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Evaluating the Influences of Complementary Corrections/Adjustments on Improving the Accuracy of BEMT in Large-Scale Wind Turbine Power Prediction

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Abstract— Advancements in technology and the increasing global demand for renewable energy have intensified interest in the development of large-scale megawatt wind turbines. Compared to small-scale turbines, these systems significantly improve energy production efficiency and reduce associated costs, making them a key solution for sustainable energy generation. This study evaluates the accuracy of power prediction using the Blade Element Momentum Theory (BEMT) when applied to large-scale wind turbines. To enhance prediction precision, several modifications are applied to the classical BEMT, including rotor tip loss corrections and adjustments for high axial induction factors. The improved model is tested on the NASA MOD-5A horizontal-axis wind turbine. Various correction methods are compared to identify the most effective combinations. Based on the analysis, the Prandtl correction for tip loss and the Peter Smith and Buhl methods for high induction factor adjustments offer the highest accuracy, minimizing error across wind speed ranges. The results confirm that the modified BEMT, when incorporating these specific corrections, provides a reliable and efficient approach for accurate power estimation in large-scale wind turbine design and analysis.

Keywords-wind turbine; blade element momentum theory; power prediction; tip loss correction; induction factor

I. INTRODUCTION

Large-scale megawatt wind turbines improve energy capture and reduce costs through optimized aero-structural designs, particularly in offshore environments with limited space [1]. Accurate power estimation is essential for optimizing performance by enabling precise adjustment of parameters like blade pitch and rotor speed [2].

Blade Element Theory (BET) models wind turbine performance by dividing blades into small sections to calculate local forces based on wind speed and blade geometry. By integrating these forces, overall turbine output is estimated. BET is effective for power prediction and aerodynamic analysis, using variables like angle of attack, lift, and drag coefficients [3,4]. Tip loss corrections, such as Prandtl's factor, improve the accuracy of wind turbine performance predictions by accounting for reduced lift at blade tips due to vortex formation, especially at low tip speed ratios [5]. Okulov and Sørensen [6] reviewed various rotor efficiency models, including the Betz limit, Goldstein's vortex theory, and advanced methods like Theodorsen's and Chattot's models, to further enhance tip loss estimation.

The high induction factor in blade element momentum theory (BEMT) presents significant challenges in accurately predicting aerodynamic performance. When the axial induction factor exceeds 0.5, discrepancies between theoretical predictions and experimental data become pronounced, necessitating the application of correction models to enhance accuracy. This issue is particularly relevant in the design and analysis of wind turbine blades, where innovative approaches are being explored to optimize performance [7]. Laalej, et al. [8] presented a simplified numerical code combining BEMT with 2D Computational Fluid Dynamics (CFD) to improve predictions of wind turbine performance, particularly the aerodynamic loads on rotor blades. Initially, the CFD simulations were conducted using the SST k-ω turbulence model for a 2D airfoil (S809 profile) to obtain aerodynamic parameters, which were then integrated into a classical BEM model via a MATLAB-Ansys interface. Initial comparisons with experimental data revealed discrepancies. To enhance accuracy, the basic BEM was replaced with a modified approach using Jonkman-Buhl stall delay and tip loss corrections. The improved model showed much better agreement with experimental data, effectively capturing the aerodynamic characteristics across different angles of attack, while also reducing computational time compared to full 3D CFD simulations [8]. Oliveira et al. [9] improved BEMT by integrating Prandtl, Shen, and Viterna-Corrigan corrections for tip/root losses, high induction, and post-stall effects. The enhanced model aligns better with experimental data and offers accurate, real-time performance predictions without CFD complexity.

In the current research some modifications are applied to the BEMT by incorporating some corrections for rotor tip losses and performing adjustments for high induction factors to increase the accuracy of the BEMT.

