RESULTS

In this section, the results of the metamodeling and sensitivity analysis will be showed. First the regression coefficient R^2 for each metamodel presented in Tab. 2 will be computed to assess the precision of the metamodels. Next, the PDF of the output of interest predicted by the metamodels will be displayed to show the uncertainty propagation from the uniform PDF of the input to the final prediction of the metamodels. Finally, the sensitivity analysis, using the Borgonovo indexes will be performed to evaluate the sensitivity of the output of interest to the thermal correction models and their respective input parameters.

3.1. The metamodels accuracy

Based on the CFD databases, the metamodels M_1, M_2 and M_3 are built for each thermal correction model. A qualitative way of assessing the accuracy of the metamodel is to look at the plot representing the metamodel predicted value Y_{PCE} versus the true CFD response Y_{CFD}. An accurate metamodel will see the scatter points of this plot close to the y = x line. Fig. 5 shows the plot Y_{PCE} versus Y_{CFD} for the metamodel M_2 evaluating the heat transfer coefficient mean value for the 2PP model. The values Y_{PCE} and Y_{CFD} are in W/m²/K.

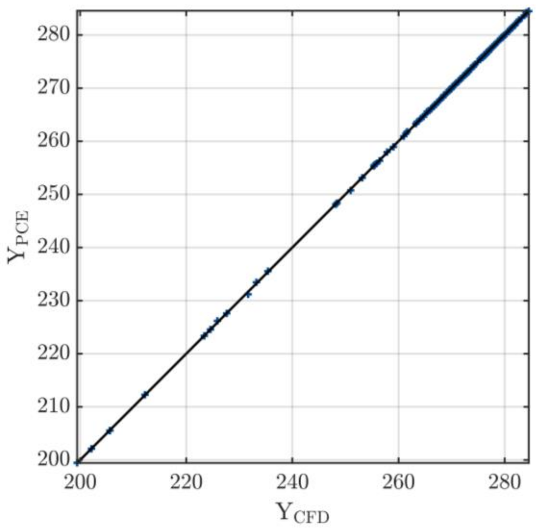

Figure 5. Y_{PCE} vs Y_{CFD} for H_c mean value for metamodel M_2 and the 2PP model

Fig. 5 shows a good qualitative accuracy for the metamodel M_2 since the scatter points are closed to be aligned on the identity line. Quantitatively, the accuracy observed is confirmed with an R^2 regression coefficient of 0.99998. The R^2 regression coefficients for all the combinations metamodel/thermal correction model are summed up in Tab. 3.

Table 3. Regression coefficients for the metamodels

Metamodel	Output of interest Y_i	R^2 coefficient	
		HAX	2PP
M_1	H_c maximum value	0.99986	0.99997
M_2	H_c mean value	0.99967	0.99998
M_3	H_c value at 8.81 cm	0.99987	0.99995

For metamodel M_3, predicting the local value at a user-defined position x at the flat plate surface, the position x = 8.81 cm has been chosen among the 100 points on the rough part of the flat plate since it is located 3.6 cm downstream the beginning of the rough zone. This point is then well in the first 15% of the total flat plate length, which is the usual zone subject to icing on an airfoil. Generally, points further near the trailing edge rarely experience ice accretions. The values gathered in Tab. 3 show a good accuracy for the metamodels generated since [11] concluded that a metamodel with $R^2 = 0.9463$ was already enough to perform a sensitivity study. The present values allow as well to observe that the 2PP model has a better predictability with the present PCE metamodeling than the HAX model. Even if the HAX model's R^2 are high for each metamodel, the regression coefficients for the 2PP model are higher and closer to 1. The accuracy observed on the metamodels is satisfying to use them with a good confidence for the remaining of the study. Furthermore, it shows that the initial sample sizes of the CFD designs of experiments (120 and 190) were enough to be able to build accurate metamodels. It is now possible to use the metamodels to evaluate the PDF of the output of interest.

