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Generalised all-optical cat correction

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E-mail: ari-john.boon@polymtl.ca**Keywords:** quantum optics, continuous variable quantum error correction, quantum communications**Abstract**

We have generalised an all-optical telecorrection protocol for the higher orders of the cat code, and show that with these higher orders we can achieve target performance at substantially reduced iteration counts at the cost of a higher mean photon-number. We also introduce a probabilistic scheme for correcting deformation of the state, which highlights two interesting abilities of telecorrection: to encode new sets of transformations, and to change the basis of the code. We find that for a target channel fidelity of 99.9% over a channel with 1dB of loss, a third-order cat code requires 70 times fewer telecorrection iterations than a first-order one, at a cost of a 3.6-fold increase in mean photon-number.

1. Introduction

Photonics is the natural platform for quantum communications, but is subject predominantly to the effects of loss, which will quickly obliterate the quantum state one wishes to transmit. As a bosonic system, optical quantum communications can benefit from the numerous bosonic codes that have been shown to suppress the effects of loss [1–8]. Among these codes, however, their performance against loss has frequently been assessed without consideration for practical photonic implementations, where the deterministic operations and measurements one has access to are considerably limited. The cat code is in the class of rotation-symmetric bosonic (RSB) codes, which has a clear mechanism for loss-correction and an explicit implementation, but relies on access to two-mode rotations [6]. Optically, the only easily accessible two-mode operation is the beamsplitter, while deterministic single-mode operations are limited to Gaussian operations. The most accessible optical nonlinearity comes from photon-number measurement, which demands cryogenic cooling for what is otherwise a room-temperature system.

More recent works have begun to consider optical implementation more explicitly, as all-optical loss-correction circuits have been proposed for the GKP [9] and cat codes [10, 11]. For cat codes in particular, because they are defined in the coherent state basis, it is possible to accurately and efficiently simulate their optical dynamics analytically [12, 13]. It has also been shown that by iteratively correcting them one can achieve seemingly arbitrary performance with a relatively simple optical circuit [10]. However, in that proposal, the cost of arbitrary performance is a steep increase in the number of iterations and an increasing mean photon-number. This means that even with its simple three-mode circuit, minimising error rates demands an exceedingly large number of photon-number measurements and ancilla states over the course of transmission.

The optical schemes proposed in [10, 11] were limited to cat codes which can only be explicitly corrected from single-photon losses. These represent only the lowest-order of loss-correcting cats [4], which have been shown to be inferior in terms of absolute performance versus loss relative to their higher-order counterparts [5, 7]. As higher-order cats retain their relatively compact and optically-convenient basis of coherent states, it is natural to assume that they too have a straightforward way of being implemented optically. Here we show that not only is this possible, but it is advantageous, as higher-order cat codes can be used to greatly reduce the number of circuit iterations one requires, albeit at the cost of working in a higher mean photon-number regime. As research into realising high-resolution

photon-number measurements [14–16] and photonic state-generation [17–22] is ongoing and fruitful, our results show a valuable new axis along which to distribute the resource cost of telecorrection.

The outline of our paper is as follows. In section 2, we describe the structure and properties of the cat code, in particular the origin of their loss capacity and sources of inherent error. In section 3.1, we provide the circuit used for cat telecorrection, and write the generalised syndrome which indicates logical X and Z errors and quantifies additional errors in the state. In section 3.2, we numerically compare the performance of different orders of the cat code in terms of the mean channel fidelity, and find the parameter requirements for each code order to achieve certain target fidelities. In section 4, we propose a scheme to probabilistically correct additional errors in the cat states that cannot be removed deterministically, for which we outline two interesting properties of the telecorrection scheme in section 4.1: the ability to encode new transformations, and to change the basis of our input state. In section 5, we summarise our results, highlighting the experimental necessities for realising higher-order cat correction based on our generalised protocol.

2. Multicomponent cat codes

The logical states of the cat code are constructed from superpositions of coherent states,

$$|e^{i\phi}\alpha\rangle = e^{-\alpha^2/2} \sum_{n=0}^{\infty} \frac{(e^{i\phi}\alpha)^n}{\sqrt{n!}} |n\rangle, \quad (1)$$

for $\alpha \in \mathbb{R}_{>0}$, where $|n\rangle$ is the Fock state for photon-number n . Coherent states are the eigenstates of the annihilation operator,

$$\hat{a}|e^{i\phi}\alpha\rangle = e^{i\phi}\alpha|e^{i\phi}\alpha\rangle, \quad (2)$$

leading the mean photon-number to be

$$\langle e^{i\phi}\alpha | \hat{n} | e^{i\phi}\alpha \rangle = |\alpha|^2, \quad (3)$$

where $\hat{n} = \hat{a}^\dagger \hat{a}$. The number of coherent states in a cat state determines the degree of discrete loss events that can be recovered from [4]. For a cat code with L -photon loss capacity, we define the logical states as

$$|\tilde{0}_L\rangle = \frac{1}{\sqrt{\mathcal{N}_0}} \sum_{m=0}^{2L+1} |e^{im\phi_L}\alpha\rangle, \quad (4)$$

$$|\tilde{1}_L\rangle = \frac{1}{\sqrt{\mathcal{N}_1}} \sum_{m=0}^{2L+1} (-1)^m |e^{im\phi_L}\alpha\rangle, \quad (5)$$

where $\phi_L \equiv \pi/(L+1)$, a tilde indicates the logical basis, and L the code order, though we will leave the order implicit when possible. The loss capacity of the cat code arises from the Fock distance between the codewords, which can be re-written as

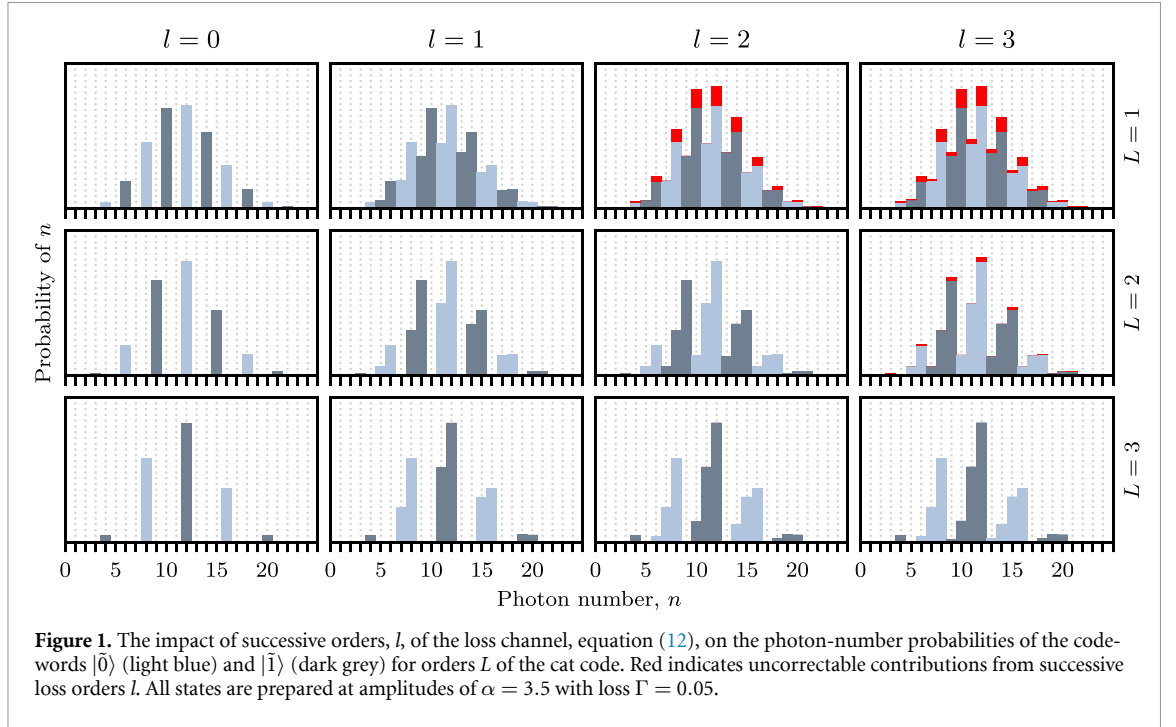
$$|\tilde{0}_L\rangle \propto \sum_{n=0}^{\infty} \frac{\alpha^{2n(L+1)}}{\sqrt{[2n(L+1)]!}} |2n(L+1)\rangle, \quad (6)$$

$$|\tilde{1}_L\rangle \propto \sum_{n=0}^{\infty} \frac{\alpha^{(2n+1)(L+1)}}{\sqrt{[(2n+1)(L+1)]!}} |(2n+1)(L+1)\rangle, \quad (7)$$

where we see that our codewords are separated in Fock space by a distance $(L+1)$, and that at a given order the qubit will be distinguished by support on every $n \equiv 0 \pmod{(L+1)}$. Therefore photon losses up to L can be identified by the Fock support of the lossy qubit. In the $L=1$ code, for instance, there is a tolerance for up to a single-photon loss, with even Fock support corresponding to the lossless state, and odd Fock support corresponding to the state after the single-photon loss.

We can re-frame this in terms of the Knill–Laflamme conditions [23], which stipulate that we can recover our logical state from a set of errors $\{\hat{E}_i\}$ if they satisfy

$$\langle \tilde{J} | \hat{E}_i^\dagger \hat{E}_j | \tilde{J}' \rangle = \epsilon_{ij} \delta_{J,J'}, \quad J, J' \in \{0, 1\}, \quad (8)$$



where $\epsilon_{i,j}$ is independent of J and J' . If our errors are loss events, then we can identify the recoverable sets for different orders of the cat code,

$$L = 1 : \{\hat{E}_i\} = \{\mathcal{I}, \hat{a}\}, \quad (9)$$

$$L = 2 : \{\hat{E}_i\} = \{\mathcal{I}, \hat{a}, \hat{a}^2\}, \quad (10)$$

$$L = 3 : \{\hat{E}_i\} = \{\mathcal{I}, \hat{a}, \hat{a}^2, \hat{a}^3\}, \quad (11)$$

and seek to reduce the effects of higher-order losses outside of these sets through our parameter choices.