II. METHODOLOGY OF CODE

The theory of BEM in the current work is taken from the Aerodynamics of Wind Turbine book by Martin O. L. Hansen [3]. The vortices induce axial and tangential velocities on the blade, which result in a change in the effective angle of attack. Generally, the induced velocity of the vortices is in the opposite direction of the wind flow and the blade rotation, as expressed in the following equations:

$$V_a=(1-a)V_0,\ V_r=(1+a')r\omega \quad (1)$$

In the above equations, V_0 is the wind speed (m/s), a is the axial induction factor, V_a is the axial velocity (m/s), r is the distance from the root of the wind turbine blade (m), ω is the rotational speed of the turbine (rpm), a' is the tangential induction factor, and V_r is the tangential velocity (m/s). The induced velocities result in a change in the effective angle of attack, which, in turn, generates different aerodynamic forces. The velocity triangle for a section of the wind turbine blade and the effect of the induction factors are shown in the figure below. Based on this figure, the relative velocity can be expressed in terms of its velocity components according to the following equations:

$$V_{rel}\sin(\phi)=V_0(1-a) \quad (2)$$
$$V_{rel}\cos(\phi)=r\omega(1+a') \quad (3)$$

In these two equations, ϕ is the aerodynamic angle between the axial and tangential induced velocities, and V_{rel} is the relative velocity (m/s). A schematic of induced velocity triangle for a section of a wind turbine blade is shown in Figure 1.

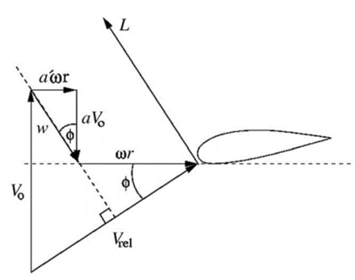


Figure 1. Induced velocity triangle for a section of a wind turbine blade.

The normal and Tangential forces (P_N and P_T) are shown in the following equations. In these equations C_N and C_T are the coefficients of the normal and tangential forces on the wind turbine blade, respectively. These coefficients are resulted by the division of forces using the term $0.5\rho V_{rel}^2 c$ according to the following equations:

$$P_N=L\cos(\phi)+D\sin(\phi)\rightarrow C_N=C_l\cos(\phi)+C_d\sin(\phi) \quad (4)$$
$$P_T=L\sin(\phi)-D\cos(\phi)\rightarrow C_T=C_l\sin(\phi)-C_d\cos(\phi) \quad (5)$$
$$C_N=P_N/0.5\rho V_{rel}^2 c,\ C_T=P_T/0.5\rho V_{rel}^2 c \quad (6)$$

The normal force and moment for each blade element are expressed differentially in the following equations.

$$dT=BP_N dr,\ \ dM=BP_T dr \quad (7)$$

In the two equations above, the parameters dT and dM represent the differential elements of thrust force (perpendicular to the blade) and the moment of each element, respectively. Additionally, B represents the number of turbines blades, which is equal to 3. Finally, the differential normal force can be obtained as follows.

$$dT=0.5\rho B\left(V_0^2(1-a)^2\right)\left(\sin^2(\phi)\right)^{-1} cC_n dr \quad (8)$$

Similarly, the moment generated on the blade section can be calculated differentially as follows.

$$dM=0.5\rho B\left(V_0(1-a)\right)\omega r(1+a')\left(\sin(\phi)\cos(\phi)\right)^{-1} cC_t dr \quad (9)$$

By simplifying and combining the above-mentioned equations, the axial and tangential induction factors can be obtained in terms of the aerodynamic angle of attack and the coefficients of axial and tangential forces.

$$a=1/\left(\left(4\sin^2(\phi)(\sigma C_N)^{-1}\right)+1\right) \quad (10)$$
$$a'=1/\left(\left(4\sin(\phi)\cos(\phi)(\sigma C_T)^{-1}\right)-1\right) \quad (11)$$

$\sigma(r)$ represents the solidity of the blade, indicating the fraction of the area covered by each section of the blade, and it is calculated as follows:

$$\sigma(r)=c(r)B/2\pi r \quad (12)$$

Also, for better expression of this algorithm, the flowchart of BEMT algorithm is shown in Figure 2.

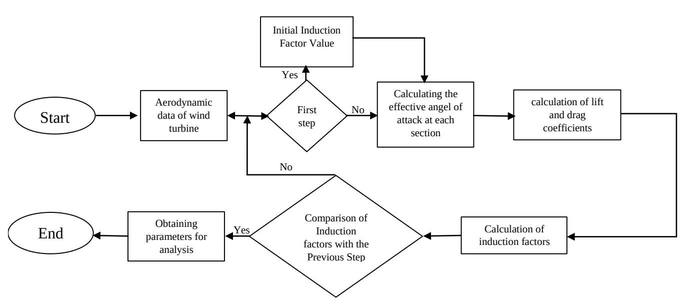


Figure 2. Flowchart of Blade element theory

III. DIFFERENT ADJUSTMENTS ON BEM

In the next sections different corrections and correlations which are used to enhance and adjust the original BEMT are described.

