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Obstacle Avoidance and Trajectory Tracking Coordinated Control of Multi-Axle All-Wheel Steering Vehicles Based on the Ackerman Principle

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Abstract—Multi-axle vehicles with autonomous driving capabilities often face challenges in achieving high trajectory tracking accuracy, particularly in complex road scenarios, and the steering angles of individual wheels may deviate from the Ackerman principle. This paper investigates multi-axle all-wheel steering vehicles based on the Ackerman principle. By utilizing the kinematic model of the vehicle, the functional relationships between the steering center position, the steering angles of individual wheels, and the specific motion trajectories of each wheel under low-speed conditions for a given trajectory are derived. Additionally, the relationships between the steering angles of each axle and the turning radius under varying numbers of axles with a fixed total wheelbase are analyzed. Furthermore, a critical issue arises in high-speed steering scenarios where obstacle avoidance and trajectory tracking must be coordinated when obstacles are present ahead. To address this issue, this study establishes a three-degree-of-freedom dynamic model comprising lateral, longitudinal, and yaw motions. A nonlinear MPC (Model Predictive Control) controller incorporating obstacle avoidance functionality and an objective function is proposed for local re-planning of the global path when obstacles are detected. The parameters from the re-planned trajectory are then fed into a linear MPC controller to control the front-wheel steering angles, thereby achieving a coordinated control strategy for obstacle avoidance and trajectory tracking.

Keywords- Ackerman principle; multi-axle all-wheel steering; trajectory tracking; MPC; local planning; obstacle avoidance

Introduction

Ackerman steering geometry, proposed in the 20th century, forms the theoretical basis for traditional vehicle steering control.[1][2] It describes the geometric relationship between the wheels and the vehicle's rotation center, ensuring good path-tracking accuracy in single-axis steering systems[3].Rajamani et al.[4]proposed a vehicle dynamics model based on Ackerman steering geometry for path tracking and stability control of single-axle vehicles. However, for performance or special-purpose autonomous vehicles (e.g., heavy-duty trucks, autonomous transport vehicles, off-road vehicles), single-axis steering methods struggle to meet the accuracy, pass ability, and maneuverability requirements in complex conditions. Therefore, multi-axle vehicles require the integration of Ackerman principles and multi-degree-of-freedom dynamics models to achieve path tracing and obstacle avoidance control.[5][6]

In low-speed driving conditions, precise trajectory control can be achieved by appropriately distributing the steering angles across axles based on Ackerman geometric relationships. Guo et al.[7] designed a trajectory controller for autonomous trucks using Ackerman principles and verified the effectiveness of the controller in low-speed conditions. However, this method is not suitable for high-speed dynamic environments requiring high-precision trajectory control. To address this issue, this paper introduces a path-tracking method based on Model Predictive Control (MPC). However, this method faces challenges in high-speed control. Therefore, a Model Predictive Control (MPC)-based path-tracking method is introduced to tackle the challenges posed by high-speed dynamic environments. Fa et al.[8]proposed a robust MPC-based path-tracking control strategy by online linearization of nonlinear vehicle models, achieving precise steering control and obstacle avoidance in complex environments. Xu et al.[9] proposed a path-tracking method based on a three-degree-of-freedom vehicle model, using a neural network to adaptively adjust MPC control parameters, thereby enhancing control performance. Additionally, the application of multi-axle all-wheel steering systems in obstacle avoidance control has also attracted attention. Wang et al.[10] proposed a hybrid control strategy combining obstacle avoidance and path tracking, dynamically adjusting the steering angle by sensing the distance between the vehicle and obstacles in real-time, achieving safe obstacle avoidance for multi-axle vehicles. Zhang et al.[11] researched multi-sensor fusion methods for obstacle detection and path planning in complex environments, providing foundational support for multi-axle vehicle obstacle avoidance control. However, the coordination problem between obstacle avoidance and trajectory tracking control remains unresolved. Li et al.[12] pointed out that multi-objective optimization methods that consider both obstacle avoidance and trajectory tracking in dynamic environments still face challenges in computational complexity and real-time performance. This paper uses nonlinear MPC to perform local planning of the global path, enabling coordinated obstacle avoidance and trajectory tracking control for multi-axle vehicles at different speeds.

