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PERFORMANCE OF PRODUCTION CELLS WITH SCRAP AND REWORKS: ANALYTICAL AND SIMULATION APPROACH

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Abstract

This study proposes an analytical framework for evaluating manufacturing systems subject to random failures and quality disruptions, where each defective part may be scrapped or reworked for a specific number of times. System behaviour is modelled using a discrete-time Markov chain, allowing the derivation of several key performance indicators (KPIs) including throughput, efficiency, yield, and the required raw material. A simulation model complements the analytical approach, validating the results and offering deeper insights into system behaviour. The close agreement between simulation and analytical results confirms the model's accuracy, with negligible errors across diverse scenarios considering discrete and continuous distributions of the system lifetimes and repairs. The present research also demonstrates that, in contrast to the majority of studies in the literature, which estimate the quality rate by the yield, this substitution results in a significant overestimation of the system throughput by up to 25 %, which has severe effects on budgeting, production planning, and customer satisfaction.

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Key Words: Unreliable Manufacturing Cell, Multi-Rework, Predefined Number of Rework Attempts, Scrapped Parts, KPIs Measures, Analytical-Simulation Modelling

1. INTRODUCTION

In today's industrial landscape, production systems are increasingly expected to be efficient, flexible, and sustainable [1]. Despite the adoption of advanced technologies, random disruptions as breakdowns and defectives remain common, leading to delays and performance losses. Lean methods are therefore essential, enabling the integration of Industry 4.0 technologies while enhancing sustainability by minimizing defects and optimizing non-conformance management [2]. Quality-related issues are particularly critical where non-compliant parts can be scrapped or reworked [3]. In sectors like automotive, aerospace, pharmaceutical, and electronics, rework is often preferred over scrapping, as it supports both economic and environmental goals [4].

During the last decade, an increasing number of studies have explored the impact of rework on production performance. For example, Chao et al. [5] used absorbing Markov chains to model systems with rework and scrap, aiming to estimate production costs. Ni et al. [6] and Jia et al. [7] analysed transfer lines incorporating rework loops, considering finite buffer capacities, and machine failures; however, their models do not account for scrap. Hajji et al. [8] proposed an approximate effectiveness formula for additive manufacturing 3D printers that incorporates breakdowns, rework, scrap, and repair. In all these studies, every part is inspected, and non-conforming items are systematically reworked within the same process, with the assumption that each defective item undergoes only a single rework attempt before being deemed conforming, and no parts are discarded. While this assumption simplifies the analysis, it may be inadequate for addressing complex or persistent defects.

Some studies have gone beyond this limitation by allowing multiple rework attempts. For instance, Peng and Khasawneh [9] and Nesaee and Nezhad [10] developed absorbing Markov models to optimize process targets in reliable systems involving both rework and scrap. Hadjinicola [11] investigated a serial production line in which defects detected at a given station are reworked at the previous one, but machine failures and repairs were not considered. Colledani and Angius [12] proposed a framework for evaluating multi-stage production lines with finite intermediate buffers, leveraging product traceability through in-line inspection and identification technologies. Hajji et al. [13] studied the throughput of a manufacturing machine involving rework and rejection. A common assumption in these studies is that defective parts can undergo an unlimited number of rework attempts. However, this assumption overlooks practical constraints, where excessive rework may lead to higher costs and cycle times than scrapping.

Moreover, Overall Equipment Effectiveness (*OEE*) is widely used as a prevalent metric for evaluating equipment performance, as defined by standards such as ISO 22400:1 [14], and SEMI E79 [15]. *OEE* is traditionally computed as the product of availability, performance, and quality rates [16]. In most applications, however, the quality rate is equated to the yield – that is, the ratio of conforming products to total inputs – thus neglecting reworked items and the performance loss due to rework operations. Some authors, such as de Groote [17] and Wudhikarn [18], have noted this limitation and advocated for the inclusion of both scrapped and reworked items in quality assessments, though without offering explicit methodologies. To overcome these limitations, de Ron and Rooda [19] proposed an alternative definition of *OEE* that incorporates scrap and defines quality as yield. De Ron and Rooda [20] introduced an empirical formula focused solely on rework, while excluding scrap and machine failures. In their approach, the quality rate (*QR*) is also equated to the yield (*Y*), while considering the impact of rework on availability.

To address these gaps, this paper develops comprehensive analytical and simulation models for a manufacturing cell subject to random failures and repairs, while accounting for both scrap and rework process. The analytical model is based on a discrete-time Markov chain and allows evaluating key performance indicators: throughput, efficiency, availability, quality rate, yield as well as the number of good, scrapped, and raw parts. It considers various rework strategies including limited, unlimited, or even no rework. A key contribution lies in its ability to explicitly capture the effect of the number of rework attempts on system performance, without relying on empirical approximations. Moreover, the proposed model generalizes several classical formulations from the literature as special cases. To validate the analytical results, a simulation model is developed for discrete lifetime and repair time distributions. Compared to simulation results, the analytical models have also been tested using continuous distributions, confirming its applicability and robustness in both discrete and continuous settings. Finally, a comparative analysis is carried out to highlight the superiority and accuracy of the proposed approach compared to alternative models widely used in both academic and industrial contexts.