3.2. Probability distribution of the outputs

The metamodels having successfully showed their accuracy, it is now possible to compute a high number of results without the need of running additional CFD cases. The fast-computing feature of the metamodels will be used to compute 10 000 output values for each combination metamodel/thermal correction model. The 10 000 input triplets (HAX) or doublets (2PP) for each thermal correction model are again generated by Latin hypercube sampling. The metamodels are then evaluated on the 10 000 samples to estimate the PDF of the output of interest. Fig. 6 shows the PDF of H_c maximum value (W/m²/K) as predicted by the metamodel M_1 on the large sample for the HAX model. The X-axis of Fig. 6 represents the H_c value while the Y-axis is the raw counts among the 10 000 predictions.

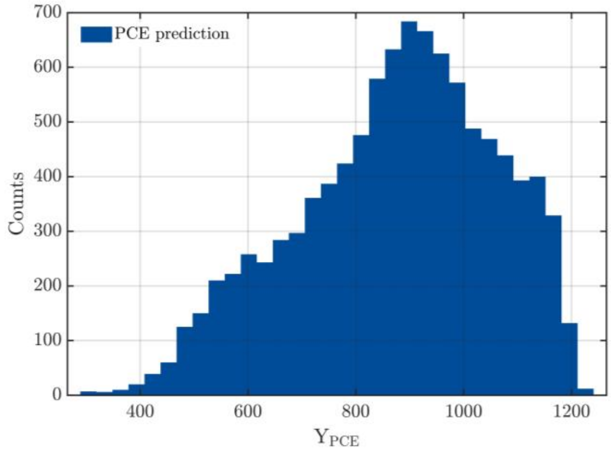

Figure 6. PDF prediction of H_c maximum value by metamodel M_1 for the HAX model

The graphical visualization of the PDF shows that even if the input parameters k, k_s and S_{corr} have uniform distributions, the output has a distribution close to a Weibull distribution, which is a classical distribution in statistical analysis as well as in reliability engineering. Nevertheless, when plotting the same PDF but for the 2PP model, it is possible to observe a behaviour different from the HAX model. As illustrated on Fig. 7, the PDF of the maximum H_c value for the 2PP model presents a different pattern. The axes on Fig. 7 are the same as on Fig. 6. The PDF observed for the 2PP model is slightly different and close to a Gumbel distribution.

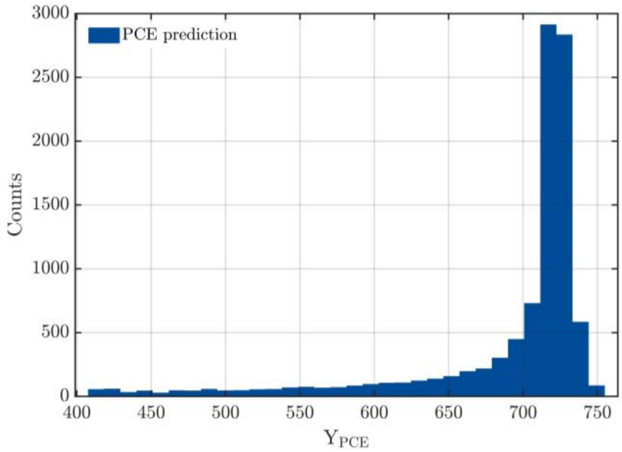

Figure 7. PDF prediction of H_c maximum value by metamodel M_1 for the 2PP model

On Fig. 7, the highest PDF value is in a narrower range than for the HAX model. It is also possible to observe that globally, the 2PP model gives H_c maximum values smaller than the HAX model: the HAX model highest probability is to have a maximum H_c about 900 W/m²/K while this value fell to about 725 W/m²/K for the 2PP model. The origin of these large differences still has to be investigated, but the HAX model, with three input parameters, may have more causes of uncertainty compared to the 2PP model with only two input parameters. This led to a wider range of possible H_c values when the HAX model is used.

The metamodeling of the heat transfer coefficient behaviour allowed highlighting the differences between the HAX model and the 2PP model, even for metamodels M_2 and M_3 not illustrated: the 2PP model is more predictable with the current PCE formulation. This higher predictability is due to the narrower highest probability zone of the 2PP model while the HAX model has a wider range of highly possible values. For instance, Fig. 7 allows estimating a fine range for the highly possible H_c maximum value, with a peak between roughly 715 W/m²/K and 735 W/m²/K (range about 20 W/m²/K wide). On the other hand, Fig. 6 shows a range about 200 W/m²/K wide for the HAX model (peak between 800 W/m²/K and 1000 W/m²/K). The types of distributions obtained for all combinations metamodel/thermal correction model are showed in Tab. 4.