This paper establishes vehicle kinematic and dynamic models and proposes a multi-axle steering trajectory tracking control method that combines Ackerman steering geometry with modern control theory, designing a steering angle allocation strategy. The specific content includes optimization strategies designed for different vehicle speed conditions: at low speeds, smooth and efficient steering control is achieved based on the vehicle's Ackerman geometric relationship; at medium to high speeds, a linear MPC algorithm is used to design the vehicle trajectory tracking controller, and a nonlinear MPC algorithm is used to design the trajectory reconstructor, thus achieving coordinated control of path tracking and obstacle avoidance for multi-axle all-wheel steering vehicles at high speeds. The effectiveness of the control algorithm is verified through simulation experiments.

I. Ackerman Steering Principle and Establishment of the Kinematic Model

A. *Ackerman Steering Principle*

The Ackerman steering principle is a geometric design approach that ensures the trajectories of all wheels of a multi-wheeled vehicle (especially front-wheel-drive vehicles) converge at a single center point during turning. This minimizes tire slip and unnecessary wear.During a vehicle's turn, each wheel follows a distinct trajectory, resulting in different turning radius. The core idea of the Ackerman steering principle is that there exists a geometric relationship between the steering angles of the front wheels and their respective turning radius. For instance, if the turning radius of the front-right wheel is fixed, the turning radius of the other wheels can be determined as follows:

$$\begin{aligned} R_{11} &= R \times sin(\delta_{i2}) \ / \ sin(\delta_{11}) \\ R_{12} &= R \\ R_{i1} &= R_{11} \times cos(\delta_{11}) \ / \ cos(\delta_{i1}) \\ R_{i2} &= R \times cos(\delta_{12}) \ / \ cos(\delta_{i2}) \end{aligned} \tag{1.1}$$

Where R_{i1} is the turning radius of the left wheel of the i-th axle,R_{i2} is the turning radius of the right wheel of the i-th axle, δ_{i1} is the steering angle of the left wheel of the i-th axle, δ_{i2} is the steering angle of the right wheel of the i-th axle.

B. *Ackerman Steering Geometry Principle*

Ackerman's principle requires that the turning radius of all wheels should intersect at the same point (turning center). The turning center is the turning of all wheels (i.e., their turning turns) should converge to a virtual point when turning - usually when the vehicle is driving at low speed, the turning center is located on the axis of the rear axle, according to the position of the turning center, the steering angle of each axle of the multi-axle vehicle can be obtained, this article takes the three-axle vehicle as an example:

$$\begin{aligned} &\delta 1 = asin((L_1 + L_2) / R) \\ &\delta 2 = (L_2 / (L_1 + L_2)) \times \delta 1 \\ &\delta 3 = 0 \end{aligned} \tag{1.2}$$

The equation of motion of a tire is:

$$\begin{aligned} X_{i+1} &= X_i + v \times dt \times \cos\theta \\ Y_{i+1} &= Y_i + v \times dt \times \sin\theta \end{aligned} \tag{1.3}$$

Where L_i is the distance from the steering center to the i-th axle, and $X_{(i+1)}$ is the x-coordinate of the wheel's position at the next time step,$X_{(i)}$ is the x-coordinate at the current time step, $Y_{(i+1)}$ is the y-coordinate of the wheel's position at the next time step, $Y_{(i)}$ is the y-coordinate at the current time step, v is the longitudinal velocity, θ is the heading angle, and dt is the time interval.

The trajectory of the right front wheel is designed as a straight line of 5m, the quarter circle of radius of 15m and the quarter circle of radius of 10m are spliced together, and the movement trajectory of each wheel can be obtained as shown in Figure 1.