The remainder of the paper is structured as follows: Section 2 describes the manufacturing cell and notations. Section 3 proposes the general analytical Markovian model and illustrates how existing models can be derived as particular cases. Section 4 develops the simulation model used for validating and extending analytical model in both discrete and continuous settings. Section 5 discusses the influence of quality-related parameters on system performance and compares the proposed model with existing approaches. Finally, Section 6 concludes the paper and suggests directions for future research.

2. MANUFACTURING CELL DESCRIPTION AND NOTATIONS

The focus of this study is the evaluation of a manufacturing cell that integrates an automated machine able of handling production, inspection, and rework processes. This cell is designed to produce a single type of product, with each part being inspected for quality conformance after production. The inspection process is integrated within the automated machine (Fig. 1).

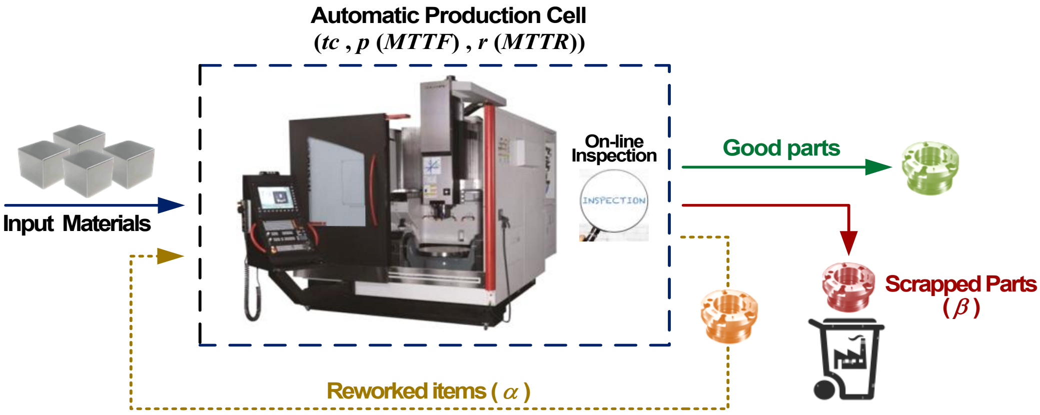


Figure 1: Manufacturing cell subject to breakdowns, scrap, and multi-rework.

Inspection time is incorporated into the processing cycle. After the inspection, a produced item may end up in one of three possible states:

- Accepted: If the part meets the required specifications and is deemed good.
- Reworked with probability α: If the part is non-conforming but has the potential to be restored to compliance with the required specifications.
- Rejected with probability β: If the part is found non-conforming and cannot be repaired.

A non-conforming item can be reworked up to a maximum of *NR* attempts. The rework process occurs on the same machine and requires a specific amount of time, equal to the processing time, and is denoted as *tc*.

The manufacturing cell is also subject to random breakdowns and repairs. The probability that the machine fails during a production or rework cycle is denoted by *p*, while the probability of successful repair is *r*. These probabilities are closely related to the system's reliability characteristics, where the Mean Time To Failure (*MTTF*) defines the average operational time before a failure occurs, and the Mean Time To Repair (*MTTR*) represents the average duration required to restore the machine to working condition after a breakdown.

3. PERFORMANCE MODELLING OF THE MANUFACTURING CELL

Rework helps recover defective parts, improving quality and reducing waste. This section presents a general model for evaluating the performance of unreliable manufacturing cells with scrap and multiple rework attempts.

3.1 Mathematical model

The Markov chain model proposed in Fig. 2 captures the system's stochastic behaviour by integrating failure, repair, scrap and rework. Non-conforming parts may undergo up to *NR* rework attempts, after which they are accepted.

The proposed Markov chain model comprises $2\times(NR + 2)$ states, representing the various stages of production and rework. These states are defined as follows:

- *GP* (Good Product): Represents a conforming item that has successfully passed inspection.
- *S* (Scrap): Represents a non-conforming item that cannot be reworked and is therefore discarded.
- R_i (Rework States) ($i = 1, ..., NR$): Represent an item undergoing an i^{th} rework attempt.
- F_j (Failure States) ($j = GP, S, R_i$ with $i = 1, ..., NR$): Represent failures occurring when the manufacturing system is in state *GP*, *S*, or R_i, respectively.

