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Adaptive LMI-Based Observer for Disturbed Nonlinear Systems With Unknown Lipschitz Constant

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***Abstract*—This paper presents an adaptive Linear Matrix Inequality (LMI)-based observer designed for Lipschitz continuous nonlinear systems with unknown Lipschitz constants. Moreover, the observer is able to handle state dependent disturbances affecting the system dynamics. In contrast to traditional LMI-based methods that rely on prior knowledge of the Lipschitz constant or bounds on nonlinear functions for observer design, this paper assumes no such information. Instead, the proposed observer adaptively adjusts its gain to accommodate the disturbances and the unknown Lipschitz constant, which can be difficult or even impossible to determine for complex and highly nonlinear functions. The adaptability makes the observer particularly useful for applications where precise knowledge of nonlinear system characteristics is lacking. The observer's asymptotic stability is verified using the Lyapunov stability approach. Finally, the proposed observer is applied to a robotic application, where it is shown to provide accurate state estimation through simulations.

***Keywords*—adaptive nonlinear observer; unknown Lipschitz constant; LMI-based observer**

I. INTRODUCTION

Although numerous control design techniques assume the availability of system states, the inability to measure all states in practice necessitates the design of observers for state estimation. Observer design for nonlinear systems has attracted considerable attention in the past decades, motivated by its growing applications in emerging technologies such as robotics, aerospace, and intelligent transportation systems [1].

Several observer design methods have been proposed for different classes of nonlinear systems. Among them, the Linear Matrix Inequality (LMI)-based approach stands out due to its computational efficiency and ability to systematically handle system constraints [2]. However, LMIs often provide conservative stability and performance conditions. In particular, for the class of Lipschitz nonlinear systems, they are typically used to establish sufficient conditions for stability and bounded performance metrics. Therefore, several studies have tried to introduce advanced LMI-based methods aimed at reducing conservatism and enhancing the applicability of these conditions to Lipschitz nonlinear systems [3], [4], [5], [6].

Many of the LMI-based approaches developed in the literature rely on prior knowledge of the Lipschitz constant or at least some known bounds on the nonlinear functions. For example, in [7], a multi-output LMI-based high-gain observer was designed for a vehicle trajectory tracking application. In [8], an LMI-based observer was designed that makes use of the differential mean value theorem (DMVT) to transform the nonlinear error dynamics into a linear parameter varying (LPV) system, which has the advantage of introducing a general Lipschitz-like condition on the Jacobian matrix for differentiable systems. In [9], an LMI design was established to address the problem of circle criterion-based H_∞ observer design for nonlinear systems, which applies to both locally Lipschitz as well as monotonic nonlinear systems. While these papers use a prior known bound on the nonlinear functions in their observer design, accurately determining the Lipschitz constant can be difficult or impossible, especially for systems with complex or poorly characterized nonlinear functions [10]. To address this issue, a common approach is to estimate the upper bound of the first derivative of the nonlinear function [11]. However, this approach can lead to excessively large Lipschitz constants, resulting in conservative or potentially infeasible LMIs, which can yield suboptimal designs and analyses [12]. Furthermore, obtaining the upper bound of the first derivative is not always straightforward for complex and highly nonlinear functions. Another major challenge in practical applications is the presence of external disturbances. These disturbances may arise from various sources, such as environmental noise, unmodeled dynamics, and sensor inaccuracies, and can greatly impact the performance and robustness of state observers [13].

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Therefore, addressing these disturbances is crucial for ensuring the reliability and robustness of observer design in real-world applications, as highlighted in previous studies on disturbance rejection and robust observer design [14], [15].

