

Joint Hybrid Repair and Remanufacturing Systems and Supply Control

F. Berthaut^a, A. Gharbi^{b*} and R. Pellerin^a

^a*Department of Mathematics and Industrial Engineering,
École Polytechnique de Montréal, Montréal, Canada;*

^b*Department of Automated Production Engineering, LCCSP Laboratory,
University of Québec, École de Technologie Supérieure, Montréal, Canada*

The control of a stochastic manufacturing system that executes capital asset repairs and remanufacturing in an integrated system is examined. The remanufacturing resources respond to planned returns of worn-out equipments at the end of their expected life and unplanned returns triggered by major equipment failures. Remanufacturing operations for planned demand can be executed at different rates and costs corresponding to different replacement and repair modes. The replacement components inventory is provided by an upstream supply with random lead times. The objective is to determine a control policy for both the supply and remanufacturing activities that minimizes the average repair/replacement, acquisition and inventory/shortage total cost over an infinite horizon. We propose a sub-optimal joint remanufacturing and supply control policy, composed of a multi-hedging point policy (MHPP) for the remanufacturing stage and an (s, Q) policy for the replacement parts supply. The MHPP is based on two inventory thresholds that trigger the use of predefined remanufacturing modes. Control policy parameters are obtained combining analytical modeling, simulation experiments and response surface methodology. The effects of the distribution, mean and variability of the lead time are tested and a sensitivity analysis of cost parameters is conducted to validate the proposed control policy. We also show that our policy leads to a significant cost reduction as compared to a combination of an hedging point policy (HPP) and an (s, Q) policy.

Keywords: stochastic optimal control; remanufacturing; overhaul; unreliable supply; hedging point policy; simulation;

1. Introduction

Remanufacturing is defined as the restoration of a product to a standard as close as possible to its original condition in appearance, performance and life expectancy (Cox *et al.* (2002)). This article focuses on capital goods, which are mostly rebuilt for the purpose of being reused by the initial product owners. This type of remanufacturing coexists with maintenance and service organisations and is integrated through product life cycle management. Thus, capital goods are used, then repaired or remanufactured in an integrated system to be reinserted into a new life cycle. The system must fulfil, with the same skilled resources, both planned remanufacturing orders and unplanned repairs. The processing of unplanned repairs has priority over the processing of planned remanufacturing orders.

The primary objective of this hybrid repair and remanufacturing system is to maintain the level of serviceable capital goods above the operating firm's level in order to ensure sufficient operating performances at the lowest cost. We aim to control the level of the serviceable equipments by controlling the remanufacturing process of the system. In practice, managers have to decide whether to repair or replace worn-out equipments. Even though replacing a component is less time consuming than repairing it, it usually costs more and replacement parts shortages prevent managers from using the replacement mode. For example, when the equipment surplus exceeds a sufficient minimum level, the remanufacturing organizations will typically choose to repair most components, whereas replacing prevailed in emergency case to accelerate the serviceable equipment supply (Gharbi *et al.* 2008). Therefore, during an overhaul, the choice of operations depends on the required service level, the current level of the serviceable inventory and on the availability of replacement parts. It is important to note that the serviceable inventory is not fed by the remanufacturing system during the unplanned repair and consequently decreases with demand and that serviceable stock shortages will result in additional costs. Furthermore, the replacement parts replenishment may also undergo uncertainties on the lead time supply. The formulation of this problem is complex because of the specific conditions of each worn-out component, of the stochastic operating process and of the uncertainties in the supply. Within this context, it is critical to develop an optimal control policy that will jointly control the remanufacturing and replenishment processes.

Our paper describes a control approach to jointly optimize the hybrid repair and remanufacturing system control policy and the supply policy. Detailed scheduling of the process is not a prime concern. The repair and replacement processes are treated in an aggregate way, characterized by rates and costs. The problem is formulated as a multi-level control problem and a sub-optimal combined policy is proposed. This policy is described by inventory thresholds (z_1, z_2) which trigger the use of the repair or replacement modes and by a reorder level-reorder quantity (s, Q) policy for the replacement parts supply. These parameters describe the entire control policy and a simulation based experimental approach is used to achieve a close approximation of this optimal control policy for a numerical example with random lead times.

The following article begins with a literature review in the area of stochastic optimal control problem for remanufacturing systems (Section 2). The hybrid repair and remanufacturing problem is then presented and a control policy developed in Section 3. A resolution approach based on a simulation model, experimental design and response surface methodology is detailed in Section 4. Then, in Section 5, we present our experimental results with an illustrative example in order to study the behaviour of the proposed control policy in presence of stochastic lead times. Finally we conclude by summarizing the main results and highlight possible directions for further research in Section 6.

2. Literature review

Reuse opportunities allow managers to consider remanufacturing as an alternative to manufacturing. However, the interaction between these two supply chains limits the effectiveness of traditional management methods. The emergence of remanufacturing has recently prompted researchers to address remanufacturing operation issues. The reader is referred to Fleischman *et al.* (1997), Guide and Jayaraman (2000) and Rubio *et al.* (2008) for an overview of the remanufacturing literature. More precisely, the production and inventory control approach generally considers systems where remanufacturing processes are integrated in a single production environment (Inderfurth *et al.* 2001, Van der Laan and Salomon 1997, Van der Laan *et al.* 1999), whereby managers can fulfil demand by ordering raw materials externally and manufacturing new products, or by overhauling used products and bringing them back to “as new conditions”. Within this context of hybrid production/remanufacturing systems, the objective is to control two different inventory positions and to synchronize external component orders and internal recovery activities at a minimum cost. For stochastic models, where the demand and returns are stochastic variables, PUSH, PULL or DUAL sourcing policies have been investigated for both periodic (Kiesmuller 2003) and continuous reviews (Zanoni *et al.* 2006, Van der Laan *et al.* 1999). The latter article also studied the effects of lead time duration and variability on the total expected cost in remanufacturing and manufacturing processes.

This article proposes a different framework for repair/remanufacturing systems for capital goods, where repair and replacement are treated in the same execution system and with the same pool of resources. This leads to the adoption of different remanufacturing rates to keep an optimal serviceable inventory level.

As the hybrid repair and remanufacturing processes respond to both planned and unplanned return flows of worn-out capital goods, similarities are observed with the stochastic optimal control problem of Flexible Manufacturing Systems (FMS). Kimemia and Gerschwin (1983) introduced the concept of hedging point policy (HPP) to control the flow rates of parts through a manufacturing system prone to machine failures and repair cycles defined by Markov chains. Within such a policy, a non negative production surplus of part types, corresponding to an optimal inventory level, is maintained to compensate for future backlogs caused by machine failures. The production rate of the system is simply controlled by the inventory level. Gharbi *et al.* (2008) provided an approach to solve the hybrid repair and remanufacturing systems control policy problem, based on the multi-hedging point policy (MHPP) theory, as in Sharifnia (1988). The serviceable inventory controls the remanufacturing rate by repairing or replacing (a faster process) a worn-out component in order to minimize the remanufacturing and inventory/shortage cost per unit of time over an infinite horizon. An important assumption made in the previous paper is that the replacement components are always available. To fill this gap and study the effects of an unreliable supply on the MHPP, Berhaut *et al.* (2008) introduced a constraining probabilistic availability of replacement parts that indicates whether the replacement components can be used every time replacement is required. However, the system did not consider a supply control policy.