A. *Tip Loss Factor Correction*

The classical BEMT theory does not consider tip and hub losses, but these factors are important for analyzing turbine performance as they directly affect torque calculation. The classical method assumes that there is a sufficient number of blades for all fluid particles passing through the rotor to interact with the blades. However, with fewer blades (modern turbines are usually made with three blades), some fluid particles will interact with the blades, but most will pass between the blades, and the loss of momentum of a particle will depend on its proximity to a blade. Tip losses on the blade result from the impact of vortices generated along the blade, which reduce circulation and consequently decrease aerodynamic efficiency. Therefore, the axial-induced velocity around the blade will vary. This directly affects the torque that the blade is subjected to. Practically, the loss coefficient is always less than one and affects the power performance predictions by about 5 to 10 percent in calculations. To account for a limited number of blades in BEMT theory, a tip loss factor (F) is usually required [8].

a) Prandtl Method : The Prandtl coefficient corrects this efficiency reduction using equations that consider the effects of vortices created at the blade tips. The vortices at the blade tips reduce the airflow near the blade tips, resulting in decreased aerodynamic efficiency. The Prandtl correction factor calculates these effects and adds them to the model to achieve better results. The Prandtl correction factor for the tip (F_{tip}) and hub (F_{hub}) regions of the blade is calculated using the following equations [4] where r is the rotor radius, r_{hub} is the rotor root radius, and B is the number of blades. The total loss coefficient F_{loss}, shown in the last equation, takes into account the product of the losses at the tip and root of the blade.

$$F_{tip}=(2/\pi)\cos^{-1}\left(\exp(-0.5B(R-r)/(r\sin(\varphi))\right) \tag{13}$$

$$F_{hub}=(2/\pi)\cos^{-1}\left(\exp(-0.5B(r-r_{hub})/(r_{hub}\sin(\varphi))\right) \tag{14}$$

$$F_{loss}=F_{loss}F_{tip} \tag{15}$$

b) Effective Radius Methos: The simplest empirical method is the effective radius method, which can be calculated using the following equations. Vortices in the blade tip region have a significant impact on rotor torque and thrust force compared to the hub region. Based on this knowledge, the tip losses can be considered by defining an effective rotor radius, which is approximately 3% less than the actual tip radius [10].

$$F_{loss}=1 \quad 0<r<r_e\ ,\ F_{loss}=0 \quad r_e<r<R, r_e=0.97R \tag{16}$$

c) Betz Method M1 and Betz M2 Method: Prandtl, along with Betz, proposed several other expressions known as Betz M1 and Betz M2. In this model, Prandtl and Betz assumed a limited number of blades and considered the vortex theory in the wake flow. The difference between these methods is that in the Betz equation M1, the velocity component in the rotor plane is calculated precisely, whereas in the Betz equation M2, it is calculated approximately [11].

$$F_{loss}=\frac{2}{\pi}\operatorname{acos}\left[\exp\left(-\frac{B}{2}\left(1-\frac{\lambda_r}{\lambda}\right)\sqrt{1+\lambda^2\left(\frac{1+a'(R)}{1-a(R)}\right)^2}\right)\right] \tag{17}$$

$$F_{loss}=\frac{2}{\pi}\operatorname{acos}\left[\exp\left(-\frac{B}{2}\left(1-\frac{\lambda_r}{\lambda}\right)\sqrt{1+\lambda^2\left(\frac{\lambda}{1-a(R)}\right)^2}\right)\right] \tag{18}$$

d) Burton Method: Burton proposed a method shown in the equation below, which is similar to the Betz M1 method. The results obtained with these methods are overestimated for any range of turbine power [12].

$$F_{loss}=\frac{2}{\pi}\operatorname{acos}\left[\exp\left(-\frac{B}{2}\left(\frac{\lambda}{\lambda_r}-1\right)\sqrt{1+\left(\frac{\lambda_r}{1-a}\right)^2}\right)\right] \tag{19}$$