Despite numerous works on nonlinear observer design, the problem of state estimation for Lipschitz nonlinear systems with unknown Lipschitz constant in the presence of external disturbances has not been thoroughly explored and continues to pose a challenge in the context of LMI-based observer design. This paper addresses this challenge by proposing an adaptive LMI-based observer that achieves state estimation for nonlinear systems with unknown Lipschitz constants. The main contributions of the paper are as follows:

- This paper investigates the class of Lipschitz continuous nonlinear systems whose Lipschitz constants are unknown. The developed LMI-based observer uses an adaptation law to account for the unknown Lipschitz constant.
- The proposed observer is designed to achieve accurate state estimation even in the presence of external disturbances, which are often unpredictable and can significantly degrade system performance. By dynamically adapting its gain, the observer can effectively compensate for the impact of disturbances.
- The stability conditions of the developed adaptive observer are formulated using LMIs. Finally, the developed observer is implemented on a robotic application, and simulation results demonstrate the effectiveness of the proposed approach.

The remainder of the paper is organized as follows. Section II presents the problem statement. The proposed adaptive observer design technique is described in Section III. In Section IV, the effectiveness of the proposed observer is demonstrated through simulations for a robotic system. Finally, Section V concludes the paper.

Notation: The sets of all n-dimensional real vectors, $n \times m$ real matrices, positive real numbers and positive definite $n \times n$ real matrices are denoted by $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$, $\mathbb{R}_+$ and $\mathbb{R}_+^{n\times n}$ respectively. Moreover, $\|.\|$ is the Euclidean norm and I_n is the $n \times n$ identity matrix. Furthermore, we use $(.)^T$ for transpose, $(.)^{-1}$ for inverse, and $\triangleq$ for equality by definition.

II. Problem Statement

Consider a class of nonlinear systems described by the following form:

$$\begin{aligned} \dot{x} &= Ax + Bf(x) + B_u u + f_d(x) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the measured output, $u \in \mathbb{R}^m$ is the control input, and matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times 1}$, $B_u \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are the linear state matrix, the coupling matrix for the nonlinear function, the input matrix, and the output matrix, respectively. $f_d(x)$ is the state-dependent external disturbance that influences the system dynamics and can ideally be approximated by $f_d(x) = w^T\sigma(x)$, where $w^T \in \mathbb{R}^{n\times 1}$ is a weight vector and $\sigma(x) : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear basis function of the state. The following assumptions are made to complete the description of the system:

- The nonlinear functions $f(x)$ and $\sigma(x)$ are assumed to be Lipschitz continuous, satisfying the following Lipschitz properties:

$$\begin{aligned} \big\|f(x_1) - f(x_2)\big\| &\le \gamma_f \|x_1 - x_2\| \\ \big\|\sigma(x_1) - \sigma(x_2)\big\| &\le \gamma_\sigma \|x_1 - x_2\| \end{aligned} \tag{2}$$

  where $\gamma_f, \gamma_\sigma \in \mathbb{R}_+$ are the Lipschitz constants.
- The pair (A, C) is observable.

The objective now is to design an observer to estimate the states of (1), using the available measurement y, and without any prior information on the Lipschitz constants γ_f, γ_σ, in the presence of the unknown disturbance term $f_d(x) = w^T\sigma(x)$.

III. Adaptive Observer Design Methodology

In this section, the design approach for the proposed adaptive observer is outlined. For the system described by (1), the following observer structure is considered:

$$\dot{\hat{x}} = A\hat{x} + Bf(\hat{x}) + B_u u + \hat{w}^T\sigma(\hat{x}) + L(y - C\hat{x}) + \kappa F(y - C\hat{x}) \tag{3}$$

where $\hat{x}$ is the state estimation vector, $L, F \in \mathbb{R}^{n\times p}$ are the observer gains to be designed and $\kappa \in \mathbb{R}_+$ is an adaptive parameter that is used to account for the unknown Lipschitz constant γ_f. By subtracting (3) from (1), and then adding and subtracting the term $w^T\sigma(\hat{x})$ the dynamics of the estimation error is derived as follows:

$$\dot{e} = (A - LC)e + B\tilde{f}(x) - \kappa FCe + w^T\tilde{\sigma}(x) + \tilde{w}^T\sigma(\hat{x}) \tag{4}$$

where $\tilde{w} = w - \hat{w}$, $\tilde{f}(x) = f(x) - f(\hat{x})$, and $\tilde{\sigma}(x) = \sigma(x) - \sigma(\hat{x})$.