Table 4. Output PDF type for each PCE metamodel

Metamodel	Output of interest Y_i	Output PDF type	
		HAX	2PP
M_1	H_c max. value	Weibull	Gumbel
M_2	H_c mean value	Weibull	Gumbel
M_3	H_c at x = 8.81 cm	Weibull	Gumbel

With the metamodels created, the sensitivity analysis using the Borgonovo indexes can be performed.

3.3. Sensitivity analysis: the Borgonovo indexes

The sensitivity analysis aim is to determine the level of influence of each input parameter on the output variability. The analysis uses the metamodels previously created to determine the Borgonovo indexes. For the present study, the analysis has been set to perform 30 000 metamodel evaluations to compute the sensitivity indexes. Fig. 8 shows the Borgonovo indexes for the inputs of the 2PP model and metamodel M_3 estimating the heat transfer coefficient value at x = 8.81 cm.

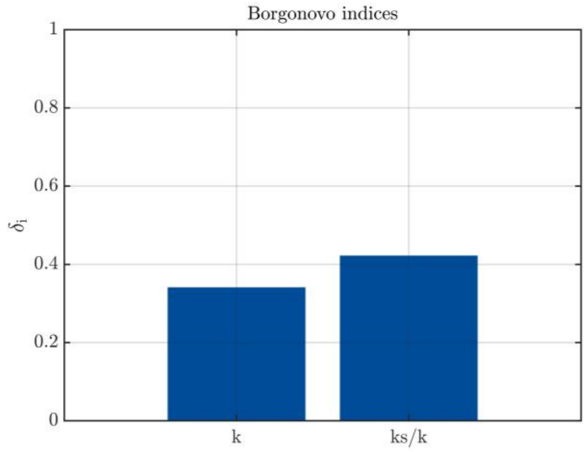

Figure 8. Borgonovo indexes for the 2PP model inputs for metamodel M_3 at x = 8.81 cm

Fig. 8 shows that the ratio k_s/k has a larger influence on the heat transfer coefficient value at x = 8.81 cm than the roughness height k alone. This implies that knowing precisely the k value is not enough to perform an accurate CFD simulation: defining with precision the k_s value is also required. The relative importance of each input parameter can vary depending on the metamodel and the H_c feature evaluated. Fig. 9 shows the Borgonovo indexes for the 2PP model for the M_1 estimating the maximum value of H_c.

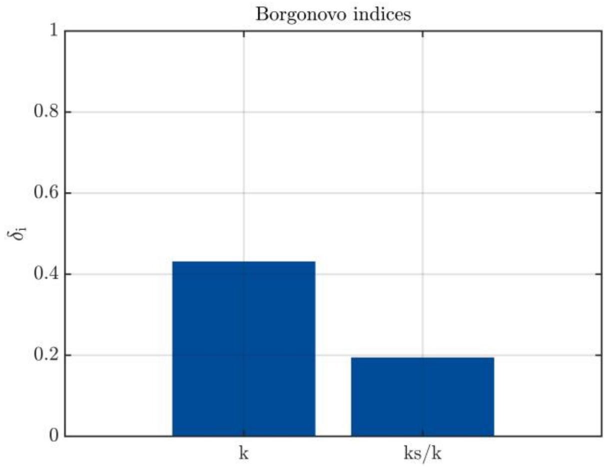

Figure 9. Borgonovo indexes for the 2PP model inputs for metamodel M_1

Fig. 9 shows that this time, the roughness height k has a larger influence on the maximum heat transfer coefficient value than the ratio k_s/k. The maximum value of H_c being at the very beginning of the rough zone, the observations between Fig. 8 and Fig. 9 indicates that the relative importance of the ratio k_s/k grows when travelling downstream while the roughness height k has a predominant role close to the leading edge of the plate. This trend is confirmed by the investigation of other points further downstream, as shown on Fig. 10

presenting the same indexes as on Fig. 8 for metamodel M_3, but at x = 11.4 cm.