B. *High Induction Factor Correction*

High Induction Factor in Blade Element Momentum (BEM) theory refers to conditions where the airflow through the rotor disk of a wind turbine decreases at a higher rate than what is assumed under normal conditions. This situation occurs when the wind turbine is operating under high load conditions and a significant amount of the kinetic energy of the air is absorbed by the blades. In such conditions, the standard BEM model, due to its simplified assumptions, cannot accurately model complex flows and nonlinear effects in the high induction region. To address this issue and improve the accuracy of BEM predictions under high induction conditions, specific corrective methods have been developed. These methods generally aim to take into account the more complex and nonlinear effects that occur under high induction conditions [8].

a) Glauert Method: Methods for refining the classical BEMT theory for high axial induction factor values are usually based on relationships derived from studies conducted by Glauert. Glauert's empirical relationship states that under turbulent flow conditions (a > 0.5), the thrust coefficient (C_T) increases up to 2.0 for a = 1. The BEMT theory fails at values of a > 1/3, and in this region, the induction factor correction is important. The proposed correction by Glauert is also applied for a > 0.4 or C_T> 0.96. Glaert fitted a parabola to some data related to rotors operating in turbulent flow to obtain these equations. This polynomial fits well with the classical momentum curve up to a = 0.4. However, a numerical problem in the classical momentum equation occurs when the loss correction factor is applied, resulting in a discontinuity between the theoretical curve and the correction method.

$$C_T=\sigma'(1-a)^2C_n/\sin^2(\phi) \tag{20}$$

$$a=(1/F_{Loss})\left(0.143+\sqrt{0.0203\pm0.6427(0.889-C_T)}\right) \tag{21}$$

b) Peter Smith Method: Peter Smith presents a model for a > 0.5. Which can be calculated using the following equations. The development of this model is based on applications with low wind speeds where the value of a is large [13].

$$a=1-\sqrt{f_t/F_t} \tag{22}$$

$$f_t=1/\left(0.11F_t^{-3}-0.7F_t^{-2}+2.15F_t^{-1}+2.15\right) \tag{23}$$

$$F_t=a/(1-a) \tag{24}$$

c) Robbert Wilson Method: Robert Wilson presents a model for a > 0.4 that can be calculated using the following equations. In low-trust areas, the results can show significant errors [14].

$$S=\sigma' C_l \cos(\phi)/8\sin^2(\phi) \quad (25)$$

$$a=2S+F_{loss}-\sqrt{F_{loss}^2+4SF_{loss}(1-F_{loss})}/2\left(S+F_{loss}^2\right) \quad (26)$$

d) Spera Method: Robert Wilson, along with Spera, also presents another modeling method for $a > a_c$ where $a_c = 0.2$, which can be calculated using the following equations. This correction involves the simple use of a straight line that is tangent to the thrust calculated through momentum theory at a critical point [10].

$$K=4F_{loss}\sin^2(\phi)/\sigma' C_n \quad (27)$$

$$a=0.5\left(2+K(1-2a_c)-\sqrt{(K(1-2a_c)+2)^2+4(Ka_c^2-1)}\right) \quad (28)$$

$$C_T=4F_{loss}(a_c^2+(1-2a_c)a) \quad (29)$$

e) Buhl Method: Marshall Buhl presents a model for a > 0.4 (equivalent to $C_T > 0.96$) that can be calculated using the following equations. This method is developed based on fitting a parabola to some information related to rotors operating in turbulent flow and is based on deriving an equation that solves the discontinuity problem described in Glover's proposed method [15].

$$C_T=8/9+(4F_{loss}-40/9)a+(50/9-4F_{loss})a^2 \quad (30)$$

$$a=18F_{loss}-20-3\sqrt{C_T(50-36F_{loss})+12F_{loss}(3F_{loss}-4)}/(36F_{loss}-50) \quad (31)$$

f) Madsen Method: Madsen has described a method for $C_T < C$ and $C_T > C$ that can be implemented respectively using the following equations [16].

$$a=k_0+k_1C_T+k_2C_T^2+k_3C_T^3 \quad (32)$$

$$a=\left(k_1+2Ck_2+3Ck_3^2\right)(C_T-C)+k_0+2.5k_1C+k_2C^2+k_3C^3 \quad (33)$$

$$k_0=-0.001701, k_1=0.251163, k_2=0.054496, k_3=0.089207 \quad (34)$$

Here, C=2.5 and the other coefficients k have been provided. In this method, the coefficients must be adjusted individually for each turbine, regardless of its nominal power. This issue makes this model non-generalizable.