Unlike the loss capacity of the cat code, the Kraus decomposition of the loss channel is not finite, where for a given magnitude of energy loss, $\Gamma \in [0, 1]$, we have

$$\mathcal{L}_\Gamma(\rho) = \sum_{l=0}^{\infty} \hat{B}_\Gamma^{(l)} \rho [\hat{B}_\Gamma^{(l)}]^\dagger, \quad (12)$$

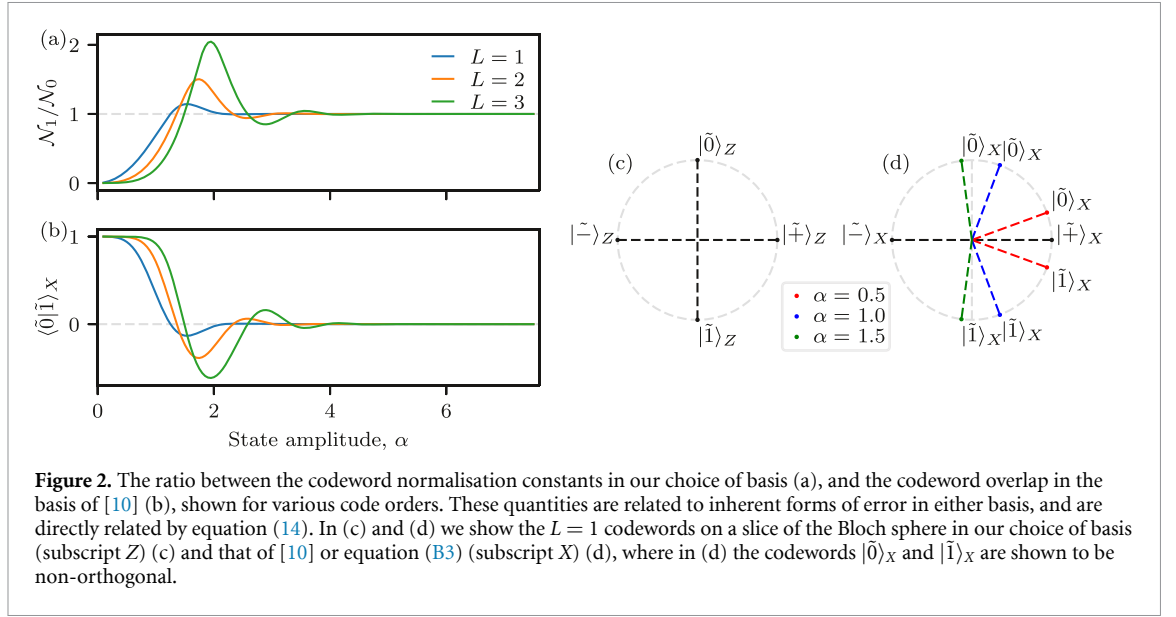
whose Kraus operators are

$$\hat{B}_\Gamma^{(l)} = \sqrt{\frac{\Gamma}{1-\Gamma}}^l \frac{\hat{a}^l}{\sqrt{l!}} \sqrt{1-\Gamma}^{\hat{n}}. \quad (13)$$

Orders $l > L$ of equation (12) will produce uncorrectable errors in an L -loss cat code, as their Fock support is degenerate with the correctable orders of loss, putting them outside of our recoverable sets $\{\hat{E}_i\}$. The amplitude of each term of equation (12) will be determined in part by Γ .

To illustrate the Fock-overlap of higher-order loss terms and the effect of increasing the code order, we show in figure 1 that for a fixed amount of loss and equivalent state amplitudes, successive orders of the cat code see significantly reduced contributions associated with beyond-capacity loss orders (those containing loss events outside of $\{\hat{E}_i\}$). One must therefore perform correction after a sufficiently small amount of loss that allows one to meet their target performance, which we show is possible in section 3.2.

However this does not make higher-order cat codes a universally better choice, as the higher orders suffer more acutely from errors inherent to the correction of the code, independent of loss. This effect is not captured by figure 1, though we will elaborate more fully on it in section 3.1. In brief, these correction errors take the form of a deformation related to the mis-match of the codeword normalisation constants, $\mathcal{N}_0$ and $\mathcal{N}_1$ (derived in appendix A), which is generally worse the lower the state amplitude [4, 5], while errors due to loss are worse with increasing state amplitude due to the Fock-damping operator $\sqrt{1-\Gamma}^{\hat{n}}$ in equation (13). This leads to optimal performance at amplitudes where these two effects



are balanced. Normalisation mis-match is shown in figure 2(a) to both be more extreme and to persist to higher amplitudes in higher-order cat codes, which in turns pushes their optimal amplitudes higher, to be seen in section 3.2.

Notably, this is complementary to the problem of codeword overlap seen in the alternate basis used by [10], whose codewords we specify in appendix B, equation (B3). Codeword overlap arises in that basis due to the non-orthogonality of its codewords $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, shown in figure 2(d), and corresponds to a degree of indistinguishability for states defined that way. This overlap in the alternate basis is plotted in figure 2(b) for increasing code order, where we see it follows similar trends as normalisation mis-match in our choice of basis. In appendix B we derive for all orders a direct relationship between codeword normalisation mis-match in our choice of basis and codeword overlap in the basis of [10],

$$\frac{\mathcal{N}_0}{\mathcal{N}_1} = \frac{1 + \langle \tilde{0} | \tilde{1} \rangle_X}{1 - \langle \tilde{0} | \tilde{1} \rangle_X}, \quad (14)$$

where $\langle \tilde{0} | \tilde{1} \rangle_X$ is the codeword overlap for the basis of [10]. With this we show that when the source of inherent error is small or large in one basis, it is necessarily also so in the other. We also show numerically that for $L = 1$ the differences in performance between these bases are relegated to the sub-optimal regime. State indistinguishability can be easily quantified via codeword overlap, and we will see in section 3.1 that the mechanism by which codeword normalisation mis-match introduces error is independent of code order. Thus, quantifying both properties as in equation (14) and figure 2 gives a reasonable notion of the severity of their associated errors.

3. Generalised cat telecorrection

Using the telecorrection protocol proposed in [10] for the $L = 1$ cat code, we find that the same optical circuit can be used to correct higher orders of the cat code up to an adjustment in the mapping of errors to the syndromes and the preparation of a higher-order ancilla. The protocol otherwise remains the same.

As illustrated in figure 3, the input mode is mixed via a real symmetric 50:50 beamsplitter with the first mode of an ancilla

$$|\tilde{\Phi}_L^+\rangle = \frac{|\tilde{0}_L \tilde{0}_L\rangle + |\tilde{1}_L \tilde{1}_L\rangle}{\sqrt{2}}, \quad (15)$$

where the ancilla is composed of order- L cat states, with its first mode prepared at amplitude $\sqrt{1 - \Gamma}\alpha$ to match that of the lossy input state, with the reduced amplitude indicated by an underbar. Photon-number measurements, (n, m) , are performed on the first two modes—that of the input state and the reduced-amplitude mode of the ancilla—and act as the syndrome for the state teleported into the logical basis of the remaining ancilla mode.

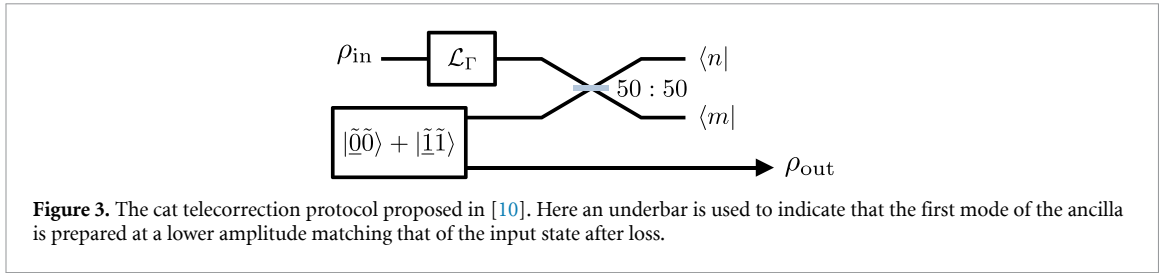


Figure 3. The cat telecorrection protocol proposed in [10]. Here an underbar is used to indicate that the first mode of the ancilla is prepared at a lower amplitude matching that of the input state after loss.

Teleportation into the ancilla's remaining mode both restores it to our desired basis and corrects the amplitude damping. Not only does this mechanism correct our state, but it is also convenient numerically, as it allows us to express for every syndrome (n, m) a direct mapping between a logical input state and a logical output state. This mapping is derived in appendix C, equation (C27).

3.1. Syndromes and leading-order transformations

Due to our choice of basis relative to [10], we see a relative exchange of syndromes for logical X and Z errors at $L = 1$. We also utilise a numerical mapping of the Pauli errors, as the relations for Z syndromes are non-trivial at orders $L > 1$.

In appendix C.2 we derive an expression for the 2×2 transformation associated with each order of the loss channel, l , and each pair of photon-number measurements, (n, m) , which constitute our syndrome. Up to order $l = L$ in the channel, due to the Fock support of each loss event up to that order being unique, a single order of loss is associated with a given pair of photon-number measurements, (n, m) , following the rule

$$l \equiv L(n + m) \bmod (L + 1). \quad (16)$$

Truncating the loss channel at this value of l allows us to calculate a single 2×2 transformation associated with each syndrome, (n, m) , up to the loss capacity of the code, which we write as follows:

$$\begin{aligned} \bar{T}_{\text{KJ}}(n, m) = \sum_{j,k=0}^{2L+1} e^{ijl\phi_L} \frac{(\sqrt{\Gamma}\alpha)^l}{\sqrt{l!}} \frac{(-1)^{j+k}}{\sqrt{2\mathcal{N}_j\mathcal{N}_K}} \frac{e^{(\Gamma-2)|\alpha|^2/2}}{\sqrt{n!m!}} \left[\sqrt{\frac{1-\Gamma}{2}} \alpha \right]^{n+m} \\ \cdot [e^{ij\phi_L} + e^{ik\phi_L}]^n [e^{ik\phi_L} - e^{ij\phi_L}]^m, \end{aligned} \quad (17)$$

where $\bar{T}_{\text{KJ}}(n, m)$ is a scalar element of $\bar{T}(n, m)$, which is a 2×2 transformation matrix defined on the indices of the logical basis, $J, K \in \{0, 1\}$, indicated by an overbar; the underbar in $\mathcal{N}_K$ indicates that the normalisation constant is that of a cat state prepared at reduced amplitude—the first mode of the ancilla. The transformations calculated this way represent the first $L + 1$ orders of the loss channel, which belong to the correctable set of errors for an L -order cat code.