Lemma 1 (The Schur complement [16]): Suppose Q and R are symmetric, i.e. $Q = Q^T$ and $R = R^T$. Then the LMI condition

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0$$

is equivalent to the quadratic inequality

$$R < 0, \quad Q - SR^{-1}S^T < 0$$

The following theorem outlines the design procedure for the adaptive observer gains L and F, as well as the adaptive parameters κ and $\hat{w}$ in (3).

Theorem: Consider the nonlinear system (1) and the observer dynamics (3). If there exist matrices $P = P^T \in \mathbb{R}_+^{n\times n}$, R and G of appropriate dimensions such that the following condition is satisfied:

$$
\begin{bmatrix} PA + A^T P - R^T C - C^T R \\ -\kappa G^T C - \kappa C^T G + (\kappa^2 + 1) I_n & PB & P\hat{B}_w^T \\ B^T P & -1 & 0 \\ \hat{B}_w P & 0 & -1 \end{bmatrix} \leq 0 \tag{5}
$$

Subject to the following adaptation laws:

$$
\begin{aligned}
\dot{\kappa} &= \gamma \|PB\| \|e\|^2 \\
\dot{\hat{B}}_w &= e^T P \|e\| \Gamma \\
\dot{\hat{w}} &= e^T P \sigma(\hat{x}) \Lambda
\end{aligned} \tag{6}
$$

where $\gamma \in \mathbb{R}$, $\Gamma = \Gamma^T \in \mathbb{R}^{n \times n}$ and $\Lambda = \Lambda^T \in \mathbb{R}^{n \times n}$ are the adaptation rates, and $\hat{B}_w$ is an auxiliary parameter and an estimate of $B_w = \gamma_\sigma w$. Then the state estimation error described by (4) asymptotically approaches zero with the observer gains given by:

$$
L = P^{-1} R^T, \quad F = P^{-1} G^T \tag{7}
$$

Proof. Consider the Lyapunov function candidate:

$$
V(e) \triangleq e^T P e + \frac{\tilde{\kappa}^2}{\gamma} + \tilde{B}_w \Gamma^{-1} \tilde{B}_w^T + \tilde{w} \Lambda^{-1} \tilde{w}^T \tag{8}
$$

where $\tilde{\kappa} = \kappa - \gamma_f$ is the estimation error of the unknown Lipschitz constant γ_f, and $\tilde{B}_w = \hat{B}_w - B_w$. Taking the derivative of the Lyapunov function along the trajectories of (4), and using (2) to construct an upper bound yields:

$$
\begin{aligned}
\dot{V}(e) \leq\ & e^T [PA + A^T P - PLC - C^T L^T P \\
& - \kappa PFC - \kappa C^T F^T P] e \\
& + 2\gamma_f \|PB\| \|e\|^2 + 2e^T P \underbrace{\gamma_\sigma w^T}_{B_w^T} \|e\| + 2e^T P \tilde{w}^T \sigma(\hat{x}) \\
& + 2\frac{\tilde{\kappa}\dot{\kappa}}{\gamma} + 2\dot{\hat{B}}_w \Gamma^{-1} \tilde{B}_w^T - 2\dot{\hat{w}} \Lambda^{-1} \tilde{w}^T
\end{aligned} \tag{9}
$$

Replacing $\gamma_f = \kappa - \tilde{\kappa}$ and $B_w^T = \hat{B}_w^T - \tilde{B}_w^T$, and using the adaptation laws in (6) results in:

$$
\begin{aligned}
\dot{V}(e) \leq\ & e^T [PA + A^T P - PLC - C^T L^T P \\
& - \kappa PFC - \kappa C^T F^T P] e + 2\kappa \|PB\| \|e\|^2 + 2\left\| P\hat{B}_w^T \right\| \|e\|^2
\end{aligned} \tag{10}
$$

by introducing $R = L^T P$ and $G = F^T P$, the asymptotic stability of the estimation error is guaranteed if the following condition is satisfied:

