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Plant Companion Lighting System To Enhance Energy Efficient Agriculture in Remote Regions

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***Abstract*— This study investigates an energy-efficient Plant Companion LED System tailored for crops with low Daily Light Integral (DLI) requirements, such as lettuce and peas. Focusing on vertical farming in northern Canada, this research aims to optimize LED lighting by maintaining consistent light-to-crop distances, adjusting light intensity based on growth stages, and minimizing light loss using reflectors. The experimental results are expected to show improved energy efficiency and uniform crop growth, helping to mitigate food insecurity in remote regions.

Keywords-component; Controlled Environment Agriculture; LED Optimization; Vertical Farm; Energy Efficiency; Food Security

I. INTRODUCTION

Northern Canada faces significant challenges in ensuring food security due to its harsh climate, limited arable land, and geographical isolation [1]. These factors lead to high dependency on costly and infrequent food shipments, exacerbating food insecurity [1, 2]. Controlled Environment Agriculture (CEA) offers a sustainable solution by enabling year-round crop production in remote areas [3]. Among CEA methods, vertical farms (VFs) are particularly promising due to relative independence from external climatic conditions. However, the reliance on artificial lighting, accounting for over 60% of energy consumption in VFs, poses a major obstacle [4, 5]. This study introduces the Plant Companion LED System to optimize energy use by dynamically adjusting lighting parameters, addressing these energy challenges while ensuring crop productivity in northern communities.

II. BACKGROUND

A. LED Lighting for Agriculture

LED lighting systems have revolutionized Controlled Environment Agriculture (CEA) by offering highly customizable and energy-efficient solutions [6]. Blue light (400–500 nm) strengthens stems and leaves, while red light (600–700 nm) enhances photosynthesis and flowering [6-8]. Combining these wavelengths supports different growth stages, while white light mimics natural sunlight for holistic development [9, 10]. Despite their efficiency, current LED setups face challenges such as uneven light distribution, high energy consumption, and fixed positioning, which limit productivity [11, 12].

B. Inverse Square Law and Light Efficiency

Light intensity diminishes exponentially with distance, as described by the Inverse Square Law *[13]*. This principle highlights the need for optimal LED placement to maximize light efficiency [14]. Advanced systems address this by dynamically adjusting light intensity and distance to maintain consistent Photosynthetic Photon Flux Density (PPFD) levels across growth stages. Monitoring the Daily Light Integral (DLI), which measures the total light a plant receives daily, is also crucial to optimizing crop growth.

C. Challenges in Current Vertical Farms

Vertical farms (VFs) rely on artificial lighting, which accounts for over 60% of their energy consumption [4, 5]. The fixed positioning of lights and indiscriminate light distribution lead to significant energy waste. Early-stage plants receive less light compared to mature ones closer to the source, resulting in uneven growth. These inefficiencies necessitate adaptive lighting systems capable of optimizing light intensity, distribution, and distance.

D. Research Objectives

This research aims to develop an energy-efficient Plant Companion LED system tailored for low-DLI crops such as lettuce and peas. The proposed system will dynamically adjust light parameters to minimize energy usage while supporting consistent crop growth. By addressing energy challenges in VFs, this study seeks to enhance food security in energy-constrained regions like northern Canada.

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III. METHODOLOGY

The proposed system integrates sensors, servo motors, and reflectors to adjust LED height, angle, and intensity dynamically. This setup aims to reduce wasted energy by maintaining a controlled distance between the light source and the crop throughout the growth stage and by focusing the direction of the LEDs towards the crop.

A. Experimental Setup

1) Plant Companion LED System

This study utilizes the Arduino UNO as the primary automation component to monitor and control the distance between the crop and the LED lights. Considering the challenges of soil-based agriculture in harsh remote regions, the proposed system incorporates a hydroponic setup. To measure the distance between the crops and the LEDs, an ultrasonic distance sensor is employed.

A vertically driven system is developed using a compliance mechanism, a linear motion system, and motors. Unlike traditional flat LED panels, the proposed system addresses the issue of light loss from the edges of the panels by integrating a hinge mechanism, allowing angular adjustment of the edge LEDs. This ensures the edge LEDs focus their light more effectively on the crops. Furthermore, a dimmer is implemented to regulate light intensity, enabling independent control of the central panel LEDs and the edge panel LEDs.

