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PARAMETER-BASED METHOD FOR PREDICTING STRAIN RATE SENSITIVITY IN SOFT MATERIALS

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***Abstract*—Investigating the mechanical behavior of soft materials through numerical techniques such as finite element analysis, require accurate mathematical models to capture their strain-rate-dependent responses. Stress-based modeling approaches, which represent stress at a given time as a combination of elastic and viscoelastic components, are often complex and typically require numerous parameters for the Prony series representation of the relaxation function. Energy-based methods employing a viscous potential are appealing, however, they have been shown to inadequately capture the rate sensitivity of soft materials with complex stress-strain curves. In this work, we introduce a parameter-based approach based on the principle that material stiffness increases with an increasing rate of loading. A functional relationship between hyperelastic parameters and strain rates is established, allowing the determination of parameters for a given strain rate. The effectiveness of this method is demonstrated by applying it to capture the rate sensitivity of self-healing hydrogel.

***Keywords*—Soft materials; Strain rate; Constitutive model; Hyperelastic**

I. INTRODUCTION

Soft materials, including elastomers and biological tissues, exhibit large nonlinear elastic stress-strain behavior that is strongly influenced by the rate of applied loading. Elastomer-based components, used in fields such as automotive, biomedical, and aerospace, often experience deformation under varying strain rates due to diverse loading conditions. Modeling the strain rate sensitivity of soft biological tissues is crucial in biomechanics, particularly for designing medical devices and implants, as well as developing biomechanical models to simulate surgical procedures [1].

Upadhyay et al. [5] outlined the two primary approaches commonly used in the literature to model the stress-strain sensitivity of soft materials to loading rate. Stress-based formulations incorporate relaxation functions or Prony series into hyperelastic frameworks, describing the stress at a given time as a combination of elastic and viscoelastic responses [3]. Energy-based formulations, derived from thermodynamic potentials, are further categorized into three types: (i) those based on extended hyperelasticity, (ii) those employing multiplicative decomposition of the deformation gradient, and (iii) those driven by external state variable-based viscous potentials [5]. Petiteau et al. [6] compared these two approaches and observed significant differences in their performance at intermediate strain rates.

The parameter-based method introduced in this study belongs to the extended hyperelasticity framework within energy-based approaches. Somarathan et al. [10] utilized a functional relationship between strain energy density at elevated strain rates and that at a reference strain rate to model strain rate-dependent of elastomers. Anssari-Benam and Hossain [2] presented a modeling framework in which the parameters of a hyperelastic model evolve with the deformation rate. For simplicity, they assumed that a linear relationship between the parameters and deformation rate.

Our formulation is based on the principle that a material's stiffness increases with deformation rate, which is captured through the material's parameters. By selecting a hyperelastic model (refer to the comprehensive reviews by Melly et al. [8] and Khaniki et al. [9] for an overview of hyperelastic models) and determining the parameters for each strain rate through optimization, a weighted exponential function is derived to describe the relationship between the parameters and strain rates. This function is subsequently utilized to determine the parameters of the hyperelastic model based on the strain rate. An experimental dataset for self-healing hydrogel described by [12] is used to validate the effectiveness of our model.

II. MODEL DEVELOPMENT

A. Elastic Potential

The elastic potential, W, is a scalar function that describes the response of hyperelastic materials to deformation. It can be expressed as a function of the deformation gradient, $W = W(\mathbf{F})$, or as a function of the right Cauchy-Green deformation tensor, $W = W(\mathbf{C})$, where $\mathbf{C} = \mathbf{F}^{\mathrm{T}} \cdot \mathbf{F}$. In this study, we adopt the invariant-based approach while accounting for the material's compressibility. The elastic potential is expressed as the sum of isochoric and volumetric components, which are functions of the invariants of the distortional right Cauchy-Green deformation tensor, $\mathbf{C}^* = J^{-\frac{2}{3}}\mathbf{C}$, and the volume ratio, $J = \det(\mathbf{F})$, respectively [14], as presented in (1).

$$W = W^{\mathrm{iso}}\left(I_1^*, I_2^*\right) + W^{\mathrm{vol}}\left(J\right) \tag{1}$$

The distortional invariants are defined as shown in (2).