Recent research has shown that integrated control systems that combine manufacturing and raw material procurement give better performance in terms of average total cost than when control is performed separately (Lee, 2005). Similarly, Brezavscek and Hudoklin (2003), Huang *et al.* (2008) included spare provisioning policy in preventive maintenance models and Hajji *et al.* (2008a, 2008b) developed an integrated production and supply policy for a three levels flexible supply chain system. Since the resolution of the problem is difficult, Hajji *et al.* (2008b) proposed a numerical approach that led to a modified and simplified policy that combines an (s, Q) policy and HPP and adopted a simulation based experimental approach to achieve a close approximation of this control policy.

Moreover, shortages in material capacity of the supplier, unexpected breakdowns, process adjustments, strikes, etc., make the treatment of supply uncertainty an important issue in the analysis of stochastic inventory problems (Gullu *et al.* 1999). This uncertainty takes the form of a stochastic lead time in our problem. During replacement parts stock outs, demand from the remanufacturing stage cannot be met by the

replacement mode and consequently the repair mode is executed. Such a situation whereby the demand not immediately met is lost is known in the supply control field as the lost sales scenario. The lost sales scenario for stochastic supply problem is more difficult to model and has received less attention than backorders case (Hadley and Within 1963). [Bensoussan *et al.* \(1983\)](#), [Cheng and Sethi \(1999\)](#) proved the optimality of (s, S) type policies for periodic review inventory problem with lost sales and stochastic demand, using a dynamic programming approach and the concept of K -convexity. In the context of continuous review, inventory systems with lost sales are in general intractable and neither an (s, S) nor an (s, Q) policy will be optimal. Nevertheless, best policies among (s, S) and (s, Q) have a cost which is generally close to that of the optimal policy ([Hill and Johansen 2006](#)), assuming a compound Poisson demand, fixed lead times and at most a single order to be outstanding at any time. The variable lead time case has been investigated, among others, by [Mohebbi and Posner \(1998\)](#), who presented an exact cost minimization formulation for an (s, Q) policy. Determination of the control parameters is then achieved through a minimization procedure or heuristics, such as the well-known Hadley and Within iterative procedure. They studied the effects of the lead time variability on the control policy parameters and on the associated cost. In addition, (s, S) and (s, Q) policies are attractive to managers due to their simplicity and ease of implementation.

The main contribution of the present paper is to jointly solve the control problems associated with hybrid repair and remanufacturing systems and replacement parts supply in a two-stage remanufacturing system. We formulate the problem as a multi-level control problem and propose in the next section a multi-hedging point policy based on two thresholds and an (s, Q) policy to control the remanufacturing and supply processes.

3. Problem statement

3.1 Model assumptions and notations

The system studied (figure 1) consists of an integrated hybrid repair and remanufacturing system and an unreliable upstream supplier. The entire system faces a single product type demand. The system must meet a demand of serviceable equipments by treating planned and unplanned returns. The planned returns are defined as the foreseeable returns of used equipments at the expected end of life and are disturbed by the unplanned returns that are triggered by major equipment failures and that must be processed right away. When an unplanned return occurs, all remanufacturing resources are pre-empted to treat it, while the serviceable equipment level decreases with equipment demand. Managers of the remanufacturing system choose to repair or replace parts of the worn-out equipments for executing the planned demand. The replacement depends on the replacement parts inventory, and thus indirectly depends on the supply policy. When the replacement is executed, the level of the replacement parts inventory decreases. Whether the replacement parts inventory is starved, replacing is not yet available until the reception of a previously placed order and repair mode is executed instead.

The remanufacturing system is designed to perform three different execution modes, noted by $\alpha \in \{0,1,2\}$ and described by a cost per unit of time $c_{u\alpha}$ and a repair rate u_α , as follows:

- a repair mode, characterized by (u_0, c_{u0}) ;
- an accelerated repair mode, characterized by (u_1, c_{u1}) , with $u_1 > u_0$ and $c_{u1} > c_{u0}$;
- a replacement mode, characterized by (u_2, c_{u2}) , with $u_2 > u_1$ and $c_{u2} > c_{u1}$, and by the replacement parts consumption. Note that the replacement part acquisition cost is included in c_{u2} .

In order to ensure the feasibility of the replacement mode in the long run, managers have to supply the replacement parts inventory by ordering a Q_i lot of replacement parts with an ordering cost K . This order is then delivered at instant θ_i after a stochastic delay τ . The replacement parts holding cost per unit of time is c_I^+ .

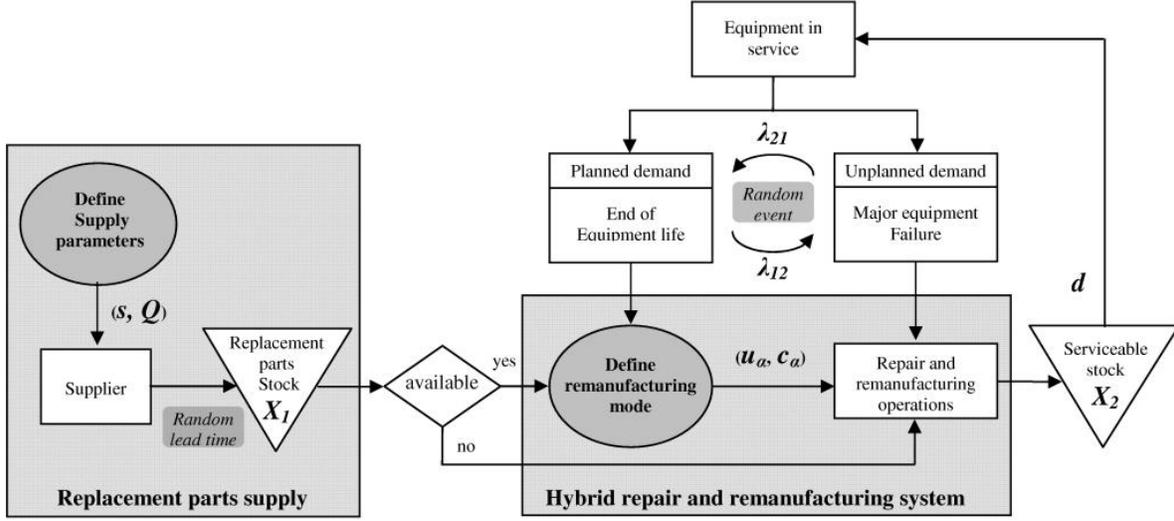


Figure 1. Joint Remanufacturing and replenishment activities control problem.

When the number of serviceable equipment is insufficient, the operating firm is significantly penalized by having to cancel operations or renting equipments. c_2^- denotes the cost to be paid per equipment per unit of time for failing to meet the service level. Similarly, c_2^+ denotes the cost for keeping inventory at a higher level than the service level. These backlog and holding costs are such that $c_2^- > c_2^+ > c_1^+$.