2) Arduino UNO

The Arduino UNO serves as the central microcontroller for the automated control system [15]. Its versatility and ease of programming make it ideal for managing multiple inputs and outputs necessary for this experiment. The Arduino UNO processes data from the ultrasonic sensor to dynamically adjust the LED panel height, ensuring optimal light exposure for crop growth. Additionally, it controls the dimmer circuits, allowing precise regulation of light intensity based on crop requirements and growth stages.

3) Ultrasonic sensor

An ultrasonic distance sensor was utilized to accurately measure the distance between the LED panels and the crop canopy. Operating at a frequency range typically between 40 kHz and 50 kHz, this sensor provided precise, non-contact measurements crucial for the vertical adjustment mechanism [16]. By continuously monitoring the growth height of crops, the sensor data enabled dynamic adjustments of the LED panels, maintaining an optimal light distance and maximizing photosynthetic efficiency. The integration of the ultrasonic sensor with the Arduino UNO allowed for seamless data processing and real-time control.

4) Li-Cor Light Sensor and Illumination Sensor

To quantify photosynthetic photon flux density (PPFD), a Li-Cor light sensor (model LI-190R Quantum Sensor) was initially used to determine the optimal lighting recipe under varying conditions, such as the distance between the canopy and crops, LED panel angles, and the number of reflectors [17]. The Li-Cor sensor provided high-resolution data on photosynthetically active radiation (PAR) levels, ensuring precise monitoring and analysis of light distribution and energy efficiency. However, during the actual experiment, an illumination sensor was used instead to measure light intensity at the crop level. This sensor transmitted data to the Arduino UNO, allowing real-time adjustments to LED intensity and angles, ensuring optimal lighting conditions and energy savings.

5) Response Surface Methodology (RSM) and Central Composite Design (CCD)

To optimize the lighting parameters, Response Surface Methodology (RSM) is employed. RSM is a statistical technique used for modeling and analyzing problems where multiple variables influence a response variable [18]. In this study, it is used to determine the optimal LED height, angle, and intensity to maximize energy efficiency and crop growth. A Central Composite Design (CCD) is implemented to systematically vary these parameters and develop a predictive model [19]. CCD provides a robust experimental design by incorporating factorial points, axial points, and center points, allowing the response surface to be estimated with high accuracy.

The motivation for using RSM and CCD in this study is to identify the optimal conditions for maintaining a PPFD range of 170-190 μmol/m²/s while minimizing energy consumption. The below are selected factors.

i. LED height (cm) – Adjusted dynamically based on crop growth.
ii. LED angle (degrees) – Optimized to minimize light loss.
iii. Light intensity (%) – Controlled using a dimmer to adjust the intensity of light that exceeds the distance to be approached.
iv. Reflectors – Used to enhance light distribution and efficiency.

A total of 31 CCD tests will be conducted to evaluate these factors systematically. The collected data will be analyzed using RSM to determine the optimal parameters. Following the statistical analysis, these optimal parameters will be applied to compare Butterhead Lettuce growth between the Plant Companion LED System and a conventional fixed LED system. The comparative analysis will include metrics such as fresh weight, dry weight, color, and texture to assess crop performance under optimized lighting conditions.

TABLE I. CENTRAL COMPOSITE DESIGN PARAMETERS

Factor	Type	Levels	Values
Distance	Fixed	2	50, 100, 150
LED Intensity	Fixed	2	10, 55, 100
LED Angle	Fixed	2	15, 30, 45
Reflectors	Fixed	2	0, 2, 4

IV. RESULTS AND DISCUSSION

A. Analysis of Variance (ANOVA) for PPFD

The analysis of variance (ANOVA) results for PPFD, based on the general linear model, provide insight into the significance of each experimental factor [20]. The factors analyzed include distance, LED intensity, LED angle, and reflectors. The ANOVA results indicate that distance (P-value = 0.003) and LED intensity (P-value = 0.000) are the two most significant factors influencing PPFD, as their P-values are well below the 0.05 significance threshold. This confirms that both parameters have a statistically significant impact on PPFD.

In contrast, LED angle (P-value = 0.297) and reflectors (P-value = 0.780) do not show statistically significant effects, suggesting that their influence on PPFD is relatively minor in the tested conditions. The high R-squared value (94.39%) and adjusted R-squared value (92.35%) indicate that the model explains most of the variability in PPFD, reinforcing the reliability of the findings.