We find that $\bar{T}(n, m)$ can be expressed in the form

$$\bar{T}(n, m) = c(n, m) \bar{D}(n, m) \bar{X}^{f(n, m)} \bar{Z}^{g(n, m)}, \quad (18)$$

where the syndrome is associated with a scalar $c(n, m)$, a diagonal deformation $\bar{D}(n, m)$, and the possible presence of Pauli X and Z errors, which are mapped by $f(n, m) \in \{0, 1\}$ and $g(n, m) \in \{0, 1\}$, respectively. For deformations, we can write

$$\bar{D}(n, m) = \begin{pmatrix} 1 & 0 \\ 0 & d(n, m) \end{pmatrix}, \quad (19)$$

and will refer to $d(n, m)$ as the deformation ratio. Further, we observe three categories of $\bar{T}(n, m)$: explicit transmission failure cases where the logical information is lost, deformed off-diagonal transformations (associated with X errors), and deformed diagonal transformations; we show examples of these here, respectively, for $L = 1$, $\alpha = 2$, and $\Gamma = 0$:

$$\bar{T}(0, 0) = \begin{pmatrix} \frac{8\sqrt{2}}{\sqrt{\mathcal{N}_0\mathcal{N}_0}e^4} & 0 \\ 0 & 0 \end{pmatrix}, \quad (20)$$

$$\bar{T}(2, 0) = \begin{pmatrix} 0 & \frac{16}{\sqrt{\mathcal{N}_1\mathcal{N}_0}e^4} \\ \frac{16}{\sqrt{\mathcal{N}_0\mathcal{N}_1}e^4} & 0 \end{pmatrix} = c(2, 0) \bar{D}(2, 0) \bar{X}, \quad (21)$$

$$\bar{T}(2,2) = \begin{pmatrix} \frac{32\sqrt{2}}{\sqrt{\mathcal{N}_0\mathcal{N}_0}e^4} & 0 \\ 0 & -\frac{32\sqrt{2}}{\sqrt{\mathcal{N}_1\mathcal{N}_1}e^4} \end{pmatrix} = c(2,2) \bar{D}(2,2) \bar{Z}. \quad (22)$$

Equation (20) corresponds to complete loss of the qubit, while for the latter two we have also written them in the expanded form of equation (18), where we see for (2,0) an X error and no Z error,

$$f(2,0) = 1, \quad g(2,0) = 0, \quad (23)$$

and for (2,2) a Z error and no X error,

$$f(2,2) = 0, \quad g(2,2) = 1, \quad (24)$$

with the two outcomes' respective deformations being

$$\bar{D}(2,0) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\frac{\mathcal{N}_0\mathcal{N}_1}{\mathcal{N}_1\mathcal{N}_0}} \end{pmatrix}, \quad (25)$$

$$\bar{D}(2,2) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\frac{\mathcal{N}_0\mathcal{N}_0}{\mathcal{N}_1\mathcal{N}_1}} \end{pmatrix}, \quad (26)$$

which we can further reduce to the deformation ratios,

$$d(2,0) = \sqrt{\frac{\mathcal{N}_0\mathcal{N}_1}{\mathcal{N}_1\mathcal{N}_0}}, \quad (27)$$

$$d(2,2) = \sqrt{\frac{\mathcal{N}_0\mathcal{N}_0}{\mathcal{N}_1\mathcal{N}_1}}. \quad (28)$$

The mappings of logical X and Z errors, $f(n,m)$ and $g(n,m)$, extracted this way from equation (17) are shown in figure 4, and are used to determine the appropriate Pauli corrections to apply. Pauli-correction does not correct deformation, which we see is influenced by ratios of normalisation constants, $\mathcal{N}_j$ and $\mathcal{N}_K$.

Deformation occurs here as a result of weak Pauli Z measurements, similar to the weak Pauli X measurements originally observed in [10], but more prevalent here due to the normalisation constant mismatch. However, because our basis does not suffer from non-orthogonality, this is not comparatively detrimental. Deformation constitutes a violation of the Knill–Laflamme conditions, equation (8), as although we can identify and correct Pauli errors, a remaining deformation has the following effect:

$$\langle \tilde{0} | \bar{D}(n,m) | \tilde{0} \rangle \neq \langle \tilde{1} | \bar{D}(n,m) | \tilde{1} \rangle, \quad (29)$$

as weak Pauli measurement causes particular outcomes to be biased depending on the input states. In particular, we can see

$$\begin{aligned} \langle \tilde{0} | \bar{D}(n,m) | \tilde{0} \rangle &= \left\langle \tilde{0} \left| \begin{pmatrix} 1 & 0 \\ 0 & d(n,m) \end{pmatrix} \right| \tilde{0} \right\rangle \\ &= \langle \tilde{0} | \tilde{0} \rangle \\ &= 1, \end{aligned} \quad (30)$$

$$\begin{aligned} \langle \tilde{1} | \bar{D}(n,m) | \tilde{1} \rangle &= \left\langle \tilde{1} \left| \begin{pmatrix} 1 & 0 \\ 0 & d(n,m) \end{pmatrix} \right| \tilde{1} \right\rangle \\ &= \langle \tilde{1} | d(n,m) | \tilde{1} \rangle \\ &= d(n,m), \end{aligned} \quad (31)$$

where the inner product of $|\tilde{1}\rangle$ is clearly deformed relative to that of $|\tilde{0}\rangle$ for any $d(n,m) \neq 1$. One can see this way that the deformation ratio represents the degree of change in one codeword's inner product relative to the other.

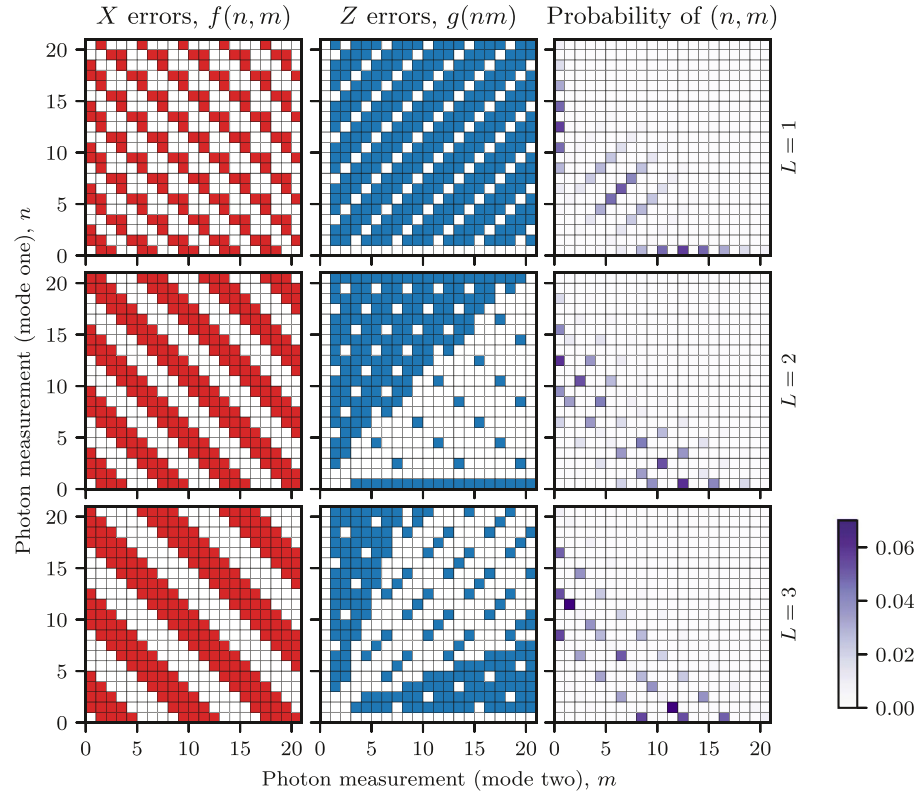


Figure 4. Maps of $f(n, m)$ and $g(n, m)$ indicating the presence of Pauli X (red) and Z (blue) errors in terms of photon-number measurements, (n, m) , on either mode, determined numerically from equation (17). A coloured tile indicates $f(n, m) = 1$ or $g(n, m) = 1$. White indicates that the respective error is not present. X and Z errors may occur simultaneously, and are always correctable. The probability of measuring (n, m) for $\alpha = 2.5$ and $\Gamma = 0$ is shown in the rightmost column.

The most obvious contribution to deformation is from the normalisation constant mis-match, as each index $\bar{T}_{\text{KJ}}(n, m)$ is weighted by $(\mathcal{N}_J \mathcal{N}_K)^{-1/2}$, which persists for all values of Γ . However it is not the sole determiner of deformation, as, for instance, at $L = 3$, $\alpha = 2$, and $\Gamma = 0$ we find

$$\bar{T}(10, 6) = \begin{pmatrix} -\frac{0.36531}{\sqrt{\mathcal{N}_0 \mathcal{N}_0}} & 0 \\ 0 & -\frac{0.16605}{\sqrt{\mathcal{N}_1 \mathcal{N}_1}} \end{pmatrix}, \quad (32)$$

which shows deformation partially independent of the normalisation constants. Regardless, we find that deformation overall is suppressed at sufficiently high amplitudes, where the contributions from weak Pauli measurement—and the sources themselves, such as normalisation mis-match—become small. In this regime the transformations overall become approximately unitary and on average $d(n, m) \rightarrow 1$. We also propose in section 4 a probabilistic method for otherwise correcting deformation.