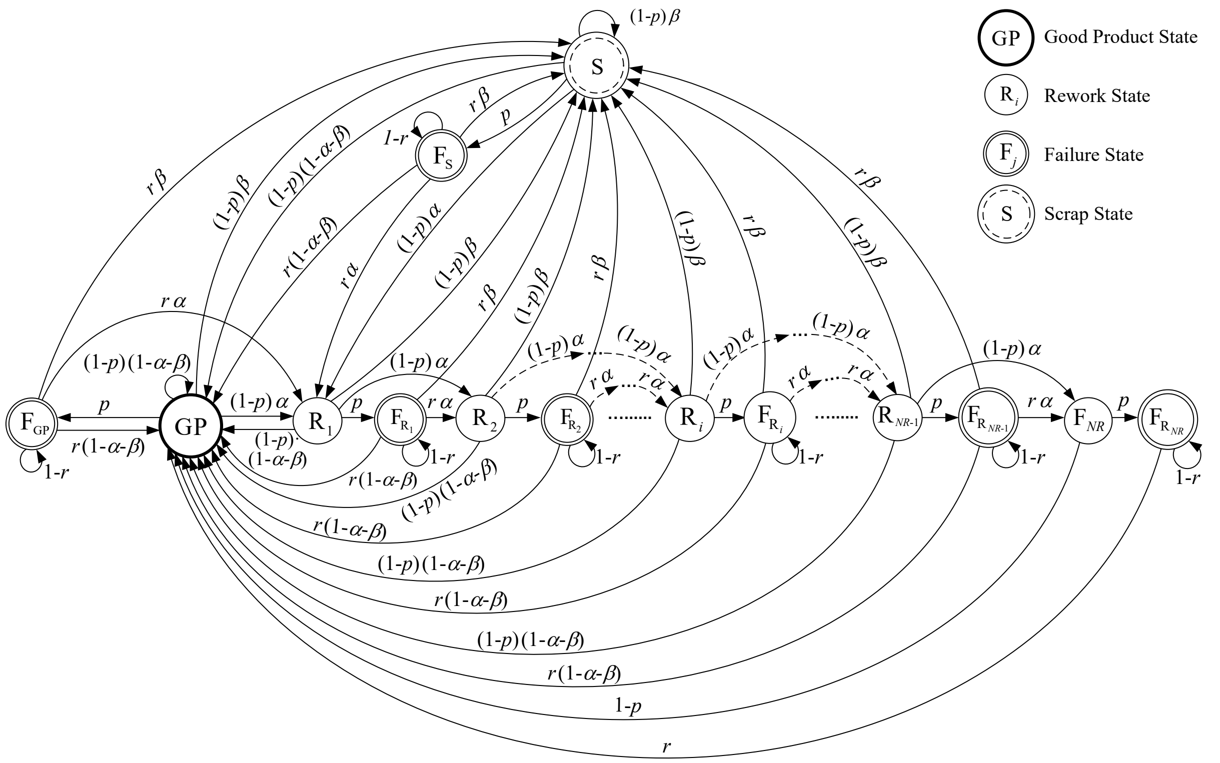


Figure 2: Markov chain model for the manufacturing cell under failures, scrap, and multiple reworks.

The state transition dynamics governing the production cell can be expressed by Eqs (1) to (7).

$$P_{GP} = (1-p)(1-\alpha-\beta)P_{GP} + r(1-\alpha-\beta)P_{FGP} + (1-p)\sum_{i=1}^{NR-1}(1-\alpha-\beta)P_{R_i} + r\sum_{i=1}^{NR-1}(1-\alpha-\beta)P_{FR_i} + (1-p)P_{R_{NR}} + r\,P_{FR_{NR}} + (1-p)(1-\alpha-\beta)P_S + r(1-\alpha-\beta)P_{FS} \tag{1}$$

$$P_{F_{GP}} = pP_{GP} + (1-r)P_{FG} \tag{2}$$

$$P_{R_1} = (1-p)\alpha P_{GP} + r\alpha P_{FGP} + (1-p)\alpha P_S + r\alpha P_{FS} \tag{3}$$

$$P_{R_i} = (1-p)\alpha P_{R_{i-1}} + r\alpha P_{FR_{i-1}};\, i = 2,..,NR \tag{4}$$

$$P_{F_{R_i}} = pP_{R_i} + (1-r)P_{FR_i};\, i = 1,..,NR \tag{5}$$

$$P_S = (1-p)\beta P_{GP} + r\beta P_{FGP} + (1-p)\sum_{i=1}^{NR-1}\beta P_{R_i} + r\sum_{i=1}^{NR-1}\beta P_{FR_i} + (1-p)\beta P_S + r\beta P_{FS} \tag{6}$$

$$P_{F_S} = pP_S + (1-r)P_{FS} \tag{7}$$

Solving these equations provides the steady-state probabilities of the different states, offering insights into the long-term performance of the manufacturing system, Eqs. (8) to (13).

$$P_{GP} = \frac{r}{r+p}\left(\frac{1}{1+\sum_{i=1}^{NR}\alpha^i} - \beta + \beta\frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{8}$$

$$P_{F_{GP}} = \frac{p}{r+p}\left(\frac{1}{1+\sum_{i=1}^{NR}\alpha^i} - \beta + \beta\frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{9}$$

$$P_{R_i} = \frac{r}{r+p}\left(\frac{\alpha}{1+\sum_{i=1}^{NR}\alpha^i}\right)\ ;\, i = 1,..,NR \tag{10}$$

$$P_{F_{R_i}} = \frac{p}{r+p}\left(\frac{\alpha}{1+\sum_{i=1}^{NR}\alpha^i}\right)\ ;\, i = 1,..,NR \tag{11}$$

$$P_S = \frac{r}{r+p}\beta\left(1 - \frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{12}$$

$$P_{F_S} = \frac{p}{r+p}\beta\left(1 - \frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{13}$$

3.2 Evaluation of Key Performance Indicators

The proposed Markov model enables the evaluation of several key performance indicators. In the literature, throughput and efficiency are the most commonly studied metrics. Throughput (*Th*) reflects the long-term average production rate and it depends on cycle time and the manufacturing system efficiency, Eq. (14) [21].

$$Th = \frac{E}{tc} \tag{14}$$

Cell efficiency measures the time the machine effectively produces conforming parts and is often linked to Overall Equipment Effectiveness (*OEE*), which combines availability, quality, and performance ratios, Eq. (15) [16].