$$
\begin{aligned}
PA + A^T P - R^T C - C^T R - \kappa G^T C - \kappa C^T G \\
+ 2\kappa \|PB\| I_n + 2\left\| P\hat{B}_w^T \right\| I_n \leq 0
\end{aligned} \tag{11}
$$

utilizing

$$
\begin{aligned}
2\kappa \|PB\| I_n &\leq \kappa^2 I_n + PBB^T P \\
2\left\| P\hat{B}_w^T \right\| I_n &\leq I_n + P\hat{B}_w^T \hat{B}_w P
\end{aligned} \tag{12}
$$

condition (11) can be rewritten as:

$$
\begin{aligned}
PA + A^T P - R^T C - C^T R - \kappa G^T C - \kappa C^T G \\
+ (\kappa^2 + 1) I_n + PBB^T P + P\hat{B}_w^T \hat{B}_w P \leq 0
\end{aligned} \tag{13}
$$

using Lemma 1, we can easily show that condition (13) is equivalent to LMI (5). Therefore, given that $V(e) > 0$, by ensuring (5) is satisfied, we get $\dot{V}(e) \leq 0$. Hence, by the Lyapunov stability criteria as well as LaSalle's invariance principle, the estimation error converges to zero and thus the nonlinear observer is asymptotically stable. This ends the proof.

□

Remark 1. Note that we begin by non-negative initial values for the parameters κ and $\hat{B}_w$ and solve LMI (5) to obtain matrices P, R and F. Then P is used to update the parameters according to (6) for the next time step. The updated values of κ and $\hat{B}_w$ are then used to solve LMI (5) and this process continues until the estimation error approaches zero and the adaptation stops.

IV. Application To a Robot

In this section, the effectiveness of the proposed adaptive observer is demonstrated through simulations using a single-link robotic manipulator application.

A. Robot Model

A single-link flexible joint robot with revolute joints, as shown in Fig. 1, is considered [17]. The state-space description of this system is given by:

$$
\begin{aligned}
\dot{\theta}_m &= \omega_m \\
\dot{\omega}_m &= \frac{k}{J_m}(\theta_l - \theta_m) - \frac{B}{J_m}\omega_m + \frac{K_\tau}{J_m} u \\
\dot{\theta}_l &= \omega_l \\
\dot{\omega}_l &= -\frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l} sin(\theta_l)
\end{aligned} \tag{14}
$$

where θ_m and θ_l are angular rotations of the motor and the link respectively, and ω_m and ω_l are their angular velocities. The parameters of this model along with their values are presented in Table I.

TABLE. I
Robot Model Parameters

System Parameter (Units)	Value
Motor inertia, J_m (kg m^2)	3.7×10^{-3}
Link inertia, J_l (kg m^2)	9.3×10^{-3}
Pointer mass, m (kg)	2.1×10^{-1}
Link length, $2b$ (m)	3.0×10^{-1}
Torsional spring constant, k (Nm rad^{-1})	1.8×10^{-1}
Viscous friction coefficient, B (Nm V^{-1})	4.6×10^{-2}
Amplifier gain, K_τ (Nm V^{-1})	8×10^{-2}

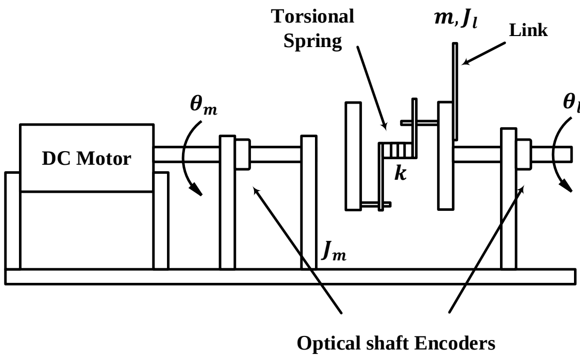

Figure. 1. Schematic of an elastic robot [17].