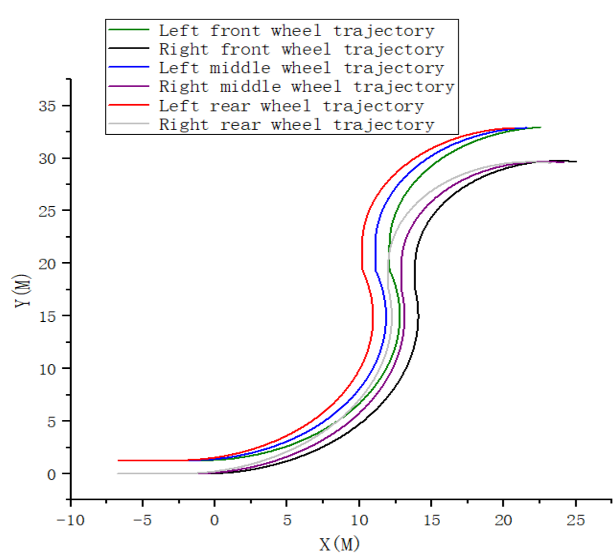

Figure 1: The movement trajectory of each wheel of the three-axle vehicle

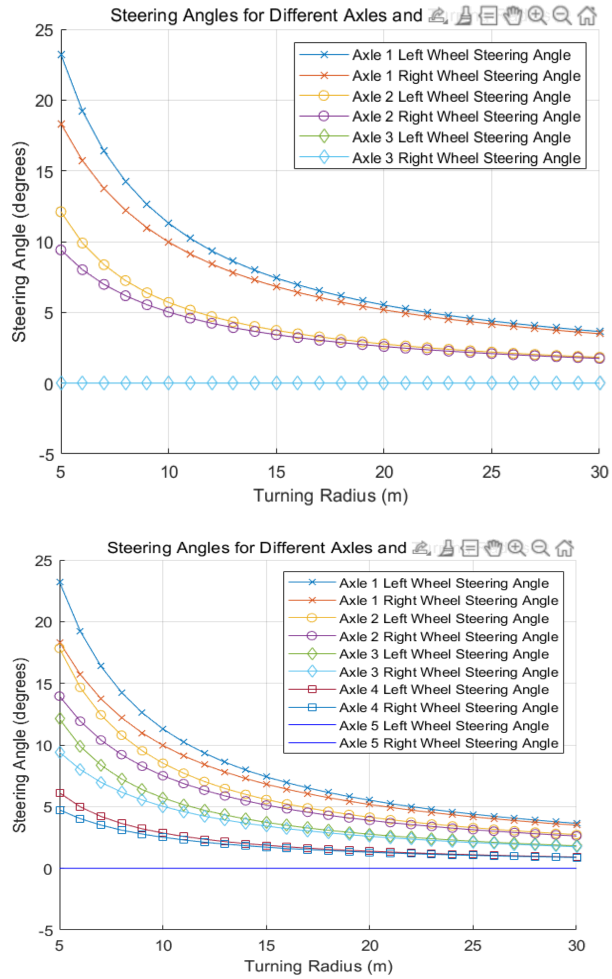

Figure 2:The relationship between the angle of each axle and the turning radius of a multi-axle vehicle

Based on the previously derived wheel angle relationships and the kinematic model, simulations were conducted for both 3-axle and 5-axle vehicles. The resulting wheel trajectories were plotted under the condition that the right front wheel's turning radius was fixed. The following observations were made in Figure 2: In the case of a fixed wheelbase, as the number of axles changes, a pattern can be observed: when the multi-axle vehicle is turning at low speed with a constant turning radius, the steering angle of the front axle does not change with the number of axles, and the same applies to the rear axle,the steering angle of the rear axle is zero. Additionally, as the turning radius increases, the steering angles of all axles continuously decrease and approach a constant value

II. Establishment of the Multi-Axle Vehicle Dynamic Model

To investigate the basic characteristics of vehicle handling stability during turning, a simplified 3-axle 2DOF vehicle model, as shown in 2.1is established and the three-axle vehicle dynamics model is shown in Figure 3.