The state of the system at time t can be described by the three following components:

- a continuous part which describes the cumulative surplus level (inventory if positive, backlog if negative), measured by $x_2(t)$;
- a piecewise continuous part which describes the replacement parts level and measured by $x_1(t)$. This part faces a continuous downstream demand (i.e., replacement rate of the remanufacturing system) and an upstream supply. When $x_1(t)$ is equal to zero, the remanufacturing system cannot perform any replacement. Let $0 \leq x_1(t) < L$ be the capacity constraint of the replacement part inventory.
- a discrete part which describes the state of remanufacturing system. This state can be classified as “producing planned demand”, denoted by $\xi(t) = 1$, or “producing unplanned demand”, denoted by $\xi(t) = 2$. This process could be modeled as a continuous time Markov chain, with time-invariant transition rates λ_{12} and λ_{21} .

The following differential equations give the dynamic of the stock levels $x_1(t)$ and $x_2(t)$:

$$\begin{aligned}
 \dot{x}_2(t) &= u(t, \alpha) - d, \quad x_2(0) = x_2, \quad \forall t \geq 0 \\
 \dot{x}_1(t) &= -u(t, 2) \cdot \text{Ind}\{\alpha = 2\}, \quad x_1(0) = x_1, \quad \forall t \in]\theta_i, \theta_{i+1}[\\
 x_1(\theta_i^+) &= x_1(\theta_i^-) + Q_i, \quad i = 1 \dots N
 \end{aligned} \tag{1}$$

$$\text{Where } \text{Ind}\{\alpha = m\} = \begin{cases} 1 & \text{if } \alpha = m \\ 0 & \text{otherwise} \end{cases}$$

Where x_1, x_2 denote the initial stock levels, d denotes the demand rate, $u(t, \alpha)$ the remanufacturing rate control in mode α , θ_i^- and θ_i^+ denote the negative and positive boundaries of the i^{th} receipt instant θ_i .

3.2 Remanufacturing and supply policies formulation

The problem formulated in the previous section is similar to the stochastic optimal control of manufacturing systems facing an unreliable upstream supply. Hajji *et al.* (2008a) considered this class of systems by a three level supply chain system responding to one-part type demand. This system was composed of an unreliable manufacturing system and of an unreliable supplier, both of which are subjected to availability and unavailability periods. In order to hedge against supply and capacity shortage, the raw material inventory x_1 and the final products surplus x_2 have to be maintained at excess levels. The decision variables were the production rate $u(\cdot)$ when the manufacturing system is up and a sequence of supply orders denoted by $\Omega = \{(\theta_0, Q_0), (\theta_1, Q_1), \dots\}$, where Q_i is the order quantity received at time θ_i . The objective was to find the optimum decisions $(\Omega, u(\cdot))$ that minimize the total cost $J(\cdot)$, which includes the manufacturing, inventory, backlog and supply costs over an infinite horizon for each system state and inventory levels. Once the problem was formulated as a dynamic programming problem, the value function was given by:

$$v(x_1, x_2, \xi) = \inf_{(\Omega, u) \in A} J(x_1, x_2, u, \Omega, \xi) \quad (2)$$

Where ξ is the state of the system (i.e. the availability state).

Hajji *et al.* (2008a) were able to establish that the value function is a viscosity solution of Hamilton-Jacob-Bellman equations that cannot be solved analytically. Since the joint production and supply policy is obtained when the value function is known, Hajji *et al.* (2008a) applied a numerical approximation method based on the Kushner iterative algorithm (Kushner and Dupuis 1992) in order to estimate the value function for discrete values of the state variables (x_1, x_2, ξ) . By observing separately the corresponding production and supply policies for the different systems states, they approximated the joint optimal control policy by a combination of a modified state dependent multi level base stock policy (MBSP) for the remanufacturing stage, and a state dependent (s, Q) supply policy. However, Hajji *et al.* (2008b) proposed a simplified approximation governed by a hedging point policy (HPP) and an (s, Q) type policy, such that:

$$u(\cdot) = \begin{cases} U_{\max} \cdot \text{Ind}\{\xi=1\} & \text{if } x_2 < Z \\ d \cdot \text{Ind}\{\xi=1\} & \text{if } x_2 = Z \\ 0 & \text{if } x_2 > Z \end{cases} \quad (3)$$

$$\Omega(\cdot) = \begin{cases} Q & \text{if } x_1(t) \leq s \\ 0 & \text{otherwise} \end{cases}$$

$$\text{with } 0 \leq Z; s < Q < L; s \geq 0.$$

Where U_{\max} is the maximum production rate and $\xi = 1$ denoted the system state 1 (manufacturing system available).

In this paper, the objective is to determine a control policy $(\Omega, u(\cdot))$ that minimizes the following average expected cost per unit of time over an infinite horizon:

$$J(x_1, x_2, u, \theta, Q, \alpha) = \lim_{T \rightarrow \infty} J_T(x_1, x_2, u, \theta, Q, \alpha)$$

$$\text{with } J_T(x_1, x_2, u, \theta, Q, \alpha) = \frac{1}{T} \left(\mathbb{E} \left[\int_0^T g(x_1, x_2, u) dt \right] + \left[\sum_{i=0}^{i=n} (K + c_R \cdot Q_j) \right] \right), \quad n: \theta_{n-1} < T \leq \theta_n \quad (4)$$

$$\text{and } g(x_1(t), x_2(t), u(t, \alpha)) = c_1^+ \cdot x_1^+(t) + c_2^+ \cdot x_2^+(t) + c_2^- \cdot x_2^-(t) + c_u \cdot u(\cdot), \quad \forall t \in]\theta_i, \theta_{i+1}[$$

Where $g(\cdot)$ denotes the instantaneous cost of maintaining the inventory/shortage $x_1(t)$ and $x_2(t)$ per unit time, $x_j^+ = \max(0, x_j)$, $x_j^- = \max(-x_j, 0)$ and c_u is the cost of remanufacturing at rate $u(\cdot)$.