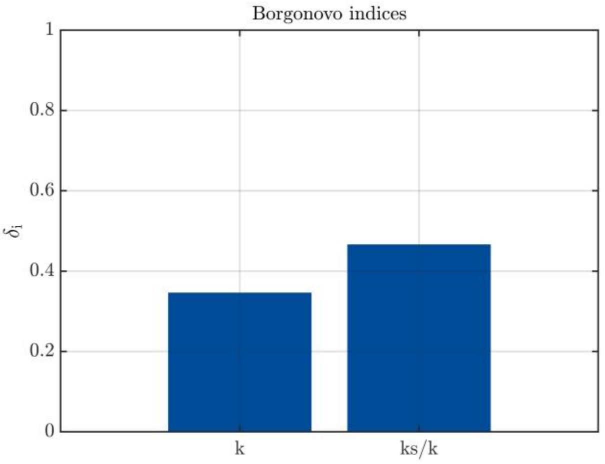

Figure 10. Borgonovo indexes for the 2PP model inputs for metamodel M_3 at x = 11.4 cm

It is possible to observe that the trend glimpsed by comparing Fig. 8 and Fig. 9 is confirmed by Fig. 10: the relative importance of the ratio k_s/k is growing when travelling downstream. This is confirmed again by computing the Borgonovo indexes at x = 15.3 cm. For concision, the figure is not included in the paper, but the indexes values are presented in Tab. 5.

Once again, the HAX model has a different behaviour regarding the relative importance of the input parameters on the output variability. Fig. 11 shows the Borgonovo indexes for the HAX model and the metamodel M_3 evaluating the H_c value at x = 8.81 cm.

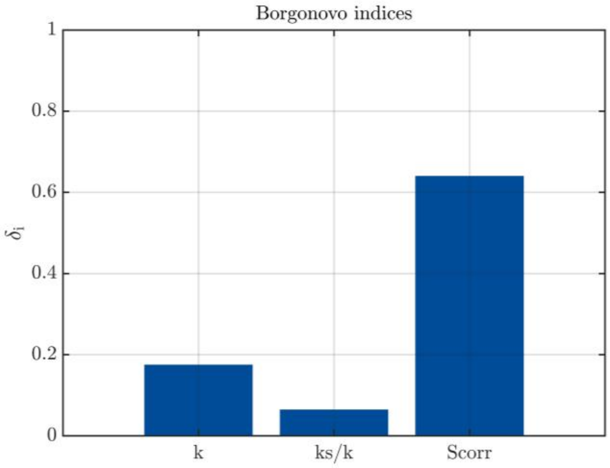

Figure 11. Borgonovo indexes for the HAX model inputs for metamodel M_3 at x = 8.81 cm

With the HAX model, the roughness height k has a larger influence on the H_c value at x = 8.81 cm than the ratio k_s/k. Nevertheless, their impact on the H_c value is relatively small compared to the S_{corr} influence on the output variability. S_{corr}, a numerical parameter depending on the roughness elements geometry by empirical correlations, then has to be carefully defined since its variations can have a large impact on the output value. Fig. 12 shows the Borgonovo indexes for the HAX model and metamodel M_1 evaluating H_c maximum value.

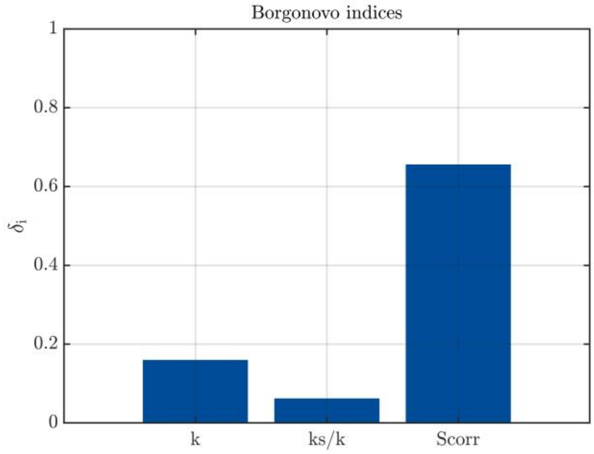

Figure 12. Borgonovo indexes for the HAX model inputs for metamodel M_1

By comparing Fig. 11 and Fig. 12, it is possible to see that, unlike of the 2PP model, the relative influence of each input parameter remains unchanged when travelling downstream. In fact, the values observed on Fig. 11 and Fig. 12 look very similar, meaning that the input parameters have the same level of influence on the output regardless of the position along the flat plate. The other Borgonovo indexes for all the combinations metamodel/thermal correction models are exposed in Tab. 5.