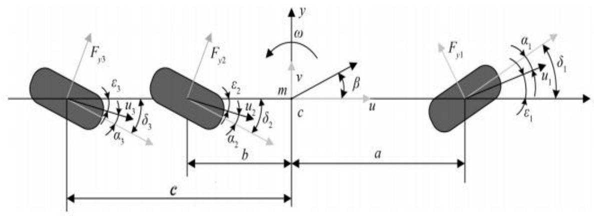

Figure 3:2DOF model of three-axle vehicle

$$
\begin{aligned}
&m(\dot{v}_x - v_y \times \dot{\varphi}) = Fx_f \times \cos\delta_f + Fx_m \times \cos\delta_m + Fx_r \times \cos\delta_r - \\
&\qquad (Fy_f \times \sin\delta_f + Fy_m \times \sin\delta_m + Fy_r \times \sin\delta_r) \\
&m(\dot{v}_y + v_x \times \dot{\varphi}) = Fy_f \times \cos\delta_f + Fy_m \times \cos\delta_m + Fy_r \times \cos\delta_r + \\
&\qquad Fx_f \times \sin\delta_f \sin\delta_f + Fx_m \times \sin\delta_m + Fx_r \times \sin\delta_r \\
&Iz \times \ddot{\varphi} = a \times (Fx_f \times \sin\delta_f + Fy_f \times \cos\delta_f) - b \times (Fx_m \times \sin\delta_m + \\
&\qquad Fy_m \times \cos\delta_m) - c \times (Fx_r \times \sin\delta_r + Fy_r \times \cos\delta_r)
\end{aligned}
\tag{2.1}
$$

Where m is the total vehicle mass, I_z is the yaw moment of inertia of the vehicle,v_x is the longitudinal velocity of the vehicle's center of mass,v_y is the lateral velocity of the vehicle's center of mass; φ is the yaw angle of the vehicle's center of mass; $\dot{\varphi}$ is the yaw rate of the vehicle's center of mass; a,b and c are the distances from the vehicle's center of mass to the respective axles; δ_f,δ_m,δ_r are the steering angles of the first, second, and third axles, respectively; Fx_f, Fx_m, Fx_r are the longitudinal forces on each wheel;Fy_f, Fy_m, Fy_r are the lateral forces on each wheel;

For the 3-axle vehicle, assuming that the steering angle of the first axle is known, based on Ackerman's steering geometry, the steering angles of the second and third axles should satisfy the following relationships:

$$
\begin{aligned}
&\frac{\tan(\delta_f)}{\tan(\delta_i)} = \frac{L_1}{L_i} \\
&k2 = \frac{\tan(\delta_m)}{\tan(\delta_f)} = \frac{\delta_m}{\delta_f} = \frac{L_2}{L_1} = \frac{L_{12} - L_1}{L_1} = 1 - \frac{L_{12}}{L_1} \\
&k3 = \frac{\tan(\delta_r)}{\tan(\delta_f)} = \frac{\delta_r}{\delta_f} = \frac{L_3}{L_1} = \frac{L_{13} - L_1}{L_1} = 1 - \frac{L_{13}}{L_1}
\end{aligned}
\tag{2.2}
$$

Where L_{1i} is the distance from the first axis to the i-th axis k2 and k3 are the ratio of the tangent of the middle and rear axle angles to the front axle.

From the expressions for the steering angle ratio between the second, third, and first axles, to calculate k2 and k3, the expression for the distance L1 from the steering center O to the front axle must first be obtained. Set the lateral slip angle β=0, and the lateral slip rate is also zero, meaning that the multi-axle vehicle is in a stable state during steering. Substituting this condition into the vehicle's differential equations gives the specific expression for L1 as (2.3) follows:

$$\begin{aligned}
L_\alpha &= (\mathrm{a}C_\mathrm{f} - \mathrm{b}C_\mathrm{m} - \mathrm{m}\mathrm{v_x}^2)(\mathrm{b}C_\mathrm{m}L_{12} + \mathrm{c}C_\mathrm{r}L_{13}) + \\
&\quad (\mathrm{a}^2C_\mathrm{f} + \mathrm{b}^2C_\mathrm{m} + \mathrm{c}^2C_\mathrm{r})(C_\mathrm{m}L_{12} + C_\mathrm{r}L_{12}) \\
L_\beta &= (\mathrm{a}^2C_\mathrm{f} + \mathrm{b}^2C_\mathrm{m} + \mathrm{c}^2C_\mathrm{r})(C_\mathrm{f} + C_\mathrm{m} + C_\mathrm{r}) - \\
&\quad (\mathrm{a}C_\mathrm{f} - \mathrm{b}C_\mathrm{m} - \mathrm{c}C_\mathrm{r} - \mathrm{m}\mathrm{v_x}^2)(\mathrm{a}C_\mathrm{f} - \mathrm{b}C_\mathrm{m} - \mathrm{c}C_\mathrm{r}) \\
L_1 &= \frac{L_\alpha}{L_\beta}
\end{aligned} \tag{2.3}$$

Where C_f, C_m, and C_r are the lateral stiffness of the front axle, center axle, and rear axle, respectively.