This paper aims to transpose the joint production and supply policy of Hajji *et al.* (2008b) to the hybrid repair and remanufacturing systems presented in the previous section. The main difference between these problems is that instead of two different modes (u_0, c_{u0}) and (u_2, c_{u2}) , the flexibility of the remanufacturing processes allows us to consider an additional repair mode (u_1, c_{u1}) , which does not consume any material of the replacement parts inventory. The control policy will be affected in two ways. First, the remanufacturing process will be managed by a multi-hedging point policy (MHPP), composed of two thresholds that trigger the execution of the three remanufacturing modes, as proposed in Gharbi *et al.* (2008). Then, the joint production policy and supply presented above entails, in the case of raw material starvation, that the production rate is stopped until the reception of a previously placed order. When the replacement parts inventory drops to 0 in our remanufacturing system, the replacement mode cannot be executed and the subsequent maximum rate (the accelerated repair rate) is applied, as it does not require any material to be attained. Concerning the replacement parts supply, we also adopt a classical (s, Q) policy to control the replenishment. Such a policy should improve the cost performance as compared to a single hedging point policy with two execution modes, which will be showed on a numerical case in the last section. Indeed, the accelerated repair mode is less expensive than the replacement mode ($c_{u1} < c_{u2}$) when the serviceable equipment inventory is plenty, and postpones possible serviceable equipment shortages, and thus additional costs ($c_2^- > c_2^+$), in the case of replacement parts unavailability ($x_1(t) = 0$). Consequently a more appropriate joint remanufacturing and supply control policy is proposed below:

$$u(x, \xi) = \begin{cases} u_0 & \text{if } x_2(t) = z_1 \text{ and } \xi = 1 \\ u_1 & \text{if } z_2 \leq x_2(t) \leq z_1 \text{ and } \xi = 1 \\ & \text{or if } x_2(t) < z_2 \text{ and } x_1(t) = 0 \text{ and } \xi = 1 \\ u_2 & \text{if } x_2(t) < z_2 \text{ and } x_1(t) > 0 \text{ and } \xi = 1 \\ 0 & \forall x \text{ and } \xi = 2 \end{cases} \quad (5)$$

$$\Omega(\cdot) = \begin{cases} Q & \text{if } x_1(t) \leq s \\ 0 & \text{otherwise} \end{cases}$$

with the constraints: $0 \leq z_2 < z_1$, $s < Q < L$, $s \geq 0$.

The remanufacturing and supply policies presented above are interrelated. Indeed, the MHPP part depends on the replacement parts availability ($x_1(t) > 0$). In a similar manner, the differential equations (1), that depicts the system dynamics, highlight the influence of the remanufacturing states (x_2, ξ) on the demand of replacement parts, thus on the replacement parts level.

Figure 2 shows the dynamics of the serviceable equipments and replacement parts inventories controlled according to the joint remanufacturing and supply control policy. As unplanned demand occurs, the remanufacturing resources are pre-empted to treat it and consequently the amount of serviceable equipments $x_2(t)$ decreases until the demand is satisfied. Then, as $x_2(t)$ drops below z_1 , the remanufacturing process is accelerated to rate u_1 . When $x_2(t)$ drops below z_2 , if replacement parts are available ($x_1(t) > 0$), the remanufacturing process is further accelerated to rate u_2 , to prevent the surplus level from crossing over a negative value or else remains at rate u_1 (for $x_1(t) = 0$). When $x_1(t)$ crosses the level s , an order of Q replacement parts is placed and received after a delay τ . During this delay, if $x_1(t)$ decreases to zero, then the replacement mode would not be available until the reception of the order. Note that the replacement parts inventory remains constant when the replacement mode is not used ($x_2(t) > z_2$ or $\xi = 2$).

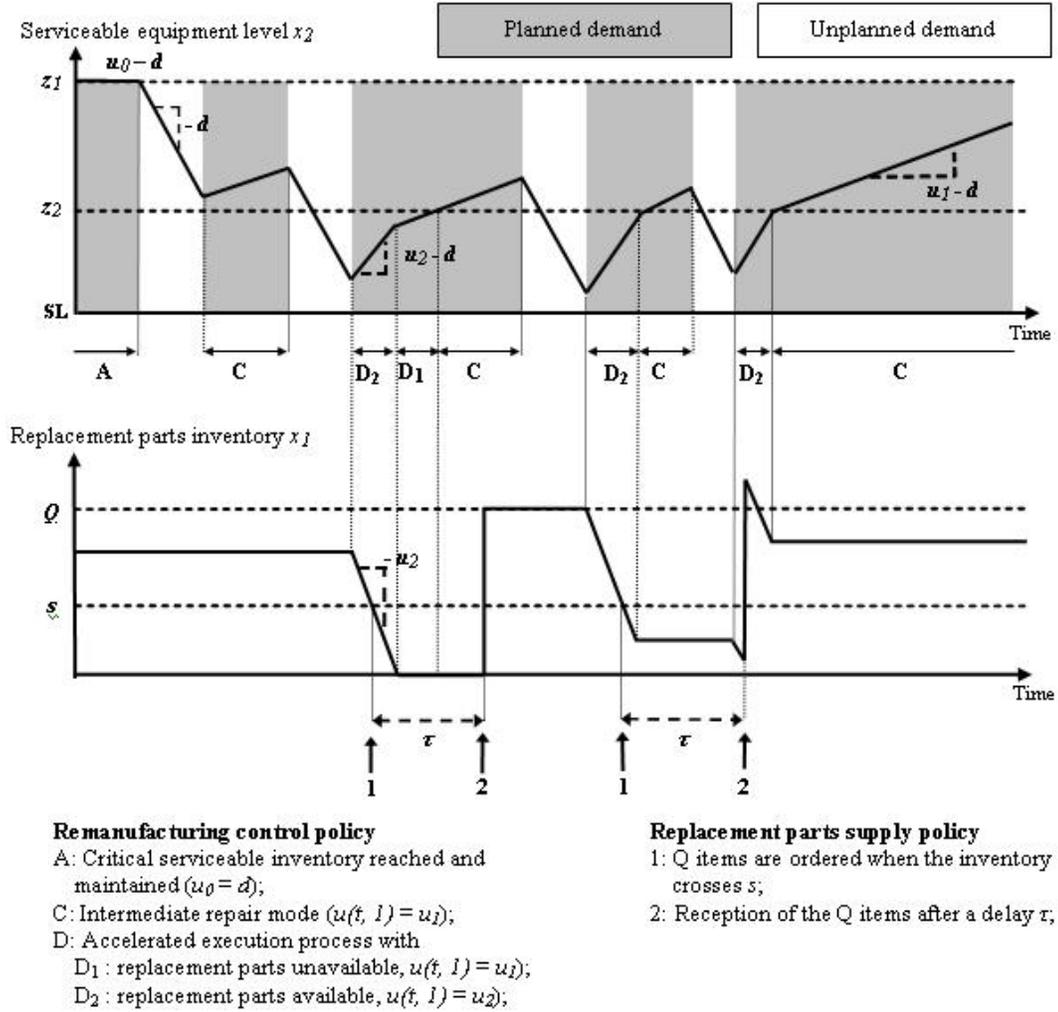


Figure 2. Evolution of the serviceable equipment inventory $x_2(t)$ and of the replacement parts inventory $x_1(t)$ under the joint remanufacturing and supply policy.

Once the values of (z_1, z_2, Q, s) are determined, the control policy is completely defined. These four control policy parameters have to be chosen in order to minimize the total cost defined in equation (4).

4. Simulation resolution and optimization

Within the sphere of control theory, especially for systems with multiple stochastic elements such as remanufacturing systems, optimal solutions are often difficult to calculate and/or are obtained under strict conditions limiting their application in real cases. Numerical methods or simulation tools are often effective approaches to understand the behaviour of a system and to obtain a close approximation of the optimal control policy. Thus, based on the work of Kenne and Gharbi (1999), we introduce a resolution approach that combines the descriptive capacities of conventional simulation models with analytical models, experimental design and response surface methodology techniques. The values of the control policy parameters are obtained by minimizing the total cost incurred by the simulation runs.