These results align with theoretical expectations. The inverse square law explains the negative effect of increasing distance on PPFD, while higher LED intensity results in increased photon flux reaching the target area. Although LED angle and reflectors do not exhibit significant direct effects, they may still serve as auxiliary parameters for optimizing energy efficiency.

TABLE II. ANALYSIS OF VARIANCE (ANOVA) FOR PPFD USING ADJUSTED SS FOR TEST

Source	DF	Seq/Adj SS	Adj MS	F	P
Distance	1	201,341	201,341	14.01	0.003
LED Intensity	1	2,439,657	2,439,657	169.79	0.000
LED Angle	1	17,253	17,253	1.20	0.297
Reflectors	1	1,180	1,180	0.08	0.780
Error	11	158,055	14,369	-	-
Total	15	2,817,485	-	-	-

B. Main Effect Analysis

The main effect plot for photosynthetic photon flux density (PPFD) illustrates clear trends based on experimental factors. As the distance between the light source and the measurement point increased, the PPFD exhibited a decreasing tendency. Conversely, when LED intensity was increased from 10 to 100, the PPFD measured at the target area significantly increased. Regarding LED angle, an increase from 15 degrees to 45 degrees resulted in a slight increase in PPFD. Similarly, as the number of reflectors increased from 0 to 4, a marginal improvement in PPFD was observed.

The decrease in PPFD with increasing distance can be attributed to the inverse square law, which states that light intensity diminishes proportionally to the square of the distance from the source. This theoretical expectation aligns well with the observed experimental results, confirming the fundamental optical principle governing light dispersion.

The increase in PPFD with higher LED intensity is a direct consequence of greater light energy being emitted by the LED source, thereby increasing the number of photons reaching the target area. This relationship is intuitive, as a higher light intensity directly corresponds to a greater PPFD value, given that all other conditions remain constant.

Although LED angle was initially hypothesized to have a notable impact on PPFD, the experimental results indicate a relatively minor influence. Preliminary tests suggested that changes in LED angle could lead to an increase of up to 25.9% in PPFD. However, in the present study, the effect was observed to be marginal. Nonetheless, adjusting the LED angle can still serve as an auxiliary mechanism to enhance energy efficiency and optimize light distribution.

Similarly, the effect of reflectors on PPFD was minimal, as confirmed by statistical analysis. While reflectors helped marginally improve PPFD by redirecting light that would otherwise be lost, their impact was not statistically significant. However, despite their limited effect, reflectors can still play a supporting role in improving light efficiency, particularly in enclosed agricultural environments.

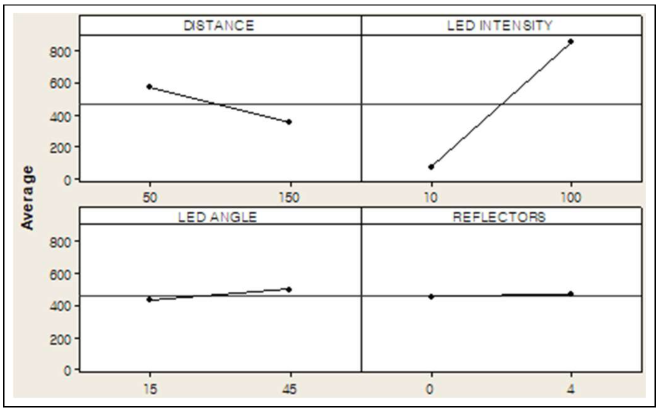


Figure 1. Main effect plot for PPFD

C. Interaction Effect Analysis

The PPFD interaction plot provides insights into how the experimental factors interact to influence light distribution. The most pronounced interaction was observed between distance and LED intensity, which exhibited a strong combined effect on PPFD. The ANOVA results further substantiate this finding, as the interaction term between distance and LED intensity had a P-value of 0.000, confirming statistical significance.