$$OEE = UTR \cdot PR \cdot QR \tag{15}$$

Here, *UTR*, *PR*, and *QR* denote the Up Time Ratio, Performance Ratio, and Quality Ratio, respectively. In this study, the manufacturing cell efficiency is deduced from the steady-state probability of the system being in the *GP* state, Eq. (8), and it is given by Eq. (16).

$$E = P_{GP} = \frac{r}{r+p}\left(\frac{1}{1+\sum_{i=1}^{NR}\alpha^i} - \beta + \beta\frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{16}$$

The availability of the cell, calculated from Eq. (17), is obtained by excluding periods when the system is in a failure state F_j (j = *GP*, *S*, and R_i).

$$UTR = 1 - \sum_j P_{F_j} = \frac{r}{r+p} \tag{17}$$

It is important to note that Eq. (16) shows that the expression for efficiency is the product of two indicators: *UTR*, Eq. (17), and *QR* which is expressed by Eq. (18).

$$QR = \frac{1}{1+\sum_{i=1}^{NR}\alpha^i} - \beta + \beta\frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i} \tag{18}$$

This confirms the *OEE* formulation in Eq. (15). Since the automated manufacturing cell operates at full capacity, *PR* is set to 100 %. Besides efficiency and availability, the proposed Markov model evaluates other key indicators, including the number of good parts, Eq. (19), rejected parts, Eq. (20), and required raw parts, Eq. (21), over a specific horizon *T*.

$$NbGP = \frac{T}{tc}P_{GP} = \frac{T}{tc}\frac{r}{r+p}\left(\frac{1}{1+\sum_{i=1}^{NR}\alpha^i} - \beta + \beta\frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{19}$$

$$NbSP = \frac{T}{tc}P_{SP} = \frac{T}{tc}\frac{r}{r+p}\beta\left(1 - \frac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{20}$$

$$NbRP = NbGP + NbSP = \frac{T}{tc}\frac{r}{r+p}\left(\frac{1}{1+\sum_{i=1}^{NR}\alpha^i}\right) \tag{21}$$

The cell yield is calculated from Eqs. (19) and (21), and is given by Eq. (22).

$$Y = 1 - \beta - \beta\sum_{i=1}^{NR-1}\alpha^i \tag{22}$$

3.3 Specific cases

The general model presented in Fig. 2 can be adapted to various industrial contexts. When the manufacturing cell produces only conforming parts (α = β = 0), the system performance depends solely on its availability, see Eq. (17). In contrast, in some industries like dental prosthetics, non-compliant parts cannot be rejected due to high costs and customization. Thus, these items are reworked repeatedly until they meet specifications (β = 0; $NR \rightarrow \infty$). Thus, the number of good parts equals the raw parts introduced. However, in other industrial contexts, non-conforming parts can undergo unlimited rework attempts until compliance ($NR \to \infty$). This is common in the automotive industry for critical components like engine blocks or cylinder heads, where defects are corrected through machining, honing, or weld repair to meet strict standards. In some production systems, only one rework attempt is allowed per defective unit before final acceptance or rejection ($NR = 1$). For example, in pharmaceutical packaging, a single correction fixes an under filled blister pack. Nevertheless, in the glass or ceramic tile manufacturing, non-conforming parts cannot be reworked ($\alpha = 0$) and are systematically rejected with probability β.

4. SIMULATION MODEL

To capture the stochastic and dynamic behaviour of a manufacturing cell subject to random breakdowns, repairs, multi-rework, and scrap, a comprehensive simulation model was developed using Arena software [22]. The model (Fig. 3) serves as a digital and virtual environment to test scenarios, evaluate performance, and support informed decision-making without disrupting real operations [23]. It allows the evaluation of several KPIs under various system configurations, including efficiency, throughput, and counts of conforming, defective, and raw material parts. A wide range of experiments was performed by varying operational, reliability and quality parameters: tc, p ($MTTF$), r ($MTTR$), α, β, and NR.

Each simulation runs for 1,000,000 time units, with an initial warm-up phase of 10,000 time units to eliminate transient effects and ensure the reliability of collected statistics. For every configuration tested, ten replications were executed.