Using the parameters of Table I, the model of the robot can be described under the form (1) using the following parameters:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}$$

The first and second states of the robot are available as measurements:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$f(x) = -15.86 \times m \times sin(x_3)$$

B. Simulation Results

Simulations are carried out in MATLAB using YALMIP toolbox, and the parameters used for simulation are presented in Table II. The rates of adaptation in (6) are tuned to balance convergence speed and robustness, and the Gaussian disturbance is introduced to the system at about $1.1s$. To demonstrate the effectiveness of the proposed adaptive observer (AO), its estimation results are compared with those obtained by the LMI-based fixed-gain observer (FGO) of [2]. Fig. 2 and Fig. 3 show the state estimation results, and the evolution of the adaptive parameters, respectively. As shown in Fig. 3, the adaptive parameters remain bounded over time without drifting. Moreover, it can be seen from Fig. 2 that despite the lack of prior knowledge about the Lipschitz constant of the nonlinear functions $f(x)$ and $\sigma(x)$, the observer effectively tracks the states of the system in the presence of disturbance. Compared to the fixed-gain observer, the proposed adaptive observer has a superior performance as demonstrated by the

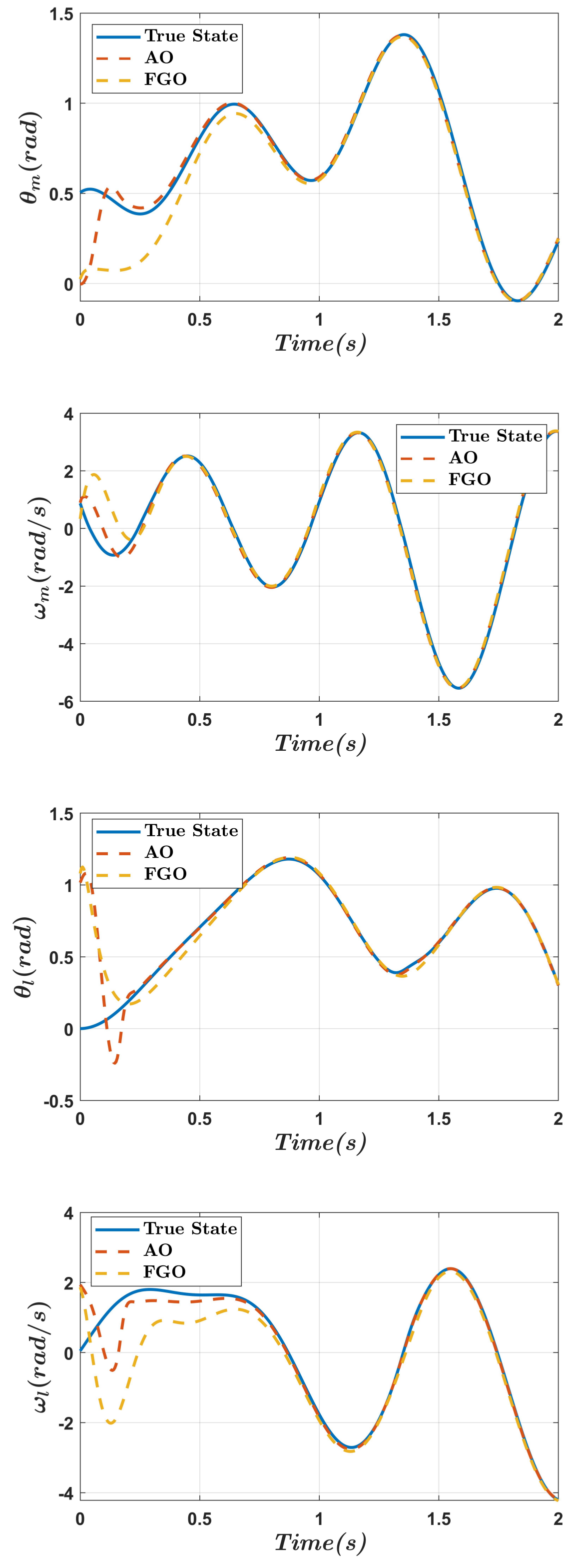

Figure. 2. State estimation results.