3.2. Performance

To quantify performance we use the channel fidelity, F_C [24]. We iterate loss and correction N times, wherein we apply the loss channel, $\mathcal{L}_\gamma$, with segment loss $\gamma = 1 - (1 - \Gamma)^{1/N}$, and correction, $\mathcal{C}$, to the first mode of an input entangled state

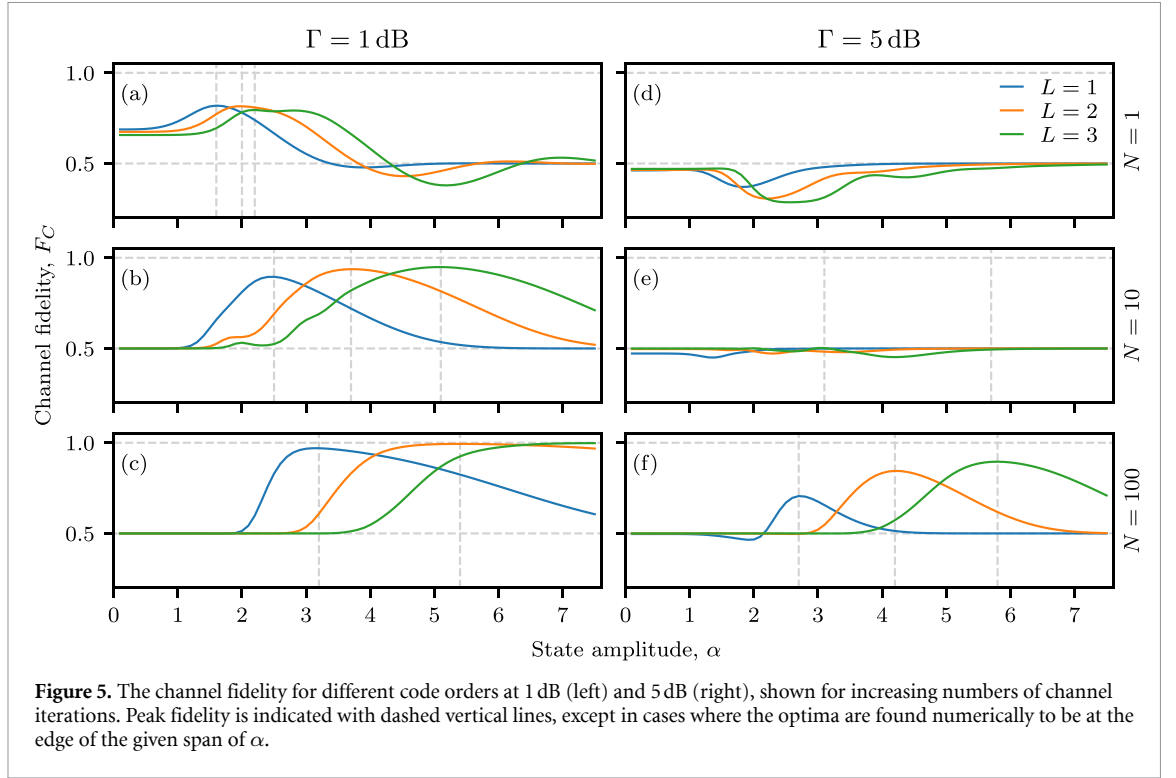
$$\rho_0 = |\tilde{\Phi}^+\rangle \langle \tilde{\Phi}^+|, \quad |\tilde{\Phi}^+\rangle = \frac{|\tilde{0}\tilde{0}\rangle + |\tilde{1}\tilde{1}\rangle}{\sqrt{2}}, \quad (33)$$

while applying the identity map, $\mathcal{I}$, to the other mode, iterating the channel as

$$\rho_{i+1} = (\mathcal{C} \otimes \mathcal{I})((\mathcal{L}_\gamma \otimes \mathcal{I})(\rho_i)), \quad (34)$$

until ρ_N is reached, then calculate the fidelity with the original state,

$$F_C \equiv \langle \tilde{\Phi}^+ | \rho_N | \tilde{\Phi}^+ \rangle. \quad (35)$$



We take $\mathcal{C}$ to include both the teleportation protocol described earlier and the correction of logical X and Z errors indicated by the syndrome measurements, with outcomes averaged over (n, m) up to an appropriate cutoff and weighted by their respective probabilities. In appendix C.1, we derive an expression, equation (C27), to calculate the effects of the combined channel in equation (34), without Pauli correction.

Although only Z is accessible through Gaussian operations, both X and Z may be accounted for via Pauli frames [25], or a probabilistic X could be implemented with this circuit similar to the proposal in [11]. Another option, which would enable a Gaussian X , would be via teleporting into the basis of the GKP code, for example, in the manner proposed in section 4.1.

Figure 5 shows that lower code orders see an upward trend in performance earlier than higher orders, as we expect from the trends in normalisation constant mis-match. However the lower orders do not generally reach the same performance levels if we continue higher in amplitude. As seen in [10], we observe that there is generally a maximum fidelity achieved at a given number of iterations, and that increasing the number of iterations also demands an increased amplitude to achieve the same fidelity. We see generally that iteration number and state amplitude must both be increased to see performance improvements. This is expected, since while smaller segments reduce the uncorrectable loss errors at each stage, each new teleportation risks deforming the state. In figure 5 we also see the effects of increasing total loss, going from 1 dB of loss on the left to 5 dB on the right. With increased total loss, we see that the maximum channel fidelity that can be achieved at a given number of iterations is reduced.

We next consider the parameters required to meet a target fidelity, shown in table 1, where we see that there is a superlinear decrease with respect to L in the number of iterations required. This comes at a cost in state amplitude, which need be higher with order, meaning that each iteration becomes more expensive, as not only do the photon-number measurements require higher resolutions, but the mean photon-number of the ancillae must also increase. However, looking to $F_{\text{target}} = 0.999$, between $L = 1$ and $L = 3$ the required number of iterations is reduced by a factor of about 70.6, while amplitude has only increased by about 1.9, or about 3.6 in terms of mean photon-number.

4. Probabilistic deformation correction

The deformations described in section 3.1 can be identified via the syndrome, but, as they are non-unitary, cannot be corrected deterministically. Here we will show that the same teleportation scheme

Table 1. The minimum number of iterations, N , and associated amplitude, α , required to achieve target channel fidelities, F_{target} , at $\Gamma = 1$ dB for different orders, L , of the cat code. For a given number of iterations we scanned over α in increments of $\Delta\alpha = 0.1$ until the fidelity threshold was satisfied. Note that the mean photon-number of a state is approximately equal to the square of its amplitude in this regime.

F_{target}	$L = 1$		$L = 2$		$L = 3$	
	N	α	N	α	N	α
0.95	44	2.9	14	3.8	11	5.0
0.99	500	3.6	68	5.1	38	6.4
0.999	10022	4.3	423	6.4	142	8.2

Table 2. Performance statistics observed over 1000 simulations of the channel with $N = 10$ iterations and loss $\Gamma = 1$ dB, averaged over inputs of the six Pauli eigenstates, with amplitudes at fractions of an optimal value, α_{opt} , derived from figure 5(b); $\alpha_{\text{opt}} = \{2.5, 3.7, 5.1\}$ for $L = \{1, 2, 3\}$. We list the average fidelity of the output state when only Pauli-correction is applied, F_{PC} , and when both Pauli- and deformation-correction are applied, F_{DC} , both at a given fraction of α_{opt} . Only successful transmissions are accounted for in the average fidelities. The percentage of failures witnessed throughout each sample is listed.

	$L = 1$			$L = 2$			$L = 3$		
	F_{PC}	F_{DC}	$\%_{\text{fail}}$	F_{PC}	F_{DC}	$\%_{\text{fail}}$	F_{PC}	F_{DC}	$\%_{\text{fail}}$
α_{opt}	93.0	93.9	0.1	96.5	97.3	1.8	96.5	97.5	0.0
$\times 0.9$	92.0	94.9	0.2	95.7	98.3	3.2	95.5	97.9	0.0
$\times 0.8$	87.8	94.4	2.2	92.1	98.5	8.9	92.2	98.6	0.0
$\times 0.7$	83.5	93.7	12.4	84.7	95.9	15.3	86.1	98.7	0.0
$\times 0.6$	81.3	94.1	47.2	73.9	87.8	17.7	77.7	95.7	0.0
$\times 0.5$	74.9	87.9	79.7	73.7	90.0	34.3	68.7	80.9	0.4

as for loss-correction can be used to probabilistically correct deformation, provided one can finely tune the ancilla. Because deformation is normally only suppressed by working in a higher-amplitude regime—exacerbating loss errors—correcting deformation explicitly allows one to achieve optimal performance at lower amplitudes, both reducing the resource cost during transmission and improving fidelity.

In table 2 we show the results of simulating transmission with and without a final deformation-correcting operation at the end, though Pauli errors are corrected in both cases. What we find is that by correcting deformation, the amplitudes showing peak performance shift downwards, which aligns with the fact that the effects of loss are reduced with lower amplitudes. In fact we see improved performance over the optima for the Pauli-correction-only cases, with the maximum gain being for $L = 3$, where the average fidelity at $0.7 \times \alpha_{\text{opt}}$ increases by 2.2%, with the amplitude reduced from $\alpha = 5.1$ to $\alpha \approx 3.6$. However, the incidence of transmission failure increases, i.e. cases such as given by equation (20), as they become more likely as $\alpha \rightarrow 0$. Thus the lower-order codes are worse off on account of their optimal Pauli-correction-only amplitudes occurring at already low amplitudes.

The falling off in performance at excessively low amplitudes is likely due to extreme deformations exacerbating loss errors, as even if those contributions are made smaller by the reduced amplitude, they will be multiplied in correcting the deformation. In particular, since deformation technically does not impact $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, but higher orders of loss can contribute to these states during transmission, in inverting the deformation these loss-error contributions can be exacerbated, worsening fidelity.

The method we will propose for implementing this type of correction is probabilistic and reliant on highly tunable ancillae. Although our proposal may be challenging to implement, its cost relative to the reduction in transmission resources and potential gains in performance is worth considering. Moreover, consider that deformation-correction could be useful when deformation has caused an unacceptable hit to state fidelity; even in the high-amplitude regime there is a possibility of selecting high-deformation outcomes. If the qubit is otherwise unusable, the costly step of attempting probabilistic correction of the deformation becomes more attractive.

4.1. Ancilla-biasing and code-switching

To implement the inverse deformation optically, one can bias the ancilla in the teleporter, where with a bias x introduced as

$$|\tilde{\Phi}^{(x)}\rangle = \frac{|\tilde{0}\tilde{0}\rangle + x|\tilde{1}\tilde{1}\rangle}{\sqrt{1 + |x|^2}}, \quad (36)$$

the transformations from equation (17) are transformed as

$$\bar{T}(n, m) \rightarrow \bar{T}^{(x)}(n, m), \quad (37)$$

where

$$\bar{T}(n, m) = \begin{pmatrix} \bar{T}_{0,0} & \bar{T}_{0,1} \\ \bar{T}_{1,0} & \bar{T}_{1,1} \end{pmatrix} \quad (38)$$

and

$$\bar{T}^{(x)}(n, m) = \sqrt{\frac{2}{1+|x|^2}} \begin{pmatrix} \bar{T}_{0,0} & \bar{T}_{0,1} \\ x\bar{T}_{1,0} & x\bar{T}_{1,1} \end{pmatrix} \quad (39)$$

as x becomes associated with the index $K = 1$ in equation (17). Note however that both $\bar{T}(n, m)$ and $\bar{T}^{(x)}(n, m)$ are either diagonal or off-diagonal, despite being written more generally here to illustrate the transformation.