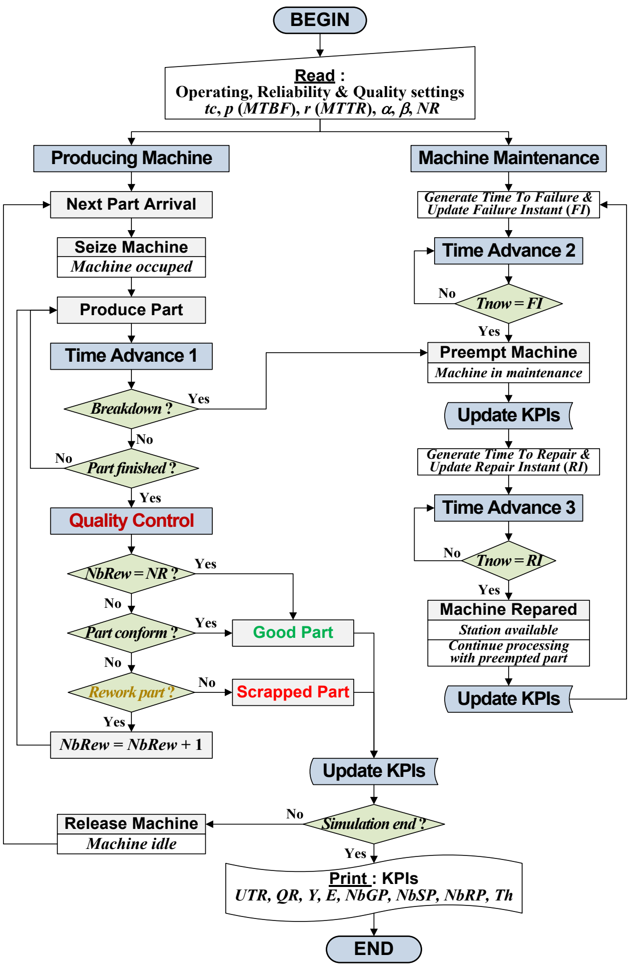


Figure 3: General simulation model for the manufacturing cell.

Fig. 3 illustrates the structure of the simulation model, which consists of seven main modules:

1. 'Read' module: defines, for each experiment, the processing time (*tc*), failure and repair distributions, rework (α) and scrap (β) probabilities per attempt, and the maximum number of rework attempts (*NR*).
2. 'Producing Machine' module: oversees part flow through the cell, including the coordination of machine usage through seize and release operations.
3. 'Quality Control' module: after completion, each item undergoes inspection, where rework or scrap decisions are made based on the number of rework attempts.
4. 'Machine Maintenance ' module: Failure and repair times for the manufacturing machine are generated based on their respective distributions.
5. 'Time Advance' module: The simulation follows a discrete-event structure, with the clock moving forward to the nearest upcoming event.
6. 'Update KPIs' module: It tracks raw parts, good outputs, scraps, and rework activities to evaluate the cell's KPIs.
7. 'Print KPIs' module: It allows the display of the main KPIs (*UTR*, *QR*, *Y*, *E*, *NbGP*, *NbSP*, *NbRP*, and *Th*) according to system parameters and simulation conditions.

When the current simulation time *TNOW* hits the simulation span, the simulation is over.

5. NUMERICAL RESULTS

5.1 Validation of proposed analytical models

To validate the robustness and precision of the proposed analytical models, hundreds of experiments were conducted using randomly generated values for operational, reliability, and quality parameters. As mentioned in Section 3, no restrictions are imposed on the types of probability distributions used to model the machine's lifetimes and repair times. This section analyses both discrete and continuous distributions for machine lifetime and repair durations, starting with geometric distributions before extending to more general distributions.

Table I lists 15 randomly generated cell configurations – defined by machine lifetime and repair time distributions, as well as cell parameters including *tc*, *MTTF*, *MTTR*, α, β, and *NR* – covering all scenarios described in Section 3.3.

Table I: Operational, reliability, and quality discrete parameters of manufacturing cell.

	Operation	Failure		Maintenance		Quality		
Case	***tc*** **(min)**	***Distribution***	***MTTF*** **(min)**	***Distribution***	***MTTR*** **(min)**	α	β	***NR***
Base	**0.31**	**Geometric (p = 0.0006)**	**516.67**	**Geometric (r = 0.0080)**	**38.75**	**0.11**	**0.05**	**3**
1	0.49	Geometric (p = 0.0030)	163.33	Geometric (r = 0.0070)	70.00	0.16	0.07	∞
2	0.84	Geometric (p = 0.0042)	200.00	Geometric (r = 0.0350)	24.00	0.13	0.03	1
3	0.58	Geometric (p = 0.0018)	322.22	Geometric (r = 0.0056)	103.57	0	0.09	0
4	0.63	Geometric (p = 0.0051)	123.53	Geometric (r = 0.0300)	21.00	0.09	0.04	4
5	0.98	Geometric (p = 0.0009)	1088.89	Geometric (r = 0.0245)	40.00	0.05	0.07	2
6	0.25	Exponential (λ = 0.0054)	185.19	Gamma (k = 2; θ = 40)	80.00	0.13	0.05	4
7	0.47	Weibull (k = 2; λ = 67)	59.38	Lognormal (μ = 2.5; σ = 0.19)	12.40	0	0	0
8	0.55	Exponential (λ = 0.0013)	769.23	Exponential (λ = 0.0141)	70.92	0.19	0.09	2
9	0.71	Normal (μ = 416.8; σ = 36.8)	416.8	Gamma (k = 3; θ = 33.4)	100.2	0.11	0.04	2
10	0.28	Exponential (λ = 0.007)	142.86	Weibull (k = 3; λ = 55.27)	49.37	0.15	0.05	4
11	0.87	Weibull (k = 3; λ = 283)	252.76	Lognormal (μ = 2.9; σ = 1.02)	30.56	0.16	0.03	∞
12	0.42	Normal (μ = 91.34; σ = 21.8)	91.34	Gamma (k = 2; θ = 17.26)	34.52	0.12	0.08	3
13	0.35	Lognormal (μ = 5; σ = 1.33)	359.4	Exponential (λ = 0.032)	31.25	0.06	0.11	5
14	0.93	Weibull (k = 3; λ = 726.8)	649.11	Weibull (k = 2; λ = 98.27)	87.04	0.1	0.05	1
15	0.48	Exponential (λ = 0.0076)	131.58	Lognormal (μ = 3.3; σ = 1.22)	57.02	0.18	0.06	2

Table II presents the manufacturing cell KPIs obtained from both the analytical and the simulation models, according to the cell configurations listed in Table I. Throughput (Th) is expressed in parts per hour. Table II also includes the mean relative errors between analytical and simulation results, Eq. (23):

$$\varepsilon\ (\%) = \frac{KPI_{sim} - KPI_{anal}}{KPI_{sim}} 100 \tag{23}$$

where KPI_{Sim} and KPI_{Anal} denote, respectively, the simulated and analytical values of a given performance indicator.