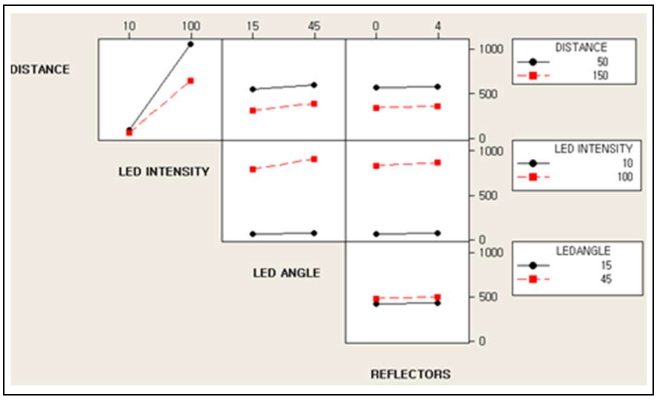


Figure 2. PPFD Interaction plot (Average)

Beyond the primary interaction, a notable effect was also observed between LED intensity and LED angle. The corresponding P-value of 0.048 suggests a significant interaction, indicating that the influence of LED intensity on PPFD is slightly modified when the LED angle is adjusted. However, other interaction terms, including those involving reflectors, did not yield statistically significant effects.

D. Regression Model Results

The quadratic regression model was applied to examine the relationship between PPFD and the key experimental factors, considering both linear and nonlinear effects. The regression analysis indicates that distance and LED intensity are the primary factors influencing PPFD, both showing statistically significant linear effects (P-value = 0.000). The negative coefficient for distance (-110.108) suggests a strong inverse relationship with PPFD, which aligns with the inverse square law of light intensity, where light diminishes as the distance increases. Conversely, the positive coefficient for LED intensity (380.522) confirms that increasing the light intensity leads to a substantial increase in PPFD.

TABLE III. ANALYSIS OF VARIANCE (ANOVA) FOR PPFD USING ADJUSTED SS FOR TEST

Term	Coefficient	SE Coeff	T-Value	P-Value
Constant	429.83	9.912	43.363	**0.000**
Distance	-110.108	7.876	-13.98	**0.000**
Intensity	380.522	7.876	48.315	**0.000**
LED Angle	34.067	7.876	4.325	**0.001**
Reflectors	8.444	7.876	1.072	0.300
Distance^2	17.002	20.742	0.82	0.424
LED Int.^{2}	-44.368	20.742	-2.139	**0.048**
LED Angle^2	30.352	20.742	1.463	0.163
Reflectors^2	28.152	20.742	1.357	0.194
Dis. × Int.	-95.397	8.354	-11.42	**0.000**
Dis. × Angle	6.895	8.354	0.825	0.421
Dis. × Ref.	1.705	8.354	0.204	0.841
Int. × Angle	25.687	8.354	3.075	**0.007**
Int. × Ref.	7.262	8.354	0.869	0.397
Angle × Ref.	1.69	8.354	0.202	0.842

A significant interaction effect was observed between distance and LED intensity (-95.397, P-value = 0.000). This result suggests that increasing both factors simultaneously does not yield a purely additive effect but rather a diminishing return, meaning that at greater distances, increasing LED intensity alone is insufficient to compensate for light loss.

The quadratic term for LED intensity (-44.368, P-value = 0.048) is statistically significant, indicating that the relationship between LED intensity and PPFD is not strictly linear. This result suggests that beyond a certain threshold, increasing LED intensity does not proportionally enhance PPFD, implying an optimal intensity level for maximizing efficiency. On the other hand, the quadratic term for distance (P-value = 0.424) is not statistically significant, suggesting that within the tested range, the impact of distance on PPFD remains largely linear.

E. Model Fit Assessment

The accuracy and reliability of the quadratic regression model were assessed using three key statistical metrics: the standard error of the regression (S), the coefficient of determination (R^2), and the adjusted R^2. These indicators provide insight into the model's ability to explain variability in PPFD and its overall predictive power.

The quadratic regression model for PPFD can be expressed as the following equation:

$$PPFD = 429.83 - 110.108(D) + 380.522(I) + 34.067(A) + 8.444(R) + 17.002(D^2) - 44.368(I^2) + 30.352(A^2) + 28.152(R^2) - 95.397(DI) + 6.895(DA) + 1.705(DR) + 25.687(IA) + 7.262(IR) + 1.690(AR) \quad (1)$$

Where D represents Distance, I represents LED Intensity, A represents LED Angle, and R represents Reflectors.

The standard error of the regression (S = 33.41) represents the average deviation between the observed PPFD values, and the predicted values generated by the model. A lower S value suggests that the model's predictions closely match the actual data, indicating a strong fit. Given that the standard error in this model is relatively small, the regression model is deemed to have high precision in predicting PPFD values.

