

The Role of Local Rotational Symmetries and Equivariance in Data-Driven Fluid Mechanics

Ryley McConkey^{1*}, Julia Balla¹, Elyssa Hofgard¹, Jigyasa Nigam¹, Tess Smidt¹

¹Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, United States

*rmcconke@mit.edu

ABSTRACT

The Navier-Stokes equations, which govern fluid flow, possess fundamental symmetries including invariance under rotation, translation, and scaling. Traditional convolutional neural networks (CNNs) naturally encode translational symmetry through their sliding filter operations, but they do not automatically preserve rotational or scaling symmetries - rotated inputs produce different feature maps than the original input. In contrast, equivariant neural networks can be designed to automatically preserve these additional symmetries in their predictions, though often at increased computational cost. This property has shown promise in several physical domains where rotational symmetry plays a key role.

We hypothesize that equivariance could be particularly valuable for fluid mechanics due to the presence of local rotational symmetries in many flows. For instance, turbulent flows contain rotating structures at different scales and orientations, from larger coherent vortices to smaller-scale eddies. These rotational features appear across various flow conditions, though their importance varies by case. An equivariant model might adapt better to these local symmetries than a CNN, potentially improving data efficiency in some scenarios by encoding aspects of the rotational structure of the flow field.

The role of local symmetries in fluid flows raises questions about model design tradeoffs. While global symmetries like translational invariance are well-studied, the practical importance of preserving local rotational symmetries in data-driven modeling is less clear. This may be especially relevant for certain turbulent flows, where rotating structures at multiple scales suggest local symmetries could influence model performance.

We will analyze the presence of local symmetries in several specific turbulent flow configurations, and investigate whether equivariance provides advantages as an inductive bias in these cases. Our study will examine both data efficiency and generalizability, comparing equivariant architectures to standard CNNs across different flow conditions. We will present this empirical investigation for several representative applications of machine learning in fluids, including super resolution, subgrid scale modelling, and turbulence closure modelling via anisotropy mappings. By evaluating these models on a range of cases, we aim to better understand where local symmetry allows symmetry-aware architectures to outperform standard architectures, helping inform the tradeoff between computational cost and potential advantages in specific flow scenarios.