g) Shen Method: Shen and colleagues have described a method with two intermediate functions, Y_1 and Y_2, which are presented in the following equations. In this method, $Y_1 \geq 2$represents an area with low trust. This condition also holds for a > 1/3 and is used to calculate the induction coefficients, as shown in the equations below. The axial induction factor is calculated as a function of the radial position and takes into account the vortices along the blade. The results show that this method predicts aerodynamic forces near the blade tip better and leads to more accurate power curves. This method was more efficient in predicting the power performance of small turbines compared to the other methods presented in this article [16].

$$Y_1=4F_{loss}\sin^2(\phi)/(\sigma' C_n) \quad (35)$$

$$Y_2=4F_{loss}\sin(\phi)\cos(\phi)/(\sigma' C_t) \quad (36)$$

$$a=2+Y_1-\sqrt{4Y_1(1-F)+Y_1^2}/\left(2(1+FY_1)\right) \quad (37)$$

$$a'=(1-a)-1/((1-aF)Y_2) \quad (38)$$

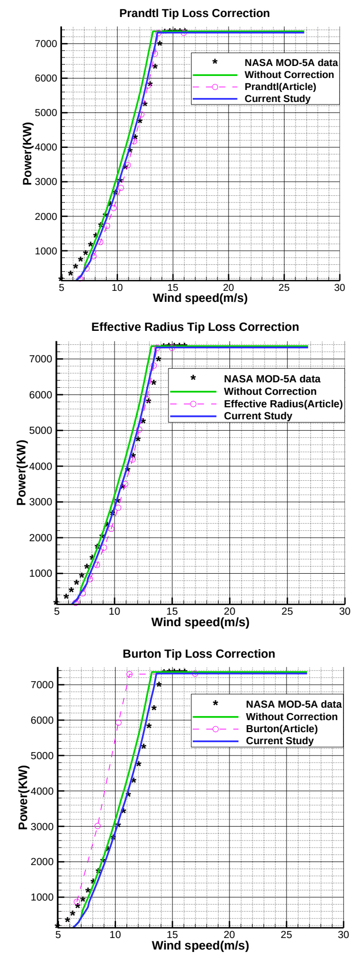
IV. Result and Discussion

In this section the effect of different corrections on the original BEMT is investigated.

A. Tip Loss Correction Results for NASA MOD-5A Wind Turbine

Various tip loss correction methods are compared for the NASA MOD-5A wind turbine to evaluate their performance under its specific large-scale conditions. Figure 3 and Table 1 present power outputs and a comparative analysis of these methods.

Based on chart analysis and error rates, the Prandtl method shows the highest accuracy in predicting the NASA MOD-5A turbine's performance, with minimal deviation across wind speeds.



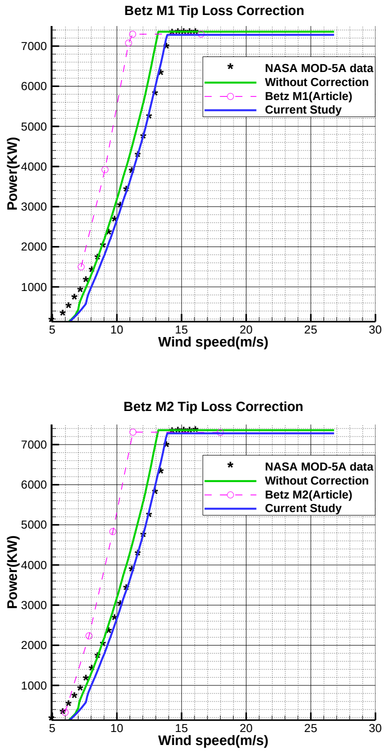
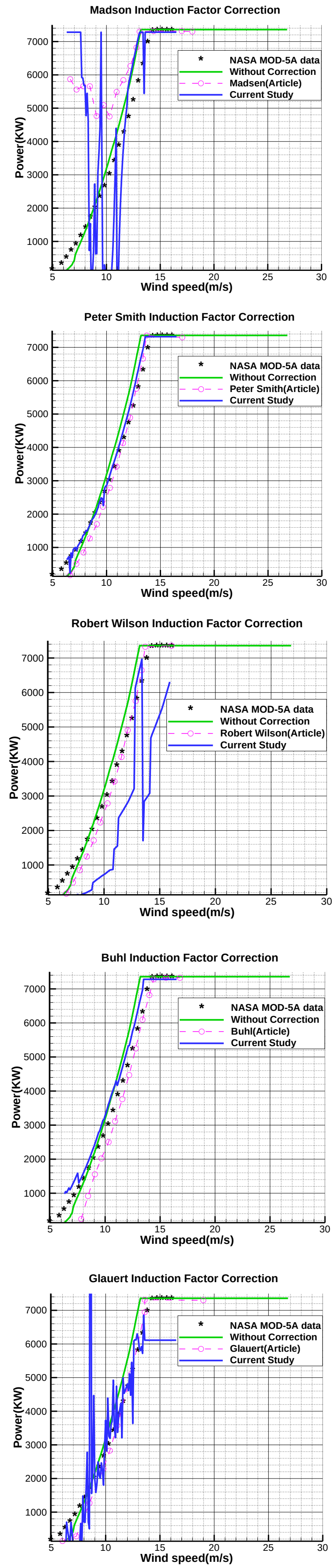