Table II: Analytical versus simulation KPIs of manufacturing cell.

Case		***UTR*** (%)	***QR*** (%)	***Y*** (%)	***E*** (%)	***NbGP***	***NbSP***	***NbRP***	***Th***
Base case	Analytical	93.02	84.02	94.39	78.16	2,495,988	148,361	2,644,349	151.272
	Simulation	93.03	84.02	94.39	78.17	2,496,301	148,368	2,644,669	151.291
	$\mathcal{E}$ (%)	0.007	0.006	0.000	0.013	0.013	0.005	0.012	0.013
1	Analytical	70.00	77.00	91.67	53.90	1,088,993	98,999	1,187,993	66.000
	Simulation	70.02	77.01	91.67	53.92	1,089,394	98,972	1,188,366	66.024
	$\mathcal{E}$ (%)	0.023	0.013	0.005	0.037	0.037	-0.028	0.031	0.037
2	Analytical	89.29	85.84	97.00	76.64	903,298	27,937	931,235	54.745
	Simulation	89.31	85.84	97.00	76.66	903,532	27,940	931,472	54.760
	$\mathcal{E}$ (%)	0.03	-0.0013	0.0005	0.03	0.026	0.011	0.025	0.026
3	Analytical	75.68	91.00	91.00	68.86	1,175,454	116,254	1,291,708	71.240
	Simulation	75.66	91.00	91.00	68.85	1,175,256	116,176	1,291,432	71.228
	$\mathcal{E}$ (%)	-0.02	0.00	0.00	-0.02	-0.017	-0.067	-0.021	-0.017
4	Analytical	85.47	87.00	95.60	74.36	1,168,509	53,721	1,222,230	70.819
	Simulation	85.51	86.99	95.60	74.39	1,168,956	53,743	1,222,699	70.846
	$\mathcal{E}$ (%)	0.047	-0.008	0.000	0.038	0.038	0.041	0.038	0.038
5	Analytical	96.46	88.03	92.65	84.91	857,758	68,047	925,805	51.985
	Simulation	96.47	88.03	92.65	84.92	857,841	68,044	925,885	51.990
	$\mathcal{E}$ (%)	0.01	0.00	0.00	0.01	0.010	-0.004	0.009	0.010
6	Analytical	69.83	82.00	94.25	57.27	2,267,739	138,235	2,405,974	137.439
	Simulation	69.90	82.01	94.27	57.32	2,269,985	138,102	2,408,087	137.575
	$\mathcal{E}$ (%)	0.096	0.003	0.011	0.099	0.099	-0.096	0.088	0.099
7	Analytical	82.72	100.00	100.00	82.72	1,742,505	0	1,742,505	105.606
	Simulation	82.74	100.00	100.00	82.74	1,742,856	0	1,742,856	105.628
	$\mathcal{E}$ (%)	0.018	0.002	0.000	0.020	0.020	0.000	0.020	0.020
8	Analytical	91.56	72.82	89.29	66.68	1,200,187	143,958	1,344,145	72.739
	Simulation	91.58	72.83	89.29	66.69	1,200,454	143,963	1,344,417	72.755
	$\mathcal{E}$ (%)	0.019	0.003	0.002	0.022	0.022	0.004	0.020	0.022
9	Analytical	80.62	85.16	95.56	68.66	957,323	44,480	1,001,803	58.020
	Simulation	80.62	85.16	95.56	68.65	957,237	44,470	1,001,707	58.014
	$\mathcal{E}$ (%)	-0.004	-0.005	0.001	-0.009	-0.009	-0.023	-0.010	-0.009
10	Analytical	74.32	80.01	94.12	59.46	2,102,342	131,326	2,233,668	127.415
	Simulation	74.37	80.01	94.12	59.50	2,103,639	131,380	2,235,019	127.493
	$\mathcal{E}$ (%)	0.066	-0.004	0.001	0.062	0.062	0.041	0.060	0.062
11	Analytical	89.21	81.00	96.43	72.26	822,303	30,456	852,759	49.837
	Simulation	89.22	81.00	96.43	72.27	822,356	30,436	852,792	49.840
	$\mathcal{E}$ (%)	0.012	-0.005	0.003	0.006	0.006	-0.065	0.004	0.006
12	Analytical	72.57	80.03	90.92	58.08	1,369,034	136,643	1,505,677	82.972
	Simulation	72.57	80.03	90.93	58.08	1,369,025	136,595	1,505,620	82.971
	$\mathcal{E}$ (%)	-0.001	0.000	0.003	-0.001	-0.001	-0.035	-0.004	-0.001
13	Analytical	92.00	83.00	88.30	76.36	2,159,909	286,253	2,446,162	130.904
	Simulation	91.99	82.99	88.30	76.35	2,159,483	286,260	2445743	130.878
	$\mathcal{E}$ (%)	-0.011	-0.008	-0.003	-0.020	-0.020	0.003	-0.017	-0.020
14	Analytical	88.18	86.36	95.00	76.15	810,653	42,666	853,319	49.131
	Simulation	88.16	86.35	95.00	76.13	810,421	42,657	853,078	49.116
	$\mathcal{E}$ (%)	-0.019	-0.010	0.000	-0.029	-0.029	-0.021	-0.028	-0.029
15	Analytical	69.77	76.64	92.92	53.47	1,102,822	84,029	1,186,851	66.838
	Simulation	69.83	76.64	92.92	53.52	1,103,846	84,099	1,187,945	66.900
	$\mathcal{E}$ (%)	0.091	0.002	0.001	0.093	0.093	0.083	0.092	0.093