TABLE. II
SIMULATION PARAMETERS

Parameter	Value
u	$sin(2\pi t)$
x_0	$[0.5 \quad 1 \quad 0 \quad 0]^T$
$\hat{x}_0$	$[0 \quad 0 \quad 1 \quad 2]^T$
$\sigma(x)$	$e^{-\frac{x_2^2}{2}}$
w^T	$[1 \quad 2 \quad 1.5 \quad 1]^T$
$\hat{w}_0^T$	$[0 \quad 0 \quad 0 \quad 0]^T$
$\hat{B}_{w0}^T$	$[0 \quad 0 \quad 0 \quad 0]^T$
κ_0	0
γ	0.2
Γ	$2I_4$
Λ	$3I_4$

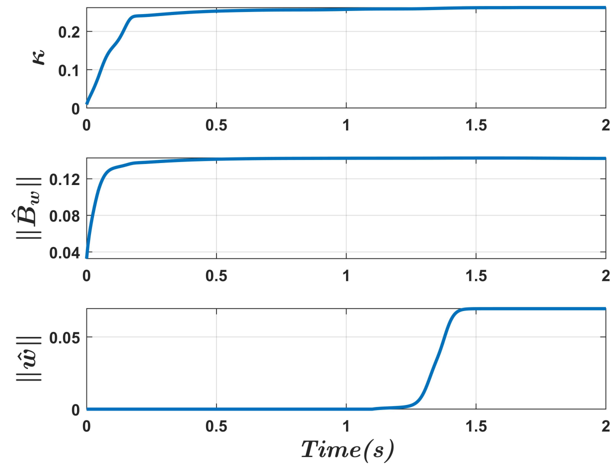

Figure. 3. Evolution of the adaptive parameters.

RMSE values in Table III. This is because traditional approaches that rely on a fixed Lipschitz constant, usually require solving more conservative LMIs. These approaches typically use a Lipschitz constant that is larger than the actual growth rate of the nonlinearity encountered during normal system operation. This results in unnecessarily conservative LMIs and subsequently higher observer gains, as well as more sensitivity to high-frequency measurement noise. On the other hand, by using an adaptive parameter instead of a predefined constant, the observer is able to dynamically adjust the upper bound on the growth of the nonlinearity based on the actual system behavior.

C. Discussion

As demonstrated by the simulation results, the proposed adaptive LMI-based observer presented in this paper offers notable advantages. First, the observer operates without any prior knowledge about the Lipschitz constant of the nonlinearities, which can be difficult to estimate accurately in many systems. This makes the observer more versatile and applicable to real-world applications where the nonlinearity bounds are not precisely known. Moreover, the gain of the proposed observer is adjusted based on the estimation error, allowing it to react dynamically to changes in the system's behavior, which not only enhances the performance but also allows for improved efficiency in state estimation under varying operating conditions. Furthermore, the observer can mitigate the effect of disturbances due to its adaptive nature, achieving accurate and robust state estimation.

TABLE. III
COMPARISON OF RMSE VALUES OF THE OBSERVERS

Observer	θ_m (rad)	ω_m (rad/s)	θ_l (rad)	ω_l (rad/s)
Adaptive observer (AO)	0.10	0.23	0.21	0.69
Fixed-Gain observer (FGO)	0.16	0.51	0.23	0.94

V. CONCLUSION

In this paper, an adaptive LMI-based observer was proposed for state estimation of Lipschitz nonlinear systems with unknown Lipschitz constant in the presence of disturbances. Due to its adaptive nature, the proposed observer eliminates the need for prior knowledge about the Lipschitz constant of the nonlinear functions involved. Lyapunov-based analysis was used to establish the asymptotic stability of the observer. Finally, simulations were conducted on a robotic manipulator and the results demonstrated the observer's effectiveness in providing accurate state estimation despite the presence of disturbances. Although the adaptability of the proposed method reduces conservatism in observer design, one limitation is the high computational effort in solving the LMIs in real-time. Future work will focus on further developing the proposed observer design method to reduce its computational burden.

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