$$I_1^* = \mathrm{tr}\left[\mathbf{C}^*\right], \quad I_2^* = \frac{1}{2}\left[\left(I_1^*\right)^2 - \mathrm{tr}\left(\mathbf{C}^*\right)^2\right] \tag{2}$$

The form of W^{iso} is derived from Yeoh's elastic potential [7] for its simplicity and accurate description of uniaxial loading [8] whereas the form of W^{vol} is obtained from [14] (3).

$$\begin{aligned} W^{\mathrm{iso}} &= c_{10}\left(I_1^* - 3\right) + c_{20}\left(I_1^* - 3\right)^2 + c_{30}\left(I_1^* - 3\right)^3 \\ W^{\mathrm{vol}} &= \frac{k}{4}\left(J^2 - 2\ln J - 1\right) \end{aligned} \tag{3}$$

Here, c_{10}, c_{20}, and c_{30} are material parameters to be identified through optimization, while k represents the bulk modulus determined using its relation with shear modulus, μ, and Poisson's ration, ν, (4).

$$k = \frac{2\mu(1+\nu)}{3(1-2\nu)} \tag{4}$$

B. Stress-strain relation

The total stress is obtained by summing the contributions from the volumetric and isochoric components as shown in (5).

$$\begin{aligned} \mathbf{S} &= 2\left[\frac{\partial W^{\mathrm{iso}}}{\partial \mathbf{C}} + \frac{\partial W^{\mathrm{vol}}}{\partial \mathbf{C}}\right] \\ \mathbf{S}^{\mathrm{iso}} &= J^{-\frac{2}{3}}\mathrm{Dev}\left(\mathbf{S}^*\right), \quad \mathbf{S}^{\mathrm{vol}} = J\frac{\partial W^{\mathrm{vol}}}{\partial J}\mathbf{C}^{-1} \\ \mathbf{S}^* &= \bar{\gamma}_1\mathbf{I} + \bar{\gamma}_2\mathbf{C}^* \end{aligned} \tag{5}$$

Where γ_1 and γ_2 are response coefficients given by (6):

$$\gamma_1 = 2\left(\frac{\partial W^{\mathrm{iso}}}{\partial I_1^*} + I_1^*\frac{\partial W^{\mathrm{iso}}}{\partial I_2^*}\right), \quad \gamma_2 = -2\left(\frac{\partial W^{\mathrm{iso}}}{\partial I_2^*}\right) \tag{6}$$

The Lagrangian deviatoric operator, Dev(A), is obtained for a second order tensor $\mathbf{A}$ as (7):

$$\mathrm{Dev}\left(\mathbf{A}\right) = \mathbf{A} - \frac{1}{3}\left[\mathbf{A} \colon \mathbf{C}\right]\mathbf{C}^{-1} \tag{7}$$

The Cauchy stress is then obtained through a push-forward operation (8).

$$\sigma = J^{-1}\mathbf{F} \cdot \mathbf{S} \cdot \mathbf{F}^{\mathrm{T}} \tag{8}$$

III. PARAMETER IDENTIFICATION

To optimize the model parameters, we employed Python's *scipy.optimize.minimize* function, selecting the *Nelder-Mead* algorithm. This method minimizes an objective function, which we defined as the sum of squared errors (SSE) (9), with initial parameter guesses set to 1.0.

$$L(\boldsymbol{\theta}) = \sum_{i=1}^{n}\left(\sigma_i^{\mathrm{exp}} - \sigma_i^{\mathrm{pred}}(\boldsymbol{\theta})\right)^2 \tag{9}$$

Where $\boldsymbol{\theta}$ represent the parameters to be optimized.
We validated our model with experimental data for self-healing hydrogels reported by Long et al. [12]. The material was subjected to uniaxial tension tests conducted at five different strain rates, ranging from 0.003 to 0.1 s^{-1}. Optimized parameters for each strain rate as given in Table I.

TABLE. I
HYPERELASTIC PARAMETERS AT DIFFERENT STRAIN RATES

Strain Rate s^{-1}	c_{10}	c_{20}	c_{30}
0.003	2211.6371	-355.4243	71.2467
0.01	3028.7225	-626.2145	121.7521
0.03	4360.1325	-1045.3313	200.6011
0.06	7038.7308	-5604.3276	3633.123
0.1	9080.1886	-14474.4612	22649.8009

This variation of parameters with strain rates is visualized as shown is Fig. 1.