4.1 Resolution Approach

The first step consists of representing the remanufacturing and supply processes control problem through a stochastic optimal control model based on control theory. The objective of this approach is to obtain the control variables, namely the repair rates (u_i) and the supply parameters, in order to improve the response variable (i.e., the incurred total cost). The problem is then structured in a near optimal control policy for the hybrid repair and remanufacturing system and for the replenishment as presented in the previous section. This policy consists in defining the thresholds (z_1, z_2) associated with the predefined remanufacturing modes, and in defining the (s, Q) type policy parameters. A simulation model is developed to describe the dynamics of the system under the control policy parameterized by (z_1, z_2, s, Q). These four factors and the related incurred cost are respectively considered as input and output of the model.

The experimental design approach defines how the control factors can be varied in order to determine the effects of the main factors, their quadratic effects and their interactions (i.e., analysis of variance or ANOVA) on cost with a minimal set of simulation experiments. In the next step, the response surface methodology is used to obtain the relationship between the incurred cost and the significant main factors, quadratic effects and interactions. Assuming that $g(\cdot)$ is jointly convex, it can be shown, with a proof similar to Lou *et al.* (1994), that the total cost $J(\cdot)$ is jointly convex and then that the value function is convex. This convexity assumption was also used by Yong (1989) and Hajji *et al.* (2008). For this reason, we choose a second-order model as regression model for constructing the cost value and aim to find its unknown parameters. The model is then optimized by minimizing the estimated cost in order to determine the best values of the factors, here called (z_1^*, z_2^*, s^*, Q^*), and the optimal cost value J^* for executing our joint policy.

4.2 Simulation model

A simulation model that combines discrete-continuous changes was developed using the Visual SLAM language (Pritsker 1999). This model consists of several networks and user routines, each of which describes a specific task in the system (demand generation, control policy, states of the system, threshold crossing of inventory variables..., etc). We adopt a schematic representation of the model in figure 3 to facilitate understanding with the following descriptions of the different blocks.

- (1) The *INITIALIZATION* block initializes the values of the joint remanufacturing and supply policy (z_1, z_2, s, Q), for which the simulation run is conducted and the values of parameters of the system, such as the remanufacturing rates, the planned and unplanned demand occurrences, the demand rate of serviceable items, the supply lead time.
- (2) The *UNPLANNED-PLANNED DEMAND OCCURENCES* block performs the arrival of a planned flow of equipments to be remanufactured and the arrival of an unplanned return of equipments to be repaired at each λ_{12}^{-1} units of time. The *REPAIR* block will then treat this unplanned demand (λ_{21}^{-1} units of time). Note that when unplanned demand occurs, the remanufacturing resources are pre-empted until this demand is satisfied. Planned demands are therefore not fulfilled during this period and the surplus level decreases.
- (3) The *CONTROL POLICY* block provides the remanufacturing rates and the supply orders (refer to equation (5)). Observation networks raise a *FLAG* whenever inventory levels cross one of the control policy thresholds (z_1, z_2 and s).

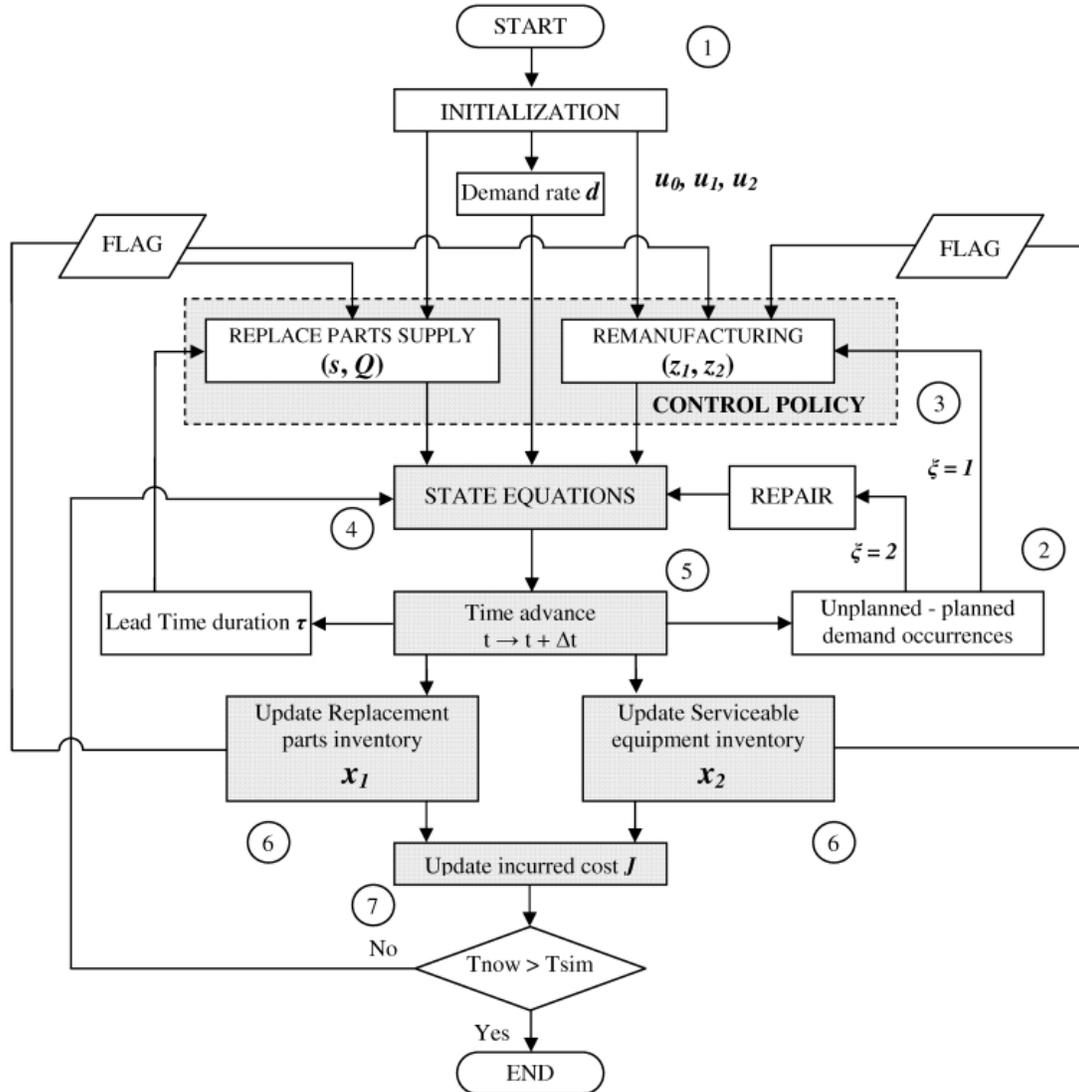


Figure 3. Diagram of simulation model.