With knowledge of the deformations $\bar{D}(n, m)$ corresponding to each $\bar{T}(n, m)$, this transformation can therefore produce a set containing an arbitrary deformation of one's choice. In effect, to correct deformation this way, we want to choose an x to satisfy

$$\bar{D}^{(x)}(n_f, m_f) = c'(n_f, m_f) \left(\prod_{i=0}^N \bar{D}(n_i, m_i) \right)^{-1}, \quad (40)$$

where our final syndrome corresponds to a biased deformation, $\bar{D}^{(x)}(n_f, m_f)$, equal to the inverse of the cumulative deformation over all previously measured syndromes (n_i, m_i) , up to a constant $c'(n_f, m_f)$ that is irrelevant after renormalisation of the state.

To assess the probability of selecting a desired transformation, one must consider that there are outcomes (n, m) with associated deformations that are degenerate up to a constant, or at least approximately so. Thus a successful outcome may be heralded by many different photon counts, and the total probability for a given deformation across these degenerate cases should be calculated. For $L = 1$ there are considerably fewer distinct deformations, meaning that if we need a particular transformation then it is statistically favourable to work at this code order. This may seem prohibitive, considering the performance gains of working at higher orders during transmission, but it is in fact possible to switch into a different basis supposing we can prepare the appropriate ancilla. The basis one teleports into is theoretically arbitrary, as one could prepare an ancilla

$$|\tilde{\Phi}_{(L,L')}^+\rangle = \frac{|\tilde{0}_L \tilde{0}_{L'}\rangle + |\tilde{1}_L \tilde{1}_{L'}\rangle}{\sqrt{2}}, \quad (41)$$

which in teleportation would take the input state from code order L to L' , although this initial code-switching teleportation would have the set of syndromes $\bar{T}(n, m)$ corresponding to L . One could similarly use this method to teleport into a cat basis of different amplitude, or into the basis of another code entirely.

In total, to implement our deformation-correction scheme, we propose two additional teleportations at the terminus of transmission:

1. first using the ancilla of equation (41) with $L' = 1$ to switch into a first-order cat code of sufficient amplitude to be in the unitary regime,
2. then using the ancilla of equation (36) with bias x whose most probable outcome has a deformation equal to the inverse of the cumulative deformation, up to some scalar factor.

As before, we will assume one has access to X and Z gates.

Using what we have described, in table 3 we see the success probability for selecting the desired transformation at different levels of bias. For no bias, $x = 1$, the probability is near 100% to select our desired outcome on account of the most probable outcomes in this regime all having approximately identical deformations of approximately identity. The sudden change to an average of 50% success probability is on account of equation (39), which has an inverse impact on the deformation ratio depending on whether an outcome includes an X error or not, i.e. whether the transformation is off-diagonal or diagonal.

If we consider, for example, the transformations of equations (21) and (22), a bias x has the following effect on them:

$$\bar{T}^{(x)}(2,0) = \sqrt{\frac{2}{1+|x|^2}} \begin{pmatrix} 0 & \frac{16}{\sqrt{\mathcal{N}_1\mathcal{N}_0}e^4} \\ x\frac{16}{\sqrt{\mathcal{N}_0\mathcal{N}_1}e^4} & 0 \end{pmatrix}, \quad (42)$$

$$\bar{T}^{(x)}(2,2) = \sqrt{\frac{2}{1+|x|^2}} \begin{pmatrix} \frac{32\sqrt{2}}{\sqrt{\mathcal{N}_0\mathcal{N}_0}e^4} & 0 \\ 0 & -x\frac{32\sqrt{2}}{\sqrt{\mathcal{N}_1\mathcal{N}_1}e^4} \end{pmatrix}, \quad (43)$$

leading to the deformations

$$\bar{D}^{(x)}(2,0) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{x}\sqrt{\frac{\mathcal{N}_0\mathcal{N}_1}{\mathcal{N}_1\mathcal{N}_0}} \end{pmatrix}, \quad (44)$$

$$\bar{D}^{(x)}(2,2) = \begin{pmatrix} 1 & 0 \\ 0 & x\sqrt{\frac{\mathcal{N}_0\mathcal{N}_0}{\mathcal{N}_1\mathcal{N}_1}} \end{pmatrix}, \quad (45)$$

and respective deformation ratios

$$d^{(x)}(2,0) = \frac{1}{x}\sqrt{\frac{\mathcal{N}_0\mathcal{N}_1}{\mathcal{N}_1\mathcal{N}_0}}, \quad (46)$$

$$d^{(x)}(2,2) = x\sqrt{\frac{\mathcal{N}_0\mathcal{N}_0}{\mathcal{N}_1\mathcal{N}_1}}. \quad (47)$$

We see that in this case, due to the X error at $(2,0)$, the bias has an inverse impact on the deformation ratio relative to $(2,2)$, and so even supposing we had $\mathcal{N}_0 \rightarrow \mathcal{N}_1$ and $\mathcal{N}_0 \rightarrow \mathcal{N}_1$, by applying bias we would see a different deformation between outcomes $(2,0)$ and $(2,2)$. A simple example of deformation correction using these transformations is provided in appendix D.

If we consider this behaviour more generally, in the high-amplitude regime where normalisation constants are equal and transformations are on average unitary without bias, introducing bias splits the outcomes into two cases corresponding to deformation ratios proportional either to x , or $1/x$ in the cases including an X error. In this regime, then, the possible values of $\bar{T}^{(x)}(n,m)$ are approximately equal, up to some scalar factor, to

$$\begin{pmatrix} 1 & 0 \\ 0 & x \end{pmatrix}, \quad \text{or} \quad \begin{pmatrix} 0 & x \\ 1 & 0 \end{pmatrix}. \quad (48)$$

Now considering the third and fourth columns of table 3, we see that the eigenstates $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ have their probabilities affected by bias, and see specifically that the probability of success is dramatically suppressed for $|\tilde{0}\rangle$. When calculating the transmission of a state for a particular outcome (n,m) , the trace of the transmitted state is equal to the probability of the outcome (n,m) . Now although the transformation $\bar{T}(n,m)$ is only used for the sake of determining leading-order errors in transmission, since it is derived from sub-capacity terms of the loss channel, we can use it to approximate the effect of bias on the probability of a given outcome. Suppose our outcome with bias is

$$\bar{T}^{(x)}(n_f, m_f) = c'(n_f, m_f) \begin{pmatrix} 0 & x \\ 1 & 0 \end{pmatrix}, \quad (49)$$

where we have included an X error to align with table 3, then the transmission of the Pauli Z eigenstates is given approximately by

$$\bar{T}^{(x)}(n_f, m_f) |\tilde{0}\rangle = c'(n_f, m_f) x |\tilde{0}\rangle, \quad (50)$$

$$\bar{T}^{(x)}(n_f, m_f) |\tilde{1}\rangle = c'(n_f, m_f) |\tilde{1}\rangle. \quad (51)$$

Thus the probability of this outcome for $|\tilde{0}\rangle$ is approximately proportional to $|x|^2$. This is, however, only for a single outcome (n_f, m_f) , and in a calculation like that for table 3 we consider all outcomes with degenerate deformation ratios to be successful outcomes, and thus the relationship is not exact. Further, if the desired outcome did not contain an X error, we would expect the relationship to be flipped.

Table 3. Probability of an outcome corresponding to the desired deformation up to second-decimal precision after a bias, x , in equation (36). The probabilities are averaged across the Pauli eigenstates to produce p_{avg} , while $p_{\bar{0}}$ and $p_{\bar{1}}$ are for $|\bar{0}\rangle$ and $|\bar{1}\rangle$ individually. X and Z errors are both assumed to be correctable, and our desired outcome corresponds to those containing X errors. The maximum bias is chosen based on the average deformation ratio for $L = 3$ and $\alpha = 0.7 \times \alpha_{\text{opt}}$ in table 2, where $d_{\text{avg}} \approx 4.76$. These calculations were performed for $L = 1$, $\alpha = 4$, and $\Gamma = 0$.

bias (x)	p_{avg}	$p_{\bar{0}}$	$p_{\bar{1}}$
1	99.9	99.9	99.9
1/2	0.5	20.0	80.0
1/3	0.5	10.0	90.0
1/4	0.5	5.9	94.1
1/5	0.5	3.8	96.2

5. Conclusion

We have generalised the protocol proposed in [10] to higher orders of the cat code, and found that these higher orders can be used to significantly reduce the number of iterations required to achieve target performance in loss-correction. The performance improvements, however, come at a cost in state amplitude, which makes the protocol's photon-number measurements more demanding, and would also likely make ancilla generation more expensive. In [15], measurement resolutions of up to 21 photons are reported at efficiencies of 90%, with higher counts possible via detector-multiplexing [14], and we hope that work such as our own would help motivate the improvement of such techniques.

Further, the mean photon-number of a state has been used as a proxy for generation cost previously [5, 6], and the high-distance Fock structure of higher-order cat codes may also impact the rate and fidelity with which they can be generated. Although substantial work has been published on protocols for generating low-order cat states or other varieties of coherent state superpositions [26–29], little literature exists on the generation of the higher-order cat states we are concerned with. The landscape is similar for cat-encoded Bell states [10, 11] outside of more general techniques [30]. Thus future work must explore the possibility of generating these states, and the resource cost relative to the performance benefits we have shown here.

Within our choice of basis, we have also identified codeword normalisation mis-match as an inherent source of error in the teleportation scheme, which is complementary to the codeword overlap seen in the basis used in the original proposal. This normalisation mis-match contributes to weak Pauli Z measurements in the protocol, but we have found that the effect is minimised at sufficient amplitudes, and moreover we are able to identify the deformation via our syndrome measurements. Further, our proposal for a probabilistic scheme to correct deformation shows that even if impractical to implement, deformation is technically reversible. It also highlights two interesting properties of the teleporter: that at sufficient amplitudes it can with high likelihood produce a set of two arbitrary but related diagonal and off-diagonal transformations, and that it can be used to change the basis of our state. Both of these techniques would require very robust ancilla generation.

