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A Compressible Liquid Inclusion of Arbitrary Shape in an Isotropic Elastic Matrix

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ABSTRACT

We use Muskhelishvili's complex variable formulation to solve the plane strain problem of a compressible liquid inclusion of arbitrary shape embedded within an infinite isotropic elastic matrix under uniform remote in-plane stresses. First, the exterior of the domain occupied by the liquid inclusion is mapped onto the exterior of a unit circle in the image plane using a mapping function that contains an arbitrary number of terms. With the aid of a modified form of analytic continuation, a set of linear algebraic equations with relatively simple structure is obtained. Once this set of linear algebraic equations is solved, the internal uniform hydrostatic stress field within the liquid inclusion and the elastic field in the matrix (characterized by a pair of analytic functions) are fully determined. We illustrate our theory by deriving a closed form solution for a hypotrochoidal liquid inclusion and comparing our results with those available in the existing literature. In addition, explicit expressions for the internal uniform hydrostatic stress within liquid inclusions with an n-fold axis of symmetry are presented graphically to examine the influence of the number of terms used in the mapping function. Finally, we determine the internal hydrostatic stress for the case of a rectangular liquid inclusion with various aspect ratios.