The results show first that the mean relative errors are negligible for all cell configurations – with discrete and continuous probability distributions – and for all KPIs; the relative errors remain under 0.1 %. This strong agreement between the two modelling approaches confirms the robustness and the accuracy of the proposed analytical approach.

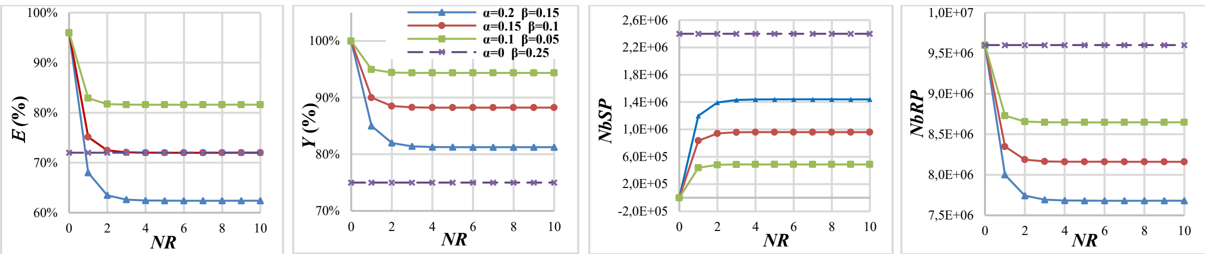
The results also show that, although the analytical model proposed in section 3.1 is based on Markov chains that consider discrete probability distributions, the proposed analytical formulations of all KPIs remain valid in the case where the lifetimes and repair distributions are of general continuous form – Exponential, Weibull, Normal, Gamma, Lognormal, ... – simply by substituting the cell availability dictated by Eq. (17), in the case of discrete distributions (geometric), by Eq. (24) taking into account the *MTTF* and *MTTR* of the cell.

$$UTR = \frac{MTTF}{MTTF + MTTR} \tag{24}$$

Overall, the close alignment between analytical and simulation results across all configurations and performance metrics validates the proposed models for both types of distributions – discrete-time and continuous distributions.

5.2 Impact of quality parameters

The numerical results presented in Table II highlight the significant impact of the number of rework attempts (*NR*) and quality parameters (α and β) on the overall performance of the manufacturing cell. Fig. 4 illustrates the influence of *NR* on the cell performance, when considering the base case in Table I. Four different combinations of the quality parameters α and β are considered: (0.2, 0.15), (0.15, 0.1), (0.1, 0.05), and (0, 0.25).



a) Effectiveness b) Yield c) Scrapped parts d) Material consumption

Figure 4: Impact of α, β and *NR* on the manufacturing cell KPIs.

As shown in Fig. 4, cell effectiveness (a), the yield (b), and the number of required raw parts (d) decrease with increasing rework attempts (*NR*), then stabilize to an asymptotic value. In contrast, the number of scrapped parts (*NbSP*) (c) increases before levelling off. These trends suggest that beyond a certain *NR*, additional rework no longer significantly impacts performance. Optimal results for efficiency, yield, and scrap are achieved with a single rework attempt. This means that while rework reduces material consumption (*NbRP*), it also introduces non-value-added time and increases the risk of scrapping, thereby reducing overall efficiency and yield. At the same time, the probabilities of rework (α) and scrap (β) are also critical. Under ideal quality (β = 0), effectiveness and yield reach 95 % and 100 %, respectively. Inversely, when rework is not allowed (α = 0), performance drops. Still, for moderate values rework remains preferable to rejection.

5.3 Throughput: proposed model versus existing formulations

Unlike most previous studies that estimate throughput based solely on yield (*QR* – ratio of conforming parts to total output) – including de Groote [17], Wudhikarn [18], de Ron and Rooda [19, 20], Nakajima [24], Jaqin et al. [25], Facchinetti and Citterio [26], Stefana et al. [27], and Liew et al. [28] – our findings challenge this assumption. In this section, we compare the throughput estimated by existing models in the literature (Th_1) with that of our proposed model (Th_2), introduced in Section 3. The analysis is conducted using the base-case configuration of the manufacturing cell (Table I), with varying values of the parameters α and NR. Fig. 5 displays the throughput values obtained from both models (Th_1 and Th_2) across different values of α and NR, along with their relative error, Eq. (25).

