Proceedings of the Canadian Society for Mechanical Engineering International Congress
32nd Annual Conference of the Computational Fluid Dynamics Society of Canada
Canadian Society of Rheology Symposium
CSME-CFDSC-CSR 2025
May 25–28, 2025, Montréal, Québec, Canada

Strong Stability Preserving Runge-Kutta Projection Methods

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Abstract

When solving nonlinear hyperbolic partial differential equations, such as compressible Euler equations, discontinuities may arise even with smooth initial conditions. To ensure convergence to the physically meaningful solution in the presence of discontinuities it is necessary to impose a nonlinear stability condition on the numerical solution. With the method of lines approach, finding the numerical solution starts with a spatial discretization to transform the problem into a system of time-dependent ordinary differential equations. Then, a time integration scheme is used to find the fully discrete solution in space and time. Significant efforts have been devoted in the past to developing high-order spatial discretizations that, when coupled with the first order forward Euler (FE) time integration scheme, are nonlinearly stable. However, combination of high order Runge-Kutta (RK) time integration schemes with these spatial discretizations may not result in a stable solution.

A family of RK schemes, known as strong stability preserving RK (SSP-RK) schemes, can be written as a convex combination of FE step solutions. Therefore, they provably preserve nonlinear stability property of FE scheme. However, SSP-RK schemes have certain limitations. Explicit SSP-RK schemes are at most fourth order accurate, and they are nonlinearly stable only under a time step size restriction, possibly becoming a limitation in terms of efficiency.

A highly efficient technique for modifying unstable RK schemes to make them nonlinearly stable is called projection method. With this method, the RK solution at each time step is corrected along a search direction such that the stability condition is enforced, ideally without reducing the order of accuracy of the base RK scheme. However, projection technique has not yet been developed to make non-SSP schemes stable similar to SSP-RK schemes.

In this work, we propose a strategy to employ projection technique to general non-SSP schemes such that they become nonlinearly stable (SSP) with minimal additional computational cost. Using this approach, we demonstrate that arbitrarily high-order non-SSP schemes become nonlinearly stable while the order of accuracy is preserved. Furthermore, for certain SSP-RK schemes with stringent step size restrictions, this projection technique allows them to remain nonlinearly stable with larger time step sizes.