- (4) The *STATE EQUATIONS* are the equations (1) expressed by a C language insert and networks. This block performs the remanufacturing activities of equipments depending on the remanufacturing rates and the supplies set by the control policies.
- (5) The *TIME ADVANCE* block change the current time according to a time step. Visual SLAM uses an algorithm to change the values of the discrete event scheduling (demand and unplanned demand rates) and continuous variables threshold crossing events.
- (6) The *UPDATE REPLACEMENT PARTS INVENTORY* x_1 and *UPDATE SERVICEABLE INVENTORY* x_2 blocks trace the real-time variations of $x_1(t)$ and $x_2(t)$. The surplus level varies as equipments are remanufactured or as a demand arrival occurs, whereas the replacement parts inventory level varies as replacement parts are consumed or as ordered items are delivered.

- (7) The UPDATE INCURRED COST block calculates the average total costs. This cost consists of the inventory/backlogs costs, which depends on the inventory levels, the remanufacturing costs and the replacement parts ordering costs.

The simulation runs until the current time T_{now} reaches the simulation horizon T_{sim} , which is the time needed to reach the steady state.

5. Numerical examples

For the following example, simulation runs are carried out and steady state cost data are collected for specific combinations of (z_1, z_2, s, Q) . Assuming that an optimal solution of the stochastic problem described in section 3 exists and taking into account the convexity property of the cost function, three levels were required for each independent variable to obtain a convex estimated cost function. For these reasons, a 3^4 experimental design and a second-order response surface model were proposed. We plan to demonstrate that this approach is efficient and robust by studying the total cost when using the proposed control policy under different conditions. First exponential supply lead times are considered and a sensitivity analysis of cost parameters is detailed. Then the effects of the distribution, mean and variability of the lead time on the optimal control policy and on the total cost are broached. Finally the control policy is compared to the combined hedging point and (s, Q) policy.

5.1 Basic case

In an illustrative example the following numerical values are considered:

- (1) planned demand rate: a constant demand inter-arrival $1/d$ with $d = 20$ units per month;
- (2) unplanned demand times:
 - the time between unplanned demand arrivals is exponentially distributed with the rate parameter λ_{12}^{-1} , with $\lambda_{12} = 4$ per month;
 - the processing time of unplanned demand is exponentially distributed with the rate parameter λ_{21}^{-1} , with $\lambda_{21} = 10$ per month;
- (3) remanufacturing rates (units per month) and costs (K \$ per unit) associated with the predefined execution modes:
 - $u_0 = d = 20; u_1 = 25; u_2 = 40;$
 - $c_{u0} = 20; c_{u1} = 40; c_{u2} = 100;$
- (4) inventories surplus and backlog costs (K \$ per unit per month): $c_2^+ = 10; c_2^- = 100; c_1^+ = 4;$
- (5) replacement parts ordering costs: $K = 200$ K \$ per order;
- (6) the order lead time is a random variable. In the next sections we will study different time distributions, whereby μ is the mean and σ the standard deviation. For the basic case, an exponential lead time distribution with the rate parameter $\lambda_l = 0.25$ per month ($\mu_l = \sigma_l = 4$) is presented.

In each case, five replications were conducted for each combination of factors (i.e., $3^4 \times 5$ simulation runs). To ensure that the total cost reaches a steady state, the duration of simulation T_{sim} was set at a value of 1,000,000 months for each replication.

The statistical analysis of the simulation data consists of a multifactor analysis of variance (ANOVA). This is accomplished using a statistical software application (STATISTICA) to determine the effects of the four independent variables (z_1, z_2, s, Q) on the dependent variable (incurred cost J). We present in the table 1 the ANOVA table of the total cost for the basic case. The effects of the main factors, their interactions and

Table 1. ANOVA table for the total cost, case I.

	<i>sum of squares</i>	<i>d.f.</i>	<i>mean square</i>	<i>F-ratio</i>	<i>P-value</i>
A : z_1	16439.754	1	16439.754	43.907	0.0000
B : z_2	60488.200	1	60488.200	161.55	0.0000
C : s	319252.78	1	319252.78	852.65	0.0000
D : Q	1181935.2	1	1181935.2	3156.7	0.0000
AA	4635.2637	1	4635.2637	12.380	0.0000
BB	30037.292	1	30037.292	80.223	0.0005
CC	50345.562	1	50345.562	134.46	0.0000
DD	394052.67	1	394052.67	1052.4	0.0000
AB	13389.009	1	13389.009	35.759	0.0000
AC	806.41102	1	806.41102	2.1537	0.1430
AD	3361.4178	1	3361.4178	8.9776	0.0029
BC	108491.61	1	108491.61	289.76	0.0000
BD	133304.96	1	133304.96	356.03	0.0000
CD	694491.02	1	694491.02	1854.8	0.0000
blocks	422.93467	4	105.73367	0.2824	0.8893
Total error	144527.01	386	374.42230		
Total (corr.)	3155981.1	404			R^2 (adj.) = 95.21 %

their quadratic effects on the dependent variable were observed. The factors, the quadratics effects and the interactions were considered significant at $p < 0.05$. An R-squared adjusted value of 0.9521 for exponentially distributed lead times, as shown in the ANOVA table, implies that 95.21% of the total variability is explained by the model (Montgomery 2001). The residual versus predicted value plots and normal probability plots were used to verify the homogeneity of the variances and the residual normality, respectively. It can be concluded that the model composed of the main factors, their quadratic effects and their interactions (except $z_{1,s}$) fit the basic case data.

We assume that there exists a function ψ of (z_1, z_2, s, Q) that provides values of the average cost corresponding to any combination of input factors: $Cost = \psi(z_1, z_2, Q, s)$. The function $\psi(\cdot)$ is called the response surface function. The non significant effect ($p > 0.05$), the third-order interactions and all other effects were ignored or added to the error ε . The estimated second-order model in the basic case with standardized factors is given by:

$$\begin{aligned}
Cost \text{ (case I)} = & 1564.99 - 7.803 \cdot z_1 - 14.97 \cdot z_2 - 66.16 \cdot Q - 34.39 \cdot s \\
& + 7.177 \cdot z_1^2 + 18.27 \cdot z_2^2 + 66.17 \cdot Q^2 + 23.65 \cdot s^2 \\
& - 8.625 \cdot z_1 \cdot z_2 + 4.321 \cdot z_1 \cdot Q + 27.21 \cdot z_2 \cdot Q + 24.55 \cdot z_2 \cdot s + 62.11 \cdot Q \cdot s + \varepsilon
\end{aligned} \tag{6}$$

The projection of the corresponding cost response surfaces onto two-dimensional planes are presented in figure 4. The minimum of the total cost function, $J^* = 1546.65$ \$, is located at $z_1^* = 48.33$, $z_2^* = 10.74$, $s^* = 32.19$, $Q^* = 84.61$. As each response surface displays the value of the total cost in function of two of the input factors (the others being fixed at their optimal values), six plots are required for four factors. These values are the sub-optimal control policy parameters defined in the previous sections and which should be applied to the remanufacturing and supply processes.