Figure 3. Output power charts considering different methods of blade tip loss correction for NASA MOD-5A wind turbine.

Table 1. Comparison of different tip loss correction methods for NASA MOD-5A wind turbine.

Current Method	Low Wind Speed Error (%)	High Wind Speed Error (%)	Suggested
Prandtl	5	2	Yes
Burton	10	20	No
Effective Radius	5	10	Yes
Betz M	15	25	No

In the investigation of the NASA MOD-5A wind turbine using various methods to correct the High Induction Factor, the primary goal is to examine the effects of strong induction flows on the aerodynamic performance of the turbine blades. High Induction Factor specifically occurs when the wind flows entering the blades significantly decrease, which can lead to reduced efficiency and even instability in the turbine's performance. In this study, various methods were employed to correct the High Induction Factor to improve modeling accuracy. The results of these corrections showed that each method exhibits different performance depending on the specific conditions of the turbine and its design features. Correcting the High Induction Factor is particularly important for the NASA MOD-5A turbine, which has large dimensions and a complex design. Figure 4 shows the output power of the current wind turbine in different wind speed conditions considering various high induction factor correction methods. Also, Table 2 shows a Comparison of different high induction factor correction methods for NASA MOD-5A.



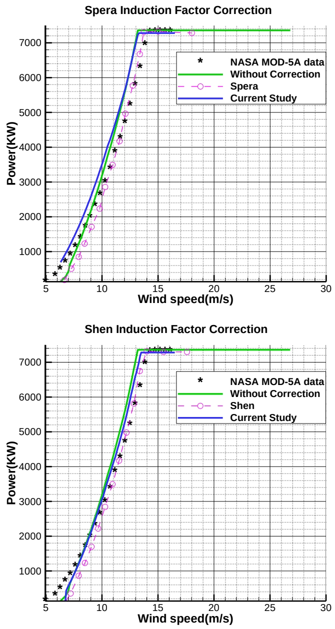


Figure 4Output power charts considering various high induction factor correction methods for NASA MOD-5A wind turbine.

Table 2Comparison of different high induction factor correction methods for NASA MOD-5A.

Current Method	Low Wind Speed Error (%)	High Wind Speed Error (%)	Suggested
Madsen	20	5	No
Robert Wilson	15	10	No
Glauert	25	10	No
Peter Smith	5	3	Yes
Buhl	5	5	Yes
Spera	5	10	No
Shen	5	7	No

V. CUNCLUSION

This study examined various methods to accurately improve the classical BEMT power prediction by incorporating it with suitable tip loss corrections and high induction factor correction. The improved BEMT was then examined by focusing on the NASA MOD-5A wind turbine, i.e., a large-scale turbine with complex design characteristics. The analysis highlighted the strengths and limitations of each method under specific operational conditions, providing valuable insights into their accuracy and effectiveness. For the tip loss correction, the Prandtl method was identified as the most accurate one, which demonstrated minimal discrepancy if compared with the actual turbine data across the most wind speed ranges. Similarly, in addressing the high induction factor, the Peter Smith and Buhl methods were shown to be the most reliable ones, offering high accuracy and better agreement with the turbine's performance data. These findings emphasize that the importance of selecting correction methods tailored to the specific design and operational characteristics of wind turbines, particularly for large and complex systems like the NASA MOD-5A. By identifying the most effective correction methods, this research contributes to improving the precision of aerodynamic performance modeling, ultimately enhancing the efficiency and stability of wind turbines under varying conditions.

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