III. Design of MPC-Based Local Path Planning and Trajectory Tracking Controllers with Obstacle Avoidance

The trajectory planning module integrates the global reference path and real-time obstacle data from sensors. Using a nonlinear Model Predictive Control (MPC) algorithm, it calculates optimized local reference path points. These discrete points are then fitted into a 5th-order polynomial curve to ensure smoothness. Finally, the polynomial coefficients are packaged and transmitted to the tracking control module for further processing.

The tracking control module receives the 5th-order polynomial coefficients from the planning layer. By inversely reconstructing the polynomial, it derives precise local path reference points. Employing its own MPC-based optimization framework, the module computes the optimal control input—specifically, the front-wheel steering angle for the next time step—to ensure accurate path following.

A. Trajectory tracking MPC design

To reduce computational demands in the planning layer, the point mass model (ignoring dimensional parameters like length and width) is adopted for vehicle dynamics representation. This abstraction significantly lowers algorithmic complexity while retaining sufficient fidelity for path planning. Furthermore, the planning layer operates on an extended control period—capable of generating trajectories spanning multiple tracking cycles within a single computation. To synchronize these processes, the planning interval is strategically configured at twice the duration of the trajectory tracking cycle, ensuring efficient resource allocation without compromising responsiveness.

B. Nonlinear model predictive control algorithms

For a nonlinear system, a discrete model is typically in the form of:

$$\begin{aligned}
&\xi(\mathrm{t}+1) = f(\xi(t), u(t)) \\
&\xi(t) \in \chi, u(t) \in \Gamma
\end{aligned} \tag{3.1}$$

Where, $f(\cdot,\cdot)=0$ is the state transition function of the system, ξ is the state vector, u is m is the control vector, χ is the state quantity constraint, and Γ is the control quantity constraint. $f(0,0)=0$ is set as a stable point of the system, and it is also the control target of the system. For any time domain N, consider the following optimization objective function $JN(\xi(t), U(t))$.

$$JN(\xi(t), U(t)) = \sum_{k=1}^{t+N-1} l(\xi(k)\mathrm{u(k)}) + P(\xi(t+N)) \tag{3.2}$$

Where $U(t) = [u(t),\cdots\cdots, u(t+N-1)]^T$ represents the control input sequence over the time domain N, and $\xi(t)$ denotes the state vector trajectory under the influence of the input vector sequence U(t). The first term $l(\cdot,\cdot)$ in the objective function characterizes the tracking capability of the desired output, while the second term $P(\cdot)$ represents the terminal constraint.

C. Dynamics-based vehicle point mass model (nonlinear, continuous)

Using the backward Euler method, the nonlinear equation (3.3) is discretized to predict the state variables at the next time step:

$$\begin{aligned}
&\ddot{\mathrm{Y}} = \mathrm{a_y},\ \ddot{\mathrm{X}} = 0,\ \dot{\varphi} = \frac{\mathrm{a_y}}{\dot{\mathrm{X}}} \\
&\dot{\mathrm{Y}} = \mathrm{v_x}\sin\varphi + \mathrm{v_y}\cos\varphi \\
&\dot{\mathrm{X}} = \mathrm{v_x}\sin\varphi + \mathrm{v_y}\cos\varphi
\end{aligned} \tag{3.3}$$

Considering the vehicle's dynamic constraints, the following constraint is added:

$$|\mathrm{a_y}| < \mu \mathrm{g} \tag{3.4}$$

Where a_y is the lateral acceleration of the vehicle, μ is the adhesion coefficient of the pavement and g is the acceleration due to gravity

Equations (3.3) and (3.4) can be simplified as follows:

$$\begin{aligned}
&\dot{\xi}(\mathrm{t}) = \mathrm{f}(\xi(\mathrm{t}), \mathrm{u(t)}) \\
&|\mathrm{u(t)}| < \mu \mathrm{g}
\end{aligned} \tag{3.5}$$

Where $\xi=[v_y,v_x,\varphi,Y,X]^T$ represents five discrete state variables: the vehicle's velocity in the y and x directions, the vehicle's heading angle, and the longitudinal and lateral coordinates of the vehicle's position. u represents the control input, which in this case is the vehicle's front wheel steering angle δ. Note that this u is not the same as the u in trajectory planning MPC, but rather the control layer's u.