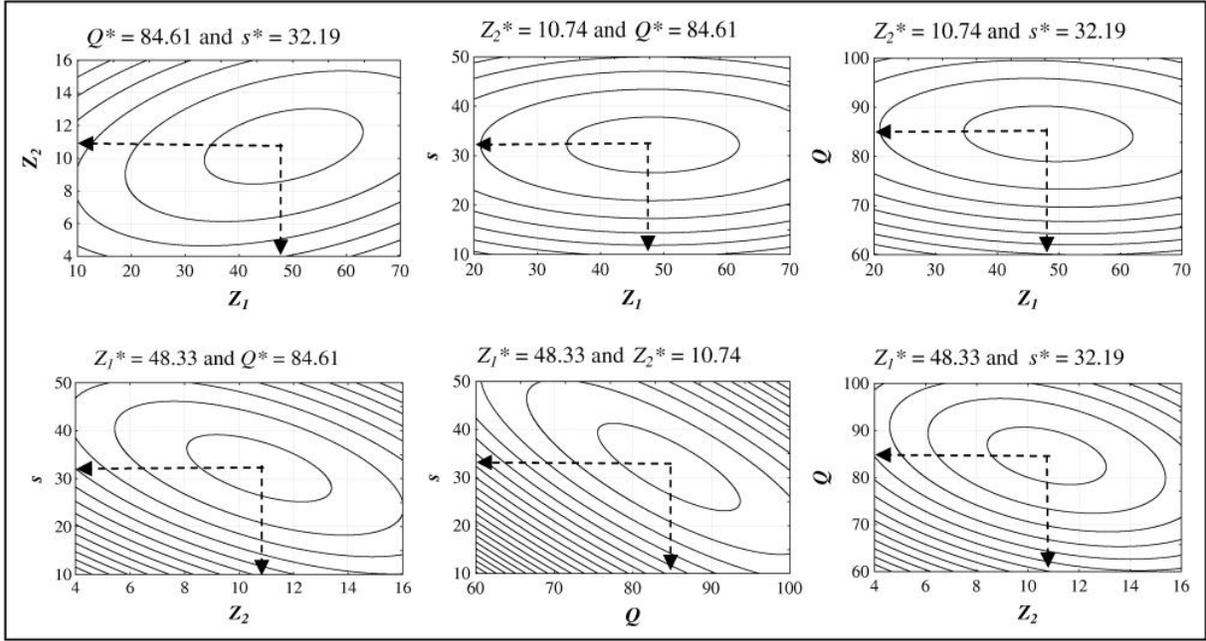


Figure 4. Cost response surfaces for an exponential lead time distribution and $(z_1^*=48.33, z_2^*=10.74, s^*=32.19, Q^*=84.61)$.

5.2 Sensitivity analysis of cost parameters

Another set of experiments is considered to measure the sensitivity of the near-optimal joint remanufacturing and supply control policy with respect to the cost parameters. Table 2 highlights the consistency between the variation of each cost parameters (i.e., c_2^+ , c_2^- , c_{u0} , c_{u1} , c_{u2} for the hybrid repair and remanufacturing system and c_1^+ and K for the supply), tested through eleven cost configurations, and the optimal parameters (z_1^*, z_2^*, s^*, Q^*) for the exponentially distributed lead time case. Results obtained, and discussed below are coherent and confirm our expectations. J^* denotes the incurred cost for the optimal values (z_1^*, z_2^*, s^*, Q^*) .

Before analysing the variation of each cost parameter, it is noted that the order point and the order quantity evolve in opposite directions for every variation of cost parameters. Indeed, the optimal supply policy is a compromise between:

- the need for low holding costs, especially when the serviceable inventory is in the region $x_2(t) > z_2$, where the replacement parts stock level remains constant and triggers holding costs,
- the need for maintaining a critical availability of replacement parts for the remanufacturing stage to prevent the serviceable equipment backlogs.

For this reason, if Q decreases (resp. increases), then s increases (resp. decreases) to ensure a proper availability of replacement parts (resp. to prevent holding costs from increasing). Hajji *et al.* (2008b) reported the same opposite directions in a sensitivity analysis for the joint production and supply control of FMS.

- *Variation of the serviceable inventory cost c_2^+ :*

When the serviceable inventory cost increases (resp. decreases), the hedging points levels decrease (resp. increase), which is intuitively predictable. At the same time, the remanufacturing stage is more subject to backlogs (resp. less), thus the replacement mode, which is the fastest mode, must be more reliable. Consequently s^* increases (resp. decreases) and Q^* decreases (resp. increases).

Table 2. Sensitivity analysis for different costs with exponential lead time distribution ($\mu = \sigma = 4$).

<i>No.</i>	c_2^+	c_2^-	c_{u0}	c_{u1}	c_{u2}	c_1^+	K	z_1^*	z_2^*	Q^*	s^*	J^*
1	10	100	20	40	100	4	200	48.33	10.74	84.61	32.19	1546.65
2	8	100	20	40	100	4	200	53.63	12.94	84.84	29.04	1522.25
3	12	100	20	40	100	4	200	42.67	8.46	84.31	35.70	1567.30
4	10	80	20	40	100	4	200	45.47	9.77	87.08	27.27	1530.84
5	10	150	20	40	100	4	200	52.28	12.09	81.19	39.07	1572.71
6	10	100	20	40	80	4	200	45.42	11.05	84.91	31.57	1434.69
7	10	100	20	40	120	4	200	48.41	10.47	84.66	32.47	1658.45
8	10	100	20	40	100	3	200	44.17	8.70	82.02	41.77	1489.71
9	10	100	20	40	100	5	200	51.89	12.54	86.64	23.96	1597.10
10	10	100	20	40	100	4	100	48.56	10.73	83.47	33.11	1540.13
11	10	100	20	40	100	4	300	48.13	10.75	85.66	31.33	1553.08

- *Variation of the backlog cost c_2^- :*

When the backlog cost increases (resp. decreases), the hedging levels increase (resp. decrease), the s^* increases (resp. decreases) and the Q^* decreases (resp. increases). Indeed one has to keep higher the stock security levels (z_1^* , z_2^* , s^*) to avoid backlogs. The rise of s^* is balanced by the fall of Q^* , because of the holding costs of the replacement parts inventory.

- *Variation of the replacement cost c_{u2} :*

When the replacement cost increases (resp. decreases), the replacement strategy is less profitable (resp. more profitable) and the control policy gives special weight to the accelerated repair mode (resp. replacement mode). Thereby the accelerated repair region, which is bounded by z_1 and z_2 , enlarges and thus z_1^* increases and z_2^* decreases. Concerning the supply parameters, the amount of needed replacement parts decreases (Q^* is then lower), and the s^* increases to keep an appropriate availability of the replacement parts inventory.

- *Variation of the replacement parts inventory cost c_1^+ :*

When the replacement parts inventory cost increases (resp. decreases), the system reacts by lowering the average replacement inventory level (resp. increases). As a result, the s^* is lower (resp. higher) and a higher Q^* is required (resp. lower). A lower average replacement inventory level also means a lower availability, thus more uncertainty for the remanufacturing stage, which is counterbalanced by higher hedging points.