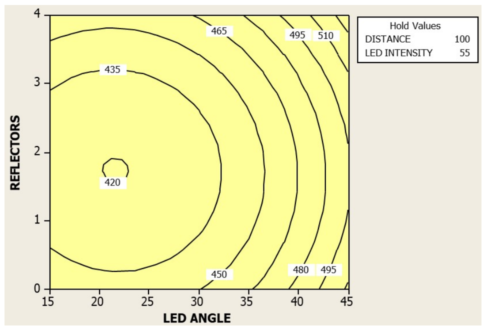
The coefficient of determination ($R^2 = 99.4\%$) quantifies the proportion of variance in PPFD that is explained by the independent variables included in the model. An R^2 value close to 1 suggests that the model captures nearly all the variability in the dependent variable. In this case, an R^2 of 99.4% indicates that the model effectively accounts for almost all fluctuations in PPFD, demonstrating a strong correlation between the experimental factors and the PPFD response.

The adjusted R^2 (98.9%) provides a refined measure of model fit by adjusting for the number of predictors. Unlike R^2, which can increase with the addition of more variables regardless of their relevance, the adjusted R^2 compensates for the degrees of freedom, preventing overestimation of model performance. The high adjusted R^2 value confirms that the inclusion of the quadratic and interaction terms enhances the explanatory power of the model without introducing unnecessary complexity.

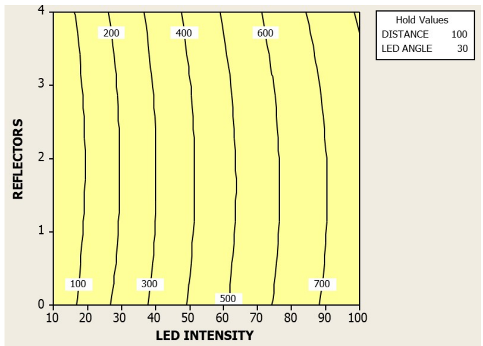
F. Contour plot analysis for PPFD

The contour plots generated using the quadratic regression model provide a graphical representation of how PPFD is influenced by different combinations of experimental factors. These plots allow for a detailed examination of the interaction effects between distance, LED intensity, LED angle, and reflectors, offering a visual interpretation of the relationships among these variables.