D. Discretization into a predictive model

In nonlinear model predictive control, the future state quantity can be predicted through the nonlinear model, the current state quantity and the control quantity sequence in the control time domain. Obviously, this is an iterative process, and since the control sequence is agnostic, it is necessary to find an explicit iterative equation to approximate the differential equation. In practical engineering, the most widely used numerical solutions include Euler's method and the fourth-order Runge-Kutta algorithm. This section uses a more single-step forward Euler method:

$$\begin{aligned} &\dot{y}(i)=\dot{y}(i-1)+T\times ay(i) \\ &\dot{x}(i)=\dot{x}(i-1) \\ &\varphi(i)=\varphi(i-1)+T\times ay(i)/\dot{x}(i-1) \\ &Y(i)=Y(i-1)+T[\dot{x}(i-1)\sin\varphi(i-1)+\dot{y}(i-1)\cos\varphi(i-1)] \\ &X(i)=X(i-1)+T[\dot{x}(i-1)\cos\varphi(i-1)+\dot{y}(i-1)\sin\varphi(i-1)] \end{aligned} \tag{3.6}$$

Where $\dot{y}(i)$ and $\dot{x}(i)$ are the transverse and longitudinal velocity of the vehicle at time i, φ(i) is the yaw angle at time i, Y(i) and X(i) are the transverse and longitudinal coordinates of the vehicle at time i Obstacle avoidance function and MPC objective function. T is the time interval. ay(i) is the lateral acceleration at time i.(i-1) is the previous moment.

E. Obstacle avoidance function and MPC objective function

1) Obstacle avoidance function function:

The design dynamically scales the penalty function intensity based on the distance deviation between obstacles and the target point: smaller deviations trigger a nonlinear increase in penalty values, compelling the planned path to avoid proximity hazards. Increasing the weight coefficient S_{obs} prioritizes obstacle avoidance, generating conservative trajectories (e.g., expanded safety margins). However, S_{obs} becomes inactive when obstacle data is unavailable (e.g., sensor failures). To address significant vehicle state estimation errors, elevating S_{obs} enhances robustness at the cost of increased tracking deviations (typical trade-off range:S_{obs} ∈[0.5,2.0], necessitating adaptive tuning via real-time state confidence evaluation.

2) Objective function:

The control goal of the trajectory planning layer is to minimize the deviation from the global reference path and realize the avoidance of obstacles. Obstacle avoidance is implemented in the form of a penalty function. The specific form is as follows:

$$J_{obs,i} = \frac{S_{obs} v_i}{(x_i - x_0)^2 + (y_i - y_0)^2 + \xi} \tag{3.7}$$

Where S_{obs} is the weight coefficient, $v_i = v_x^2 + v_y^2$, (x_i, y_i) is the position coordinate of the obstacle point in the body coordinate system, and(x_0, y_0) is the coordinate of the vehicle's centroid of mass, ξ is a small positive number, which is used to prevent the occurrence of a denominator of 0.$J_{obs,i}$ is the obstacle avoidance function of the sampling time i.

F. Polynomial fittings

The trajectory planning algorithm minimizes distance deviations from reference points within a finite time horizon. It generates discrete trajectory points across the predicted time domain. As the prediction horizon expands, the number of these local reference points increases proportionally. Directly transmitting raw discrete points to the control layer overburdens the controller's input interfaces, complicating standardized controller design. Additionally, mismatched control cycles between the planning layer and the tracking layer create synchronization challenges, hindering precise discrete-point trajectory tracking.

To ensure smooth docking between the planning layer and the control layer, it is necessary to process the local reference path generated by the trajectory planning algorithm. Curve fitting is the primary method for handling discrete points. Since the vehicle's position must be continuous, the curve must also be continuous. Additionally, the yaw angle requires the curve to be first-order continuous, while acceleration constraints demand second-order continuity. Therefore, this paper selects a 5th-order polynomial as the fitting curve.

$$\begin{aligned} Y &= a_0 t^5 + a_1 t^4 + a_2 t^3 + a_3 t^2 + a_4 t + a_5 \\ \varphi &= b_0 t^5 + b_1 t^4 + b_2 t^3 + b_3 t^2 + b_4 t + b_5 \end{aligned} \tag{3.8}$$

Where a_p=[a_0,a_1,a_2,a_3,a_4,a_5],bp=[b_0,b_1,b_2.b_3,b_4,b_5] is parameters to be found.