Table 5. Borgonovo indexes for each metamodel/thermal correction model combination

Metamodel	Output of interest Y_i	Borgonovo indexes HAX	Borgonovo indexes 2PP
M_1	H_c max. value	k: 0.1598 k_s/k: 0.0622 S_{corr}: 0.6561	k: 0.4316 k_s/k: 0.1946
M_2	H_c mean value	k: 0.2021 k_s/k: 0.0708 S_{corr}: 0.5967	k: 0.3501 k_s/k: 0.4717
M_3	H_c at x = 8.81 cm	k: 0.1754 k_s/k: 0.0647 S_{corr}: 0.6406	k: 0.3413 k_s/k: 0.4223
M_3	H_c at x = 11.4 cm	k: 0.1825 k_s/k: 0.0659 S_{corr}: 0.6320	k: 0.3463 k_s/k: 0.4664
M_3	H_c at x = 15.3 cm	k: 0.1896 k_s/k: 0.0674 S_{corr}: 0.6199	k: 0.3535 k_s/k: 0.4717

Values in Tab. 5 show that the hierarchy of influence remains the same for every metamodel when the HAX thermal correction is used: S_{corr} is the most influent parameter while the ratio k_s/k is the least influent. For the 2PP correction, the roughness height k is the most influent parameter for metamodel M_1 but the ratio k_s/k becomes predominant for metamodels M_2 and M_3. The most influent parameter of the HAX model being S_{corr}, this model seems to be the most delicate to calibrate since S_{corr} relies on the roughness elements geometry, when k is measurable and k_s has many more documented correlations to be computed [6]. The disadvantage of the HAX model, in the present application, is its strong dependency to a strictly geometrical parameter (S_{corr}). In fact, S_{corr} is a parameter used for general conical or hemispheric roughness elements. Roughness elements induced by atmospheric icing usually don't have these shapes.

This section exposed the main results of the application of the metamodeling procedure followed by the sensitivity analysis. It was showed a good accuracy of the metamodels generated and finally, the Borgonovo indexes allowed classifying the input parameters according to their level of influence on the output of interest.

4. CONCLUSION

The influence of the surface roughness of a flat plate through thermal correction models on the convective heat transfer coefficient has been studied in this paper. More precisely, the objective of performing a sensitivity analysis of the heat transfer coefficient features to the rough input parameters has been achieved. Furthermore, a comparison between two rough thermal correction models has been done, highlighting the different behaviour between both. It has been showed that the polynomial chaos expansion (PCE) metamodeling suits well for the present study. The metamodels generated present very satisfactory regression coefficients greater than 0.99967. This characteristic allows using the metamodels with confidence to perform a Borgonovo sensitivity indexes calculation. Nevertheless, the regression coefficients were better for the 2PP model compared to the HAX model. It highlights the fact that the 2PP model has a more predictable behaviour when using a PCE metamodel. The calculation of the sensitivity indexes showed that the relative influence of the input parameters of the 2PP model depends on the location along the flat plate: the roughness height k plays a preponderant role near the leading edge compared to the ratio k_s/k, but the contrary occurs when travelling downstream. For the HAX model, the hierarchy of the input parameters' influence doesn't depend on the location on the flat plate. The numerical parameter S_{corr} always has the largest influence and the ratio k_s/k the smallest. The fact that the HAX model depends strongly on a parameter which has few documentation, and which is not defined for roughness elements induced by icing is a disadvantage for the icing applications. The better accuracy with the PCE formulation of the 2PP model and the fact that the outputs depend mainly on S_{corr} for the HAX model make the 2PP model more suitable for the current applications illustrated in this paper. Future extensions of this work are to perform the same type of analysis but on the ice accretions characteristics, such as the ice thickness, as output parameters. In parallel, improving the correlations to determine S_{corr} for roughness elements induced by icing will allow refining the thermal behaviour predictions with the HAX model.

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