Ultimately, we find that if one can realise the measurements and state generation required to teleport higher-order cat states, then transmission will require far fewer correction stages.

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Data availability statement

The data that support the findings of this study are openly available at the following URL: <https://github.com/polyquantique/CatTelecorrection>.

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Appendix A. Normalisation constants

To derive general expressions for the normalisation constants, $\mathcal{N}_0$ and $\mathcal{N}_1$, of our cat states given by equations (4) and (5), we will use the following expression for the overlap of two coherent states [31]:

$$\langle \beta | \alpha \rangle = \exp \left[\frac{1}{2} (\beta^* \alpha - \beta \alpha^* - |\beta - \alpha|^2) \right], \quad (\text{A1})$$

which for coherent states of equal amplitudes but a difference in phase, simplifies to

$$\langle \alpha | e^{i\phi} \alpha \rangle = \exp \left[(e^{i\phi} - 1) |\alpha|^2 \right]. \quad (\text{A2})$$

The normalisation constants are derived from the unnormalised overlaps of our codewords.

Beginning with $|\tilde{0}\rangle$,

$$\mathcal{N}_0 = \sum_{j,j'=0}^{2L+1} \langle e^{ij\phi_L} \alpha | e^{ij'\phi_L} \alpha \rangle \quad (\text{A3})$$

$$= 2(L+1) \sum_{j=0}^{2L+1} \langle \alpha | e^{ij\phi_L} \alpha \rangle \quad (\text{A4})$$

$$= 2(L+1) e^{-|\alpha|^2} \sum_{j=0}^{2L+1} \exp [e^{ij\phi_L} |\alpha|^2], \quad (\text{A5})$$

where from (A3) to (A4) we have used the fact that the overlaps are repeated for the $2(L+1)$ superposed coherent states.

Similarly for $|\tilde{1}\rangle$, but accounting for alternating signs, we find

$$\mathcal{N}_1 = 2(L+1) e^{-|\alpha|^2} \sum_{j=0}^{2L+1} (-1)^j \exp [e^{ij\phi_L} |\alpha|^2]. \quad (\text{A6})$$

Appendix B. Choice of basis

The basis of cat codewords we chose to use in this work differs from the choice originally proposed in [10]. We will distinguish the two bases by the effect of the rotation

$$\hat{R}_L \equiv \exp (i\phi_L \hat{a}^\dagger \hat{a}), \quad (\text{B1})$$

where in our choice of basis $\hat{R}_L = \bar{Z}$, and for the basis in [10] $\hat{R}_L = \bar{X}$. On account of this, we will refer to the bases as R_Z cats,

$$|\tilde{0}\rangle_Z = \frac{1}{\sqrt{\mathcal{N}_0}} \sum_{m=0}^{2L+1} |e^{im\phi_L} \alpha\rangle, \quad |\tilde{1}\rangle_Z = \frac{1}{\sqrt{\mathcal{N}_1}} \sum_{m=0}^{2L+1} (-1)^m |e^{im\phi_L} \alpha\rangle, \quad (\text{B2})$$

and R_X cats,

$$|\tilde{0}\rangle_X = \frac{1}{\sqrt{\mathcal{N}_X}} \sum_{m=0}^L |e^{2im\phi_L} \alpha\rangle, \quad |\tilde{1}\rangle_X = \frac{1}{\sqrt{\mathcal{N}_X}} \sum_{m=0}^L |e^{(2m+1)\phi_L} \alpha\rangle, \quad (\text{B3})$$

respectively. The Pauli X eigenstates in the R_X basis are equivalent to Pauli Z eigenstates in R_Z ,

$$|\tilde{\pm}\rangle_X = |\tilde{0}/\tilde{1}\rangle_Z, \quad (\text{B4})$$

but there are key differences: in R_Z the codewords are perfectly orthogonal, $\langle \tilde{0} | \tilde{1} \rangle_Z = 0$, but have different normalisation constants, $\mathcal{N}_0 \neq \mathcal{N}_1$; in R_X the codewords are not perfectly orthogonal, $\langle \tilde{0} | \tilde{1} \rangle_X \neq 0$, but have a common normalisation constant, $\mathcal{N}_X$.

These differences have been commented on in [4], where they are noted to lead to practical differences in performance against the loss channel, as normalisation mis-match leads to deformation in correcting the code, while non-orthogonality means an inherent level of indistinguishability between the codewords—both distinct violations of the Knill–Laflamme conditions. However, both sources of error are noted to diminish as $\alpha \rightarrow \infty$, and we show here how the two are intrinsically linked, and show for $L = 1$ that their impact on performance is relegated to the sub-optimal regime.

To begin with, an R_X qubit requires additional normalisation to account for overlap [4],

$$|\tilde{\Psi}\rangle_X = \frac{\mu|\tilde{0}\rangle_X + \nu|\tilde{1}\rangle_X}{\sqrt{1 + 2\text{Re}(\mu\nu^*)\langle \tilde{0} | \tilde{1} \rangle_X}}, \quad (\text{B5})$$

where $|\mu|^2 + |\nu|^2 = 1$ and we take $\alpha \in \mathbb{R}$ and so $\langle \tilde{0} | \tilde{1} \rangle_X \in \mathbb{R}$. With this, we can define the properly normalised R_X Pauli X eigenstates,

$$|\tilde{\pm}\rangle_X = \frac{|\tilde{0}\rangle_X \pm |\tilde{1}\rangle_X}{\sqrt{2(1 \pm \langle \tilde{0} | \tilde{1} \rangle_X)}}, \quad (\text{B6})$$

and expand the numerator as follows:

$$|\tilde{0}\rangle_X \pm |\tilde{1}\rangle_X = \sum_{m=0}^L \frac{|e^{2im\phi_L}\alpha\rangle \pm |e^{(2m+1)i\phi_L}\alpha\rangle}{\sqrt{\mathcal{N}_X}} \quad (\text{B7})$$

$$= \sum_{m=0}^{2L+1} \frac{(\pm 1)^m |e^{im\phi_L}\alpha\rangle}{\sqrt{\mathcal{N}_X}}. \quad (\text{B8})$$

Now, using this and equation (B6) on the LHS of equation (B4), we find

$$\sum_{m=0}^{2L+1} \frac{(\pm 1)^m |e^{im\phi_L}\alpha\rangle}{\sqrt{2\mathcal{N}_X(1 \pm \langle \tilde{0} | \tilde{1} \rangle_X)}} = \sum_{m=0}^{2L+1} \frac{(\pm 1)^m |e^{im\phi_L}\alpha\rangle}{\sqrt{\mathcal{N}_{0/1}}} \quad (\text{B9})$$

$$2\mathcal{N}_X(1 \pm \langle \tilde{0} | \tilde{1} \rangle_X) = \mathcal{N}_{0/1}, \quad (\text{B10})$$

showing that the source of intrinsic error in one basis is directly related to that of the other, and with which we write

$$\frac{\mathcal{N}_0}{\mathcal{N}_1} = \frac{1 + \langle \tilde{0} | \tilde{1} \rangle_X}{1 - \langle \tilde{0} | \tilde{1} \rangle_X}. \quad (\text{B11})$$

Thus, we see that as codeword overlap in the R_X basis grows small, $\langle \tilde{0} | \tilde{1} \rangle_X \rightarrow 0$, the ratio between R_Z normalisation constants approaches unity, $\mathcal{N}_0/\mathcal{N}_1 \rightarrow 1$. However, as the form of these errors is distinct, we must consider how they impact performance.

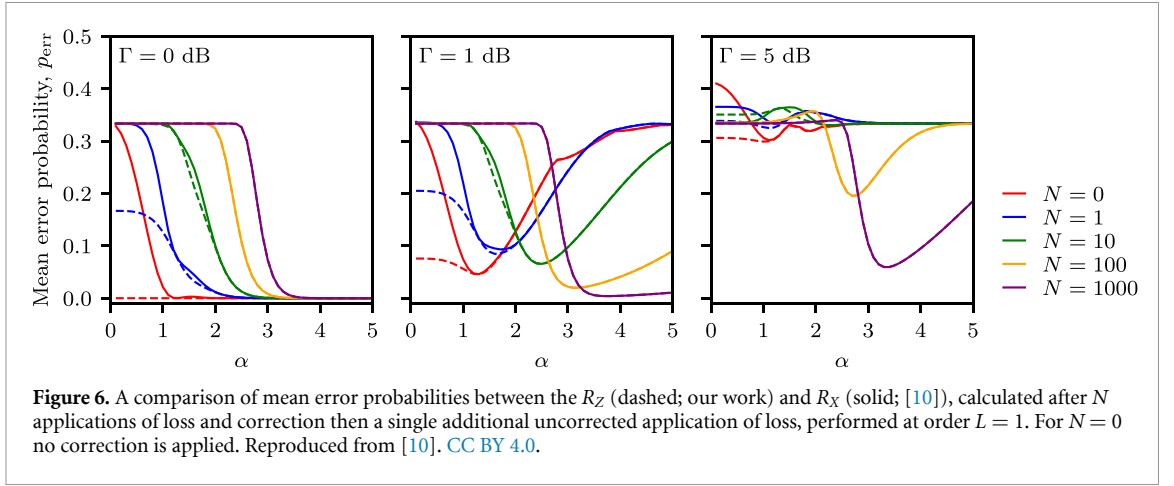
Numerically, we will make a direct comparison to the results from [10], mimicking the original proposal's choice of the mean error probability as a metric, its use of an additional segment of loss, and its choice of code order $L = 1$. The mean error probability is calculated by averaging over the trace distance of each pair of Pauli eigenstates— $\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$, $\{|\tilde{+}\rangle, |\tilde{-}\rangle\}$, and $\{|\tilde{+i}\rangle, |\tilde{-i}\rangle\}$ —after they have been passed through the channel, iterating as $\rho_{i+1} = \mathcal{C}(\mathcal{L}_\gamma(\rho_i))$ with ρ_0 an eigenstate, where for a given pair of Pauli eigenstates, e.g. $\rho_0^{X+} = |\tilde{+}\rangle\langle\tilde{+}|$ and $\rho_0^{X-} = |\tilde{-}\rangle\langle\tilde{-}|$, we have

$$p_{\text{err}}^X = \frac{1}{2} - \frac{1}{4} \|\mathcal{L}_\gamma(\rho_0^{X+}) - \mathcal{L}_\gamma(\rho_0^{X-})\|_1, \quad (\text{B12})$$

with $\gamma = 1 - (1 - \Gamma)^{1/(N+1)}$, and the average is given by

$$p_{\text{err}} \equiv \frac{p_{\text{err}}^X + p_{\text{err}}^Y + p_{\text{err}}^Z}{3}. \quad (\text{B13})$$

In figure 6 we compare performance in R_Z and R_X this way, and see that there are only significant differences in the sub-break-even regime for $L = 1$. As α increases, sources of error in both R_Z and R_X are minimised, and the bases become approximately identical in performance.