A model predictive control (MPC) framework is developed for trajectory planning, utilizing a point-mass vehicle model to generate dynamically feasible trajectories. These trajectories satisfy vehicle dynamic constraints (e.g., acceleration limits) while ensuring obstacle avoidance. For precise path tracking, a second MPC controller is implemented with a high-fidelity vehicle dynamics model, validated through Simulink/TruckSim co-simulation to account for real-world vehicle behavior. The integrated control architecture, coordinating both planning and tracking modules, is illustrated in Figure 4.

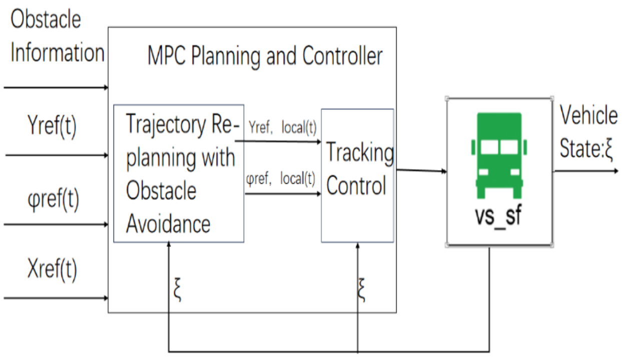

Figure 4:MPC Control Logic Diagram

IV. Experimental Verification

In this paper, the double-shift trajectory is selected as the reference trajectory[13], and an obstacle with a length of 5m and a width of 0.5 m is set up in the turning path of the double-shift line as the experimental scene, and the multi-axle vehicle travels at a speed of 20m/s, and the simulation results are shown in Figure 5.

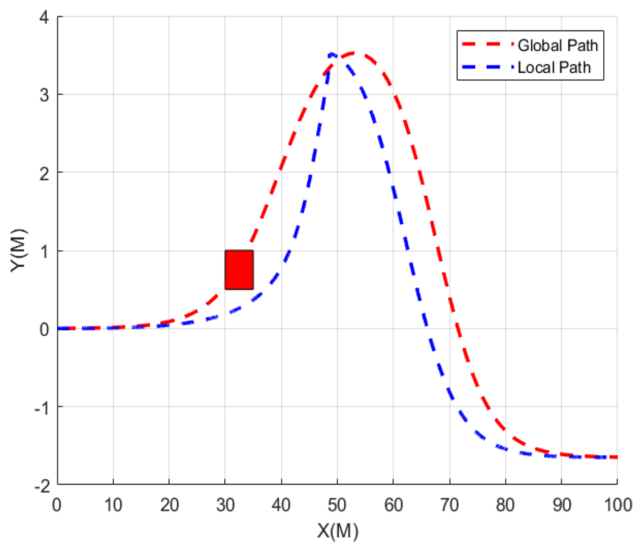

Figure 5:Comparison of the target trajectory and the actual trajectory

As can be seen from Figure 5, through the control of MPC, the multi-axle vehicle first carried out the local planning of the global path before encountering the obstacle, planned a local path that can avoid the obstacle, and tracked the target trajectory after completing the obstacle avoidance task, and finally coincided with the target trajectory, so that the cooperative control of trajectory tracking and obstacle avoidance was realized.

V. Summary

In this paper, the multi-axle all-wheel steering vehicle is modeled through the Ackerman principle, because the lateral force and lateral acceleration of the vehicle at low speed are small and negligible, so only the kinematic modeling of the multi-axle vehicle is done in the first section, and the relationship between the rotation angle of each wheel and the steering center is analyzed in the case of a given path. At the same time, the law of the change of the rotation angle of each wheel with the change of the number of axles and the turning radius of the vehicle in the case of fixed total wheelbase is obtained. Therefore, the dynamics modeling of multi-axle vehicles is carried out in the second section, and the control algorithm is introduced and the specific control effect is presented in the third and fourth sections, so as to verify the effectiveness of the control.

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