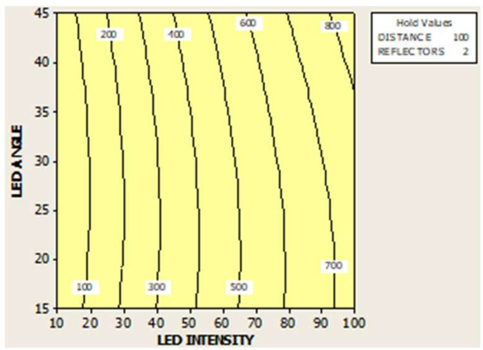
(a) Reflector * Angle



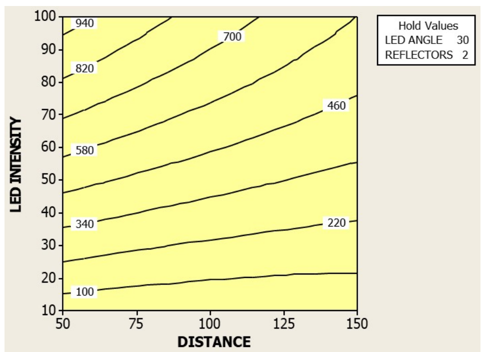
(b) Reflector * Intensity



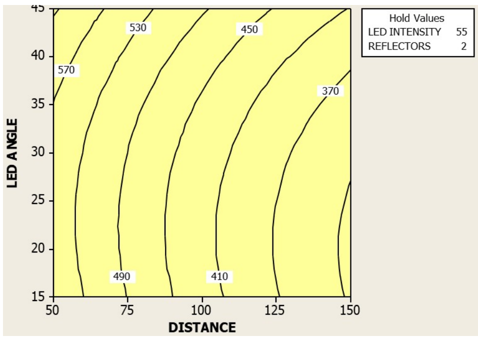
(c) Reflector * Intensity



(d) Intensity * Distance



(e) Angle * Distance



(f) Angle * Reflector

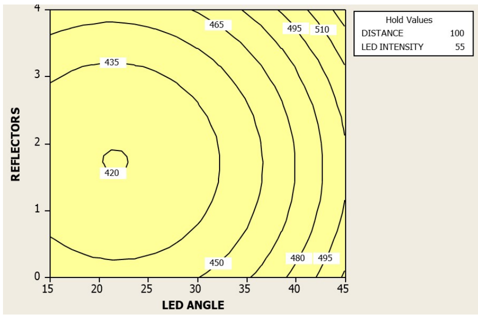


Figure 3. Contour Plot for PPFD

Similarly, the interaction between LED intensity and LED angle demonstrates that PPFD fluctuations are primarily driven by changes in LED intensity, while variations in LED angle exert a less pronounced effect. This suggests that increasing LED intensity is a more effective strategy for optimizing PPFD than modifying the LED angle alone. Additionally, the contour plots examining LED intensity and distance indicate a significant interaction effect. The results reveal that PPFD decreases more rapidly with increasing distance, particularly when LED intensity is low. However, at shorter distances, increasing LED intensity leads to a substantial rise in PPFD, whereas at longer distances, the impact of LED intensity is diminished. This highlights the importance of jointly optimizing both LED intensity and distance to achieve efficient light distribution.

The relationship between LED angle and distance follows a consistent trend, with PPFD decreasing as distance increases, irrespective of the LED angle setting. This suggests that while LED angle adjustments may have some influence on PPFD, distance remains the primary determinant of light distribution. Lastly, the interaction between LED angle and reflectors exhibits a pattern in which PPFD increases and decreases in tandem with changes in both variables. This indicates that while these factors contribute to PPFD variations, their combined effect is secondary compared to the influence of LED intensity and distance.

V. CONCLUSION SAND NEXT STEPS

A. Composite optimal solution

The optimal conditions for achieving the target PPFD were determined using a composite desirability function that considered both light distribution and energy efficiency. The analysis identified the most effective parameter settings to balance these objectives while ensuring optimal plant growth in a controlled environment.

The results indicate that the optimal combination of experimental variables is as follows: a distance of 50 mm, an LED intensity of 15, an LED angle of 45 degrees, and the use of four reflectors. Under these conditions, the predicted PPFD value is 163.6594, which closely aligns with the target value of 165.0. Additionally, the corresponding energy consumption under these settings is 2.2695, demonstrating an efficient use of energy while maintaining the desired light intensity.

The composite desirability function produced a value of 0.91062, indicating a high level of optimization across multiple responses. The results confirm that LED intensity and distance are the most influential parameters, with LED angle and reflectors contributing as fine-tuning factors for light distribution efficiency.

These findings emphasize the importance of a well-calibrated lighting system that integrates adaptive control mechanisms to maintain optimal PPFD levels while minimizing energy consumption. Future applications of this research could involve real-time adaptive lighting systems that dynamically adjust LED parameters in response to plant growth stages, further enhancing energy efficiency and improving crop yield in controlled environment agriculture.

B. Conclusions and Future Work

The findings from this study demonstrate that LED intensity and distance are the primary factors influencing PPFD, with statistically significant effects observed in both the main effect and interaction analyses. The quadratic regression model provides a strong predictive framework, as indicated by the high R^2 and adjusted R^2 values. The results highlight the need to optimize LED intensity and distance to maximize energy efficiency while ensuring uniform light distribution in controlled environment agriculture.

Future work should explore adaptive lighting systems that dynamically adjust LED intensity and distance based on plant growth stages. Additionally, further research is needed to evaluate the long-term energy efficiency of such systems and their impact on plant productivity. Investigating the influence of different LED spectra and environmental factors such as humidity and temperature could also provide deeper insights into optimizing controlled environment agriculture lighting systems.

Furthermore, the scalability and extendibility of the proposed system should be examined to assess its feasibility for large-scale applications. Future studies should consider how the lighting system can be adapted for different plant species and larger controlled environments. Another important aspect to investigate is the statistical significance of LED angle, as this factor showed minor influence in the present study but may have greater implications under different experimental conditions. To explore this further, the design and fabrication of hinged components using additive manufacturing can be implemented in future research, allowing for more flexible and adaptive lighting solutions.

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