The results clearly show that the proposed model of the throughput (Th_2) significantly deviates from the yield-based estimate (Th_1), especially when rework strategies are implemented. The discrepancy between the two can reach up to 25 % when α = 20 %, leading to substantial differences in cell productivity and potentially resulting in suboptimal decisions regarding production planning and resource allocation.

$$\varepsilon\ (\%) = \frac{Th_1 - Th_2}{Th_2}100 \tag{25}$$

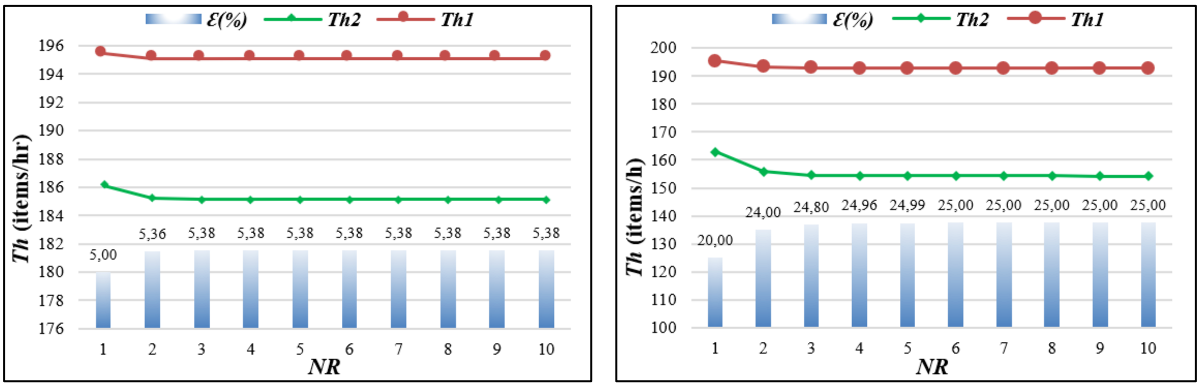
This deviation is due to the greater value of the yield, Eq. (22), over the value of the quality ratio, Eq. (18). In fact, consider the ratio between these two corresponding expressions, Eq. (26).

$$\frac{Y}{QR} = \frac{1-\beta-\beta\sum_{i=1}^{NR-1}\alpha^i}{\dfrac{1}{1+\sum_{i=1}^{NR}\alpha^i}-\beta+\beta\dfrac{\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}} \tag{26}$$

The simplification of the above expression leads to:

$$\frac{Y}{QR} = \frac{1-\beta-\beta\sum_{i=1}^{NR-1}\alpha^i}{\dfrac{1-\beta\left(1+\sum_{i=1}^{NR}\alpha^i\right)+\beta\alpha^{NR}}{1+\sum_{i=1}^{NR}\alpha^i}} = \frac{1-\beta-\beta\sum_{i=1}^{NR-1}\alpha^i}{\dfrac{1-\beta-\beta\sum_{i=1}^{NR-1}\alpha^i}{1+\sum_{i=1}^{NR}\alpha^i}} = 1+\sum_{i=1}^{NR}\alpha^i \tag{27}$$

Since, in Eq. (27), the term $1+\sum_{i=1}^{NR}\alpha^i$ is always ≥ 1 it follows that the yield (Y) is always greater than or equal to the quality ratio (QR). This observation is also supported by the simulation results in Table II, which confirm a consistent divergence between the two indicators when rework is taken into account.



a) α = 5 %; β = 5 %   b) α = 20 %; β = 5 %

Figure 5: Th_1 versus Th_2 (proposed model).

6. CONCLUSIONS

This study presents a performance evaluation model for unreliable manufacturing systems, using a Markov chain framework. The model accounts for machines that produce both conforming and non-conforming parts, the latter being either reworked or scrapped. A general analytical approach has been developed to estimate key performance indicators (KPIs), such as cell effectiveness, throughput, availability, quality rate, yield, and the quantities of good, scrapped, and raw parts.

The methodology explicitly incorporates the fixed number of authorized rework attempts (NR). From this general model, several specific scenarios have been derived to reflect diverse industrial realities, including perfect quality, rejection of defective items, infinite or single rework attempts. The model's accuracy has been validated through simulation, confirming its ability to reliably assess overall system performance. By modelling the effects of rework strategies, the proposed approach offers valuable insights into how quality-related decisions influence operational outcomes. The findings indicate that increasing the number of rework attempts generally leads to lower system effectiveness and yield, due to extended processing times and a higher likelihood of rejection. However, it also helps reduce raw material consumption by enabling the recovery of non-compliant parts.

Unlike many previous studies that approximate the quality rate using the yield, simulation results from this work demonstrate that the two metrics differ significantly when rework strategies are applied. Treating yield as a proxy for quality rate tends to overestimate system performance, which can lead to misguided decisions in production planning and resource allocation. In addition, the results show that the proposed rework-based strategy yields substantial improvements in performance compared to the conventional strategy of rejecting all non-compliant parts for the lower values of rework and scrap rates.

In the context of modern manufacturing, integrating models that encompass production, breakdowns, repairs, and rework marks a significant advancement in evaluating system performance with greater precision and efficiency. Future research could focus on adapting this approach to multi-product manufacturing cells, particularly in environments characterized by large batches or high product variety with low volumes. The model could also be extended to more complex multi-station systems, incorporating cost-driven optimization by aligning economic indicators with performance metrics. Sustainability considerations – such as energy usage, carbon footprint, and overall environmental impact – could also be embedded, supporting the transition toward Industry 5.0 principles.

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