Appendix C. Generalised expressions for cat telecorrection

C.1. Mapping of input logical states to output logical states

We will begin with the effect of the loss channel on a coherent state. Using the Kraus operators of the loss channel given in equation (13), for a coherent state of amplitude α and phase ϕ , we find

$$\hat{B}_\Gamma^{(l)} |e^{i\phi}\alpha\rangle = \sqrt{\frac{\Gamma}{1-\Gamma}} \frac{\hat{a}^l}{\sqrt{l!}} \sqrt{1-\Gamma}^{\hat{n}} |e^{i\phi}\alpha\rangle \quad (C1)$$

$$= \sqrt{\frac{\Gamma}{1-\Gamma}} \frac{\hat{a}^l}{\sqrt{l!}} e^{-\Gamma\alpha^2/2} |e^{i\phi}\sqrt{1-\Gamma}\alpha\rangle \quad (C2)$$

$$= \sqrt{\frac{\Gamma}{1-\Gamma}} e^{-\Gamma\alpha^2/2} \frac{\sqrt{1-\Gamma}^l e^{il\phi}\alpha^l}{\sqrt{l!}} |e^{i\phi}\sqrt{1-\Gamma}\alpha\rangle \quad (C3)$$

$$= e^{-\Gamma\alpha^2/2} \frac{(\sqrt{\Gamma}e^{i\phi}\alpha)^l}{\sqrt{l!}} |e^{i\phi}\sqrt{1-\Gamma}\alpha\rangle. \quad (C4)$$

Going forward we will denote the coherent states composing a particular order of cat code as follows,

$$\alpha_j \equiv e^{ij\phi_L}\alpha, \quad (C5)$$

The damped amplitude will be indicated with an underbar as

$$\underline{\alpha} \equiv \sqrt{1-\Gamma}\alpha, \quad (C6)$$

giving us

$$\underline{\alpha}_j = e^{ij\phi_L}\underline{\alpha}. \quad (C7)$$

An underbar will also be used to indicate any other quantity belonging to states prepared at a damped amplitude, such as the logical codewords, $|\underline{j}\rangle$, and normalisation constants, $\underline{\mathcal{N}}_j$.

For a state in the cat basis described by equations (4) and (5), we can write

$$\rho_{\text{in}} = \sum_{J,J'} q_{JJ'} |\tilde{J}\rangle \langle \tilde{J}'| = \sum_{J,J'} \sum_{j,j'=0}^{2L+1} q_{JJ'} \frac{(-1)^{jL+j'L'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{J'}}} |\alpha_j\rangle \langle \alpha_{j'}|, \quad (C8)$$

which will also describe any state immediately following teleportation, as here teleportation restores the state to the original basis. Now, for our cat state of equation (C8), the effects of loss are as follows:

$$\mathcal{L}_\Gamma(\rho_{\text{in}}) = \sum_{l=0}^{\infty} \hat{B}_\Gamma^{(l)} \rho_{\text{in}} [\hat{B}_\Gamma^{(l)}]^\dagger \quad (C9)$$

$$= \sum_{l=0}^{\infty} \sum_{J,J'} q_{JJ'} \hat{B}_\Gamma^{(l)} |\tilde{J}\rangle \langle \tilde{J}'| [\hat{B}_\Gamma^{(l)}]^\dagger \quad (C10)$$

$$= \sum_{l=0}^{\infty} \sum_{J,J'}^{\{0,1\}} \sum_{j,j'=0}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{J'}}} \hat{B}_\Gamma^{(l)} |\alpha_j\rangle \langle \alpha_{j'}| \left[\hat{B}_\Gamma^{(l)} \right]^\dagger \quad (\text{C11})$$

$$= \sum_{l=0}^{\infty} \sum_{J,J'}^{\{0,1\}} \sum_{j,j'=0}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{J'}}} \frac{\left[\Gamma e^{i(j-j')\phi_L} \alpha^2 \right]^l}{l!} e^{-\Gamma \alpha^2} |\alpha_j\rangle \langle \alpha_{j'}| \quad (\text{C12})$$

$$= \sum_{J,J'}^{\{0,1\}} \sum_{j,j'=0}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{J'}}} \exp \left[\Gamma \alpha^2 e^{i(j-j')\phi_L} \right] e^{-\Gamma \alpha^2} |\alpha_j\rangle \langle \alpha_{j'}|, \quad (\text{C13})$$

where from (C12) to (C13) the sum over each order of loss has been re-written as

$$\sum_{l=0}^{\infty} \frac{\left[\Gamma e^{i(j-j')\phi_L} \alpha^2 \right]^l}{l!} = \exp \left[\Gamma \alpha^2 e^{i(j-j')\phi_L} \right]. \quad (\text{C14})$$

The lossy cat state is then mixed via a 50:50 beamsplitter with the first mode of an encoded $|\tilde{\Phi}^+\rangle$ ancilla given by

$$\rho_\Phi = \frac{1}{2} \sum_{\tilde{K}, \tilde{K}'}^{\{0,1\}} |\tilde{K}\tilde{K}\rangle \langle \tilde{K}'\tilde{K}'|, \quad (\text{C15})$$

where the ancilla is composed of logical cat states, and has its first mode prepared at a reduced amplitude to match that of the lossy input state. The action of a real symmetric 50:50 beamsplitter on two modes containing coherent states is given by

$$\hat{U}|\alpha, \beta\rangle = \left| \frac{\alpha + \beta}{\sqrt{2}}, \frac{\beta - \alpha}{\sqrt{2}} \right\rangle. \quad (\text{C16})$$

Using this we find

$$\rho_M = \hat{U}_{1,2} [\mathcal{L}(\rho_{\text{in}}) \otimes \rho_\Phi] \hat{U}_{1,2}^\dagger \quad (\text{C17})$$

$$= \sum_{J,J', j,j'=0}^{\{0,1\}} \sum_{K,K'}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{J'}}} \exp \left[\Gamma \alpha^2 e^{i(j-j')\phi_L} \right] e^{-\Gamma \alpha^2} \hat{U}_{1,2} |\alpha_j \tilde{K} \tilde{K}\rangle \left\langle \alpha_{j'} \tilde{K}' \tilde{K}' \right| \hat{U}_{1,2}^\dagger \quad (\text{C18})$$

$$= \sum_{J,J', j,j', K,K' k,k'=0}^{\{0,1\}} \sum_{K,K'}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'+kK+k'K'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{\tilde{K}} \mathcal{N}_{J'} \mathcal{N}_{\tilde{K}'}}} \exp \left[\Gamma \alpha^2 e^{i(j-j')\phi_L} \right] e^{-\Gamma \alpha^2} \hat{U}_{1,2} |\alpha_j \alpha_{\tilde{K}} \tilde{K}\rangle \left\langle \alpha_{j'} \alpha_{\tilde{K}'} \tilde{K}' \right| \hat{U}_{1,2}^\dagger \quad (\text{C19})$$

$$= \sum_{J,J', j,j', K,K' k,k'=0}^{\{0,1\}} \sum_{K,K'}^{2L+1} q_{JJ'} \frac{(-1)^{j+j'J'+kK+k'K'}}{\sqrt{\mathcal{N}_J \mathcal{N}_{\tilde{K}} \mathcal{N}_{J'} \mathcal{N}_{\tilde{K}'}}} \exp \left[\Gamma \alpha^2 e^{i(j-j')\phi_L} \right] e^{-\Gamma \alpha^2} \left| \underline{\gamma}_{(j,k)} \underline{\beta}_{(k,j)} \tilde{K} \right\rangle \left\langle \underline{\gamma}_{(j',k')} \underline{\beta}_{(k',j')} \tilde{K}' \right|, \quad (\text{C20})$$

where we have used

$$\underline{\gamma}_{(j,k)} \equiv \left[e^{ij\phi_L} + e^{ik\phi_L} \right] \sqrt{1 - \Gamma \alpha} / \sqrt{2}, \quad (\text{C21})$$

$$\underline{\beta}_{(k,j)} \equiv \left[e^{ik\phi_L} - e^{ij\phi_L} \right] \sqrt{1 - \Gamma \alpha} / \sqrt{2}. \quad (\text{C22})$$

to simplify the mixed output of the beamsplitter; one can see here the importance of matching the amplitude of the ancilla's first mode to that of the lossy transmitted state, as otherwise the pairs of exponentials in either of equations (C21) or (C22) would lack a common factor.