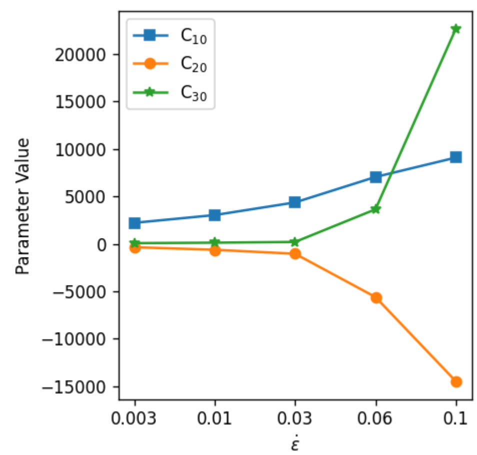

Figure. 1. Variation of parameters with strain rates

IV. PROPOSED PARAMETER-BASED METHOD

Building on the observed variation of hyperelastic parameters with strain rates, as illustrated in Fig. 1, we now propose a functional relationship between these parameters and strain rates, as expressed in (10).

$$f\left(\dot{\epsilon}\right) = \frac{a}{\dot{\epsilon}^2} + b\dot{\epsilon}^c \tag{10}$$

Where a, b, and c are parameters to be determined using an optimization method, and $f\left(\dot{\epsilon}\right)$ represents the parameter value corresponding to a specific strain rate. The parameters (see Table II) are determined using the *scipy.optimize.minimize* function with the *Powell* algorithm.

TABLE. II
FUNCTION PARAMETERS FOR EVERY HYPERELASTIC PARAMETER

Hyperelastic Parameter	a	b	c
c_{10}	0.0077	31168.6945	0.5363
c_{20}	-0.0034	-1223815.8574	1.9261
c_{30}	0.0007	88718261.5071	3.5929

V. RESULTS

By substituting the parameters from Table II into the function defined in (10), the resulting predicted parameters are illustrated in Fig. 2. The results demonstrate that the proposed function captures the variation of parameters with strain rates accurately. The predictions obtained using the parameter-based

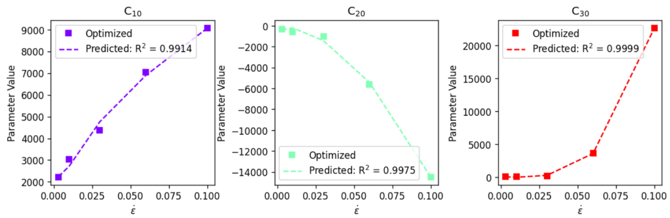

Figure. 2. Comparison of function-predicted and optimized parameters

method are compared with the experimental data shown in Fig. 3. The accuracy of the predictions was evaluated using the coefficient of determination, R^2. The results indicate a high level of accuracy, with R^2 ranging from 0.97 to 0.99, where a value of 1.0 signifies a perfect fit between the predicted and observed data.

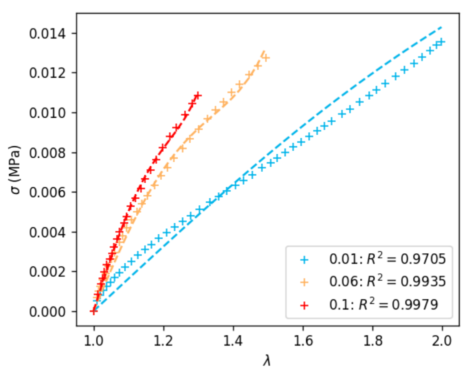

Figure. 3. Comparison between predicted and experimental stress-stretch curves

VI. CONCLUSION

The proposed parameter-based method for predicting strain-rate sensitivity in soft materials provides an efficient alternative to characterizing the viscoelastic behavior of these complex systems. It offers an accurate and straightforward alternative to conventional viscoelasticity methods, particularly when dealing with complex stress-strain curves, such as those observed in the self-healing hydrogel examined in this study. Unlike typical S-shaped curves, these curves exhibit a distinct hyperelastic response.

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