Next, two photon-number measurements are made on the first two modes: that of the input state and the reduced-amplitude mode of the ancilla. For coherent states a photon-number measurement is as follows for $\beta \in \mathbb{C}$:

$$\langle n | \beta \rangle = e^{-|\beta|^2/2} \frac{\beta^n}{\sqrt{n!}}. \quad (\text{C23})$$

Here, for brevity, we show the effect of the measurements only on the left bra-ket pair:

$$\langle nm | \gamma_{(j,k)} \beta_{(k,j)} \rangle = \frac{1}{\sqrt{n!m!}} \exp \left[-\frac{1}{2} (|\gamma_{(j,k)}|^2 + |\beta_{(k,j)}|^2) \right] [\gamma_{(j,k)}]^n [\beta_{(k,j)}]^m,$$

where we find that the exponential is a constant,

$$\begin{aligned} |\gamma_{(j,k)}|^2 + |\beta_{(k,j)}|^2 &= \frac{\alpha^2}{2} (|e^{ij\phi_L} + e^{ik\phi_L}|^2 + |e^{ik\phi_L} - e^{ij\phi_L}|^2) \\ &= \frac{\alpha^2}{2} [(e^{ij\phi_L} + e^{ik\phi_L})(e^{-ij\phi_L} + e^{-ik\phi_L}) \\ &\quad + (e^{ik\phi_L} - e^{ij\phi_L})(e^{-ik\phi_L} - e^{-ij\phi_L})] \\ &= \frac{\alpha^2}{2} [4e^0 + e^{i(j-k)\phi_L} - e^{i(j-k)\phi_L} + e^{i(k-j)\phi_L} - e^{i(k-j)\phi_L}] \\ &= 2\alpha^2. \end{aligned} \quad (C24)$$

Thus we have

$$\langle nm | \gamma_{(j,k)} \beta_{(k,j)} \rangle = \frac{e^{-(1-\Gamma)\alpha^2}}{\sqrt{n!m!}} \left[\sqrt{\frac{1-\Gamma}{2}} \alpha \right]^{n+m} [e^{ij\phi_L} + e^{ik\phi_L}]^n [e^{ik\phi_L} - e^{ij\phi_L}]^m. \quad (C25)$$

Using this, the expression for the final state corresponding to a syndrome measurement (n, m) is

$$\begin{aligned} \rho_{\text{out}}(n, m) &= (\langle nm | \otimes \mathcal{I}) \rho_M(|nm\rangle \otimes \mathcal{I}) \\ &= \sum_{\substack{J, J', \\ K, K'}}^{\{0,1\}} \sum_{\substack{j, j', \\ k, k'=0}}^{2L+1} q_{JJ'} \frac{(-1)^{jJ'+j'J'+kK+k'K'}}{\sqrt{\mathcal{N}_J \mathcal{N}_K \mathcal{N}_{J'} \mathcal{N}_{K'}}} \frac{e^{(\Gamma-2)\alpha^2}}{n!m!} \left[\frac{(1-\Gamma)}{2} \alpha^2 \right]^{n+m} \\ &\quad \cdot [e^{ij\phi_L} + e^{ik\phi_L}]^n [e^{ik\phi_L} - e^{ij\phi_L}]^m \\ &\quad \cdot [e^{-ij'\phi_L} + e^{-ik'\phi_L}]^n [e^{-ik'\phi_L} - e^{-ij'\phi_L}]^m \\ &\quad \cdot \exp[\Gamma\alpha^2 e^{i(j-j')\phi_L}] \\ &\quad \cdot |\tilde{K}\rangle \langle \tilde{K}'|. \end{aligned} \quad (C26)$$

Note that the trace of equation (C27) at a given (n, m) is equal to the probability of said outcome, and thus averaging over these values is inherently weighted by their probabilities.

From equation (C27) we can see how the two techniques used in section 4.1 work. The biased ancilla of equation (36) can be written as

$$\rho_{\Phi}(x) = \frac{1}{1+|x|^2} \sum_{K, K'}^{\{0,1\}} x^K (x^*)^{K'} |\tilde{K}\rangle \langle \tilde{K}'|, \quad (C28)$$

and it is clear that x^K remains associated with $|\tilde{K}\rangle$ in our final state even after teleportation.

The basis of $|\tilde{K}\rangle$ in equation (C27) is also evidently arbitrary, as teleportation does not depend in any way on the properties of the ancilla's second mode, and requires only that the first mode be prepared appropriately. This is similar to the arbitrary order of the third rail seen in the correction scheme proposed by [6] using two-mode rotations for the class of RSB codes, of which cat codes are a member. It may initially appear useful to switch between R_Z and R_X cats to gain access to both Gaussian Z and X operations, respectively, but even in the unitary regime each teleportation risks a full set of Pauli errors that could nullify the desired operation. However, a teleportation into the GKP code basis, for example, would enable all-optical correction of the X and Z errors via displacements.

C.2. Leading transformations corresponding to a syndrome

Here we show the origin of the expression for the sub-loss-capacity transformations associated with a syndrome, $\bar{T}(n, m)$, using an approximation of the channel that considers only pure state cat qubits as inputs and the first L orders of the loss channel. Otherwise the mechanics are the same as in the prior section, so the derivation will not be exhaustive. Beginning with the qubit,

$$|\psi_{\text{in}}\rangle = \frac{q_0|\tilde{0}\rangle + q_1|\tilde{1}\rangle}{\sqrt{2}}, \quad (C29)$$

we can write

$$|\psi_{\text{out}}\rangle \approx \sum_{J,K}^{\{0,1\}} \sum_{j,k=0}^{2L+1} q_J e^{ij\phi_L} \frac{(\sqrt{\Gamma}\alpha)^l}{\sqrt{l!}} \frac{(-1)^{J+K}}{\sqrt{2\mathcal{N}_J\mathcal{N}_K}} \frac{e^{(\Gamma-2)|\alpha|^2/2}}{\sqrt{n!m!}} \left[\sqrt{\frac{1-\Gamma}{2}} \alpha \right]^{n+m} \cdot [e^{ij\phi_L} + e^{ik\phi_L}]^n [e^{ik\phi_L} - e^{ij\phi_L}]^m |\tilde{K}\rangle, \quad (\text{C30})$$

where

$$l \equiv L(n+m) \bmod (L+1), \quad (\text{C31})$$

and so the association of q_J with $|\tilde{K}\rangle$ allows us to re-write equation (C30) as a 2×2 transformation,

$$\bar{T}_{\text{KJ}}(n, m) = \sum_{j,k=0}^{2L+1} e^{ij\phi_L} \frac{(\sqrt{\Gamma}\alpha)^l}{\sqrt{l!}} \frac{(-1)^{J+K}}{\sqrt{2\mathcal{N}_J\mathcal{N}_K}} \frac{e^{(\Gamma-2)|\alpha|^2/2}}{\sqrt{n!m!}} \left[\sqrt{\frac{1-\Gamma}{2}} \alpha \right]^{n+m} \cdot [e^{ij\phi_L} + e^{ik\phi_L}]^n [e^{ik\phi_L} - e^{ij\phi_L}]^m. \quad (\text{C32})$$

Appendix D. Deformation correction example

Here we will show a simple example of deformation correction using the transformations provided in equations (21) and (22). The parameters are, again, $L = 1$, $\alpha = 2$, and $\Gamma = 0$; we will assume the entire system is lossless, and thus deformation-correction will perfectly restore the state.

We will use the state $|\tilde{+}\rangle$ for this example, and we will assume a single instance of teleportation has been applied with the outcome $(2, 0)$. Our state is thus transformed as

$$\bar{T}(2, 0) \left(\frac{|\tilde{0}\rangle + |\tilde{1}\rangle}{\sqrt{2}} \right) = \begin{pmatrix} 0 & \frac{16}{\sqrt{\mathcal{N}_1\mathcal{N}_0}e^4} \\ \frac{16}{\sqrt{\mathcal{N}_0\mathcal{N}_1}e^4} & 0 \end{pmatrix} \left(\frac{|\tilde{0}\rangle + |\tilde{1}\rangle}{\sqrt{2}} \right) \quad (\text{D1})$$

$$= \frac{16}{\sqrt{2}e^4} \left(\frac{1}{\sqrt{\mathcal{N}_0\mathcal{N}_1}} |\tilde{0}\rangle + \frac{1}{\sqrt{\mathcal{N}_1\mathcal{N}_0}} |\tilde{1}\rangle \right) \quad (\text{D2})$$

which, after the necessary X correction, becomes

$$\frac{16}{\sqrt{2}e^4} \left(\frac{1}{\sqrt{\mathcal{N}_1\mathcal{N}_0}} |\tilde{0}\rangle + \frac{1}{\sqrt{\mathcal{N}_0\mathcal{N}_1}} |\tilde{1}\rangle \right), \quad (\text{D3})$$

where we note that the state is not normalised, but is weighted according to the probability of all prior outcomes.

Now, let us suppose for the sake of the example that $(2, 2)$ is our desired outcome, and that we will succeed. Keeping our bias, x , arbitrary for now, the effect of outcome $(2, 2)$ on the state so far will be

$$\begin{aligned} \bar{T}^{(x)}(2, 2) X \bar{T}(2, 0) \left(\frac{|\tilde{0}\rangle + |\tilde{1}\rangle}{\sqrt{2}} \right) \\ = \frac{16}{\sqrt{2}e^4} \sqrt{\frac{2}{1+|x|^2}} \begin{pmatrix} \frac{32\sqrt{2}}{\sqrt{\mathcal{N}_0\mathcal{N}_0}e^4} & 0 \\ 0 & -x \frac{32\sqrt{2}}{\sqrt{\mathcal{N}_1\mathcal{N}_1}e^4} \end{pmatrix} \left(\frac{1}{\sqrt{\mathcal{N}_1\mathcal{N}_0}} |\tilde{0}\rangle + \frac{1}{\sqrt{\mathcal{N}_0\mathcal{N}_1}} |\tilde{1}\rangle \right) \end{aligned} \quad (\text{D4})$$

$$= \frac{512}{e^8} \sqrt{\frac{2}{1+|x|^2}} \left(\frac{1}{\sqrt{\mathcal{N}_1\mathcal{N}_0\mathcal{N}_0\mathcal{N}_0}} |\tilde{0}\rangle - \frac{x}{\sqrt{\mathcal{N}_0\mathcal{N}_1\mathcal{N}_1\mathcal{N}_1}} |\tilde{1}\rangle \right), \quad (\text{D5})$$

to which we will apply a Z correction and for simplicity ignore the common scalar factors, giving us

$$\left(\frac{1}{\sqrt{\mathcal{N}_1 \mathcal{N}_0 \mathcal{N}_0 \mathcal{N}_0}} |\tilde{0}\rangle + \frac{x}{\sqrt{\mathcal{N}_0 \mathcal{N}_1 \mathcal{N}_1 \mathcal{N}_1}} |\tilde{1}\rangle \right). \quad (\text{D6})$$

From here we can see that to correct the deformation of equation (D3) x must satisfy

$$x = \frac{\sqrt{\mathcal{N}_0 \mathcal{N}_1 \mathcal{N}_1 \mathcal{N}_1}}{\sqrt{\mathcal{N}_1 \mathcal{N}_0 \mathcal{N}_0 \mathcal{N}_0}}, \quad (\text{D7})$$

and thus with said choice of x and after renormalisation we return to our initial state

$$\frac{|\tilde{0}\rangle + |\tilde{1}\rangle}{\sqrt{2}}. \quad (\text{D8})$$

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