- *Variation of the order cost K :*

When the order cost increases (resp. decreases), Q^* increases (resp. decreases) and s^* decreases (resp. increases). Indeed a higher order cost incites fewer frequent orders but an increasing number of new replacement parts per order to decrease the acquisition cost per equipment unit. This effect must be balanced by a lower order point in order to maintain a proper average replacement parts inventory level. The influence on the hedging point levels is weak.

5.3 The effects of lead time on the control policy

5.3.1 General lead time mean results

The influence of lead time distributions on the optimal parameters of the control policy is examined using gamma, normal, log-normal time distributions instead of exponential time distribution. The former distributions are often encountered in literature to represent stochastic lead times (Song and Yao 2002, Bagchi 1987, Van der Laan *et al.* 1999). For the normal distribution, a modified normal distribution that only provides positive values of lead times is used. In order to compare these different lead time distributions, the same mean and standard deviation are used in all distributions ($\mu = \sigma = 4$) and simulation experiments are conducted with the cost parameters of the basic case.

The convexity property holds and then there exists a set of factors (z_1, z_2, s, Q) that minimizes the total cost for the tested lead time distributions. In all cases, a multifactor analysis of variance (ANOVA) was conducted and lead to R-squared adjusted values close to 0.95. The residual analysis completed these studies and confirmed the adequacy of the second order model. As shown in table 3, the optimal values (z_1^*, z_2^*, s^*, Q^*) and the total cost obtained with different lead time distributions are close to the values obtained numerically with the exponential distribution.

Table 3. Estimated best values for different lead time distributions, with ($\mu = \sigma = 4$).

<i>lead time distributions</i>	z_1^*	z_2^*	Q^*	s^*	J^* (\$)	R^2 adj. (%)
Exponential	48.33	10.74	84.61	32.19	1546.65	95.21
Gamma	46.59	10.64	85.51	31.44	1547.37	95.11
Normal	51.18	10.68	81.64	34.98	1530.15	94.97
Lognormal	50.83	10.75	90.67	26.40	1558.48	95.36

5.3.2 The effects of the lead time mean

Several researchers have investigated the effects of lead time mean and variability on the cost performance. Generally, a higher average lead time as well as higher lead time variability of a supplier would cause a higher level of inventory, and thus higher holding and total cost. A large part of the literature is focused on quantifying the effects of lead time on the optimal inventory costs and policy decision variables in stochastic inventory models (among others, Mohebbi and Posner 1998, Song 1994, He *et al.* 2005), considering the lead time as an exogenous variable. Note that variance reduction is also a common theme in several theories, including total quality management (TQM) and just-in-time (JIT). However, few papers have considered the effects of lead time in a remanufacturing environment. Van der Laan, Salomon and Dekker (1999), under PUSH and PULL strategies and Tang and Grubbström (2005), for a cycle ordering policy, studied the impact of stochastic remanufacturing and manufacturing lead times in a hybrid production/remanufacturing system.

For the hybrid repair and remanufacturing presented in this paper, a set of experiments was conducted in order to study the impact of lead time mean and variability on the proposed control policy. Under the gamma distribution to model the lead time, we used the same approach and experimental design to test the control policy with respectively different lead time means and variability factors (defined as the ratio σ/μ).

The effects of the lead time mean on the optimal values of the control parameters and on the total cost are presented in figure 5. As the lead time mean increases, the reorder point s^* and the average replacement parts

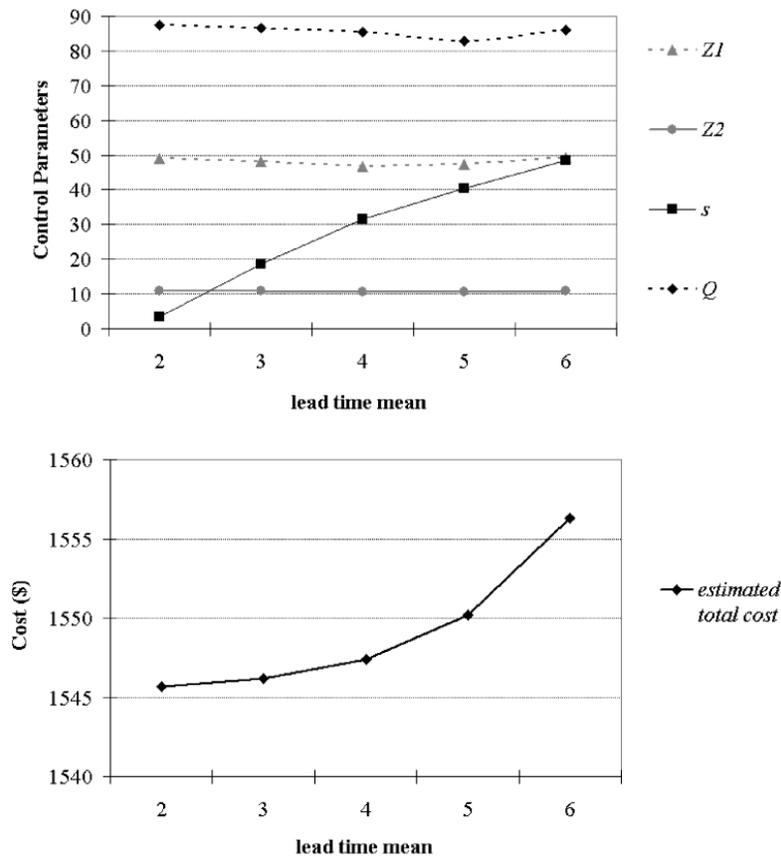


Figure 5. Effect of the lead time mean on the control policy and on the estimated cost.

inventory level increase to avoid replacement part starvation and thus to avoid serviceable equipments stock outs, whereas the other control parameters (z_1^* , z_2^* , Q^*) remain almost constant. Our simulation results show that the stationary replacement parts availability associated with the optimal control parameters is close to 90% in each case, which means that the impact of a higher lead time mean is absorbed by the supply stage and does not affect the remanufacturing stage. Furthermore, a higher lead time mean leads to a slight increase of the total cost, as the replacement parts holding costs increase.

5.3.3 The effects of the lead time variability

Figure 6 shows that the total cost and the optimal values of the supply parameters increase significantly as the lead time variability factor increases. Indeed, the more scattered the lead time is, the more uncertain is the supply. Early and late deliveries introduce waste in the form of excess cost into the remanufacturing and supply system. Early deliveries contribute to excess inventory holding costs, particularly holding costs associated to the replacement parts inventory when the replacement mode is not used ($x_2(t) > z_2$), whereas late deliveries contribute to replacement parts being out of stock ($x_1(t) = 0$), and thus to serviceable equipment backlogs ($x_2(t) < 0$). In other words, the control policy parameters increase tends to protect the system against supply uncertainty, triggering higher total costs. In addition, we observe that the reorder point s^* and the order quantity Q^* serve as adjustment variables for low and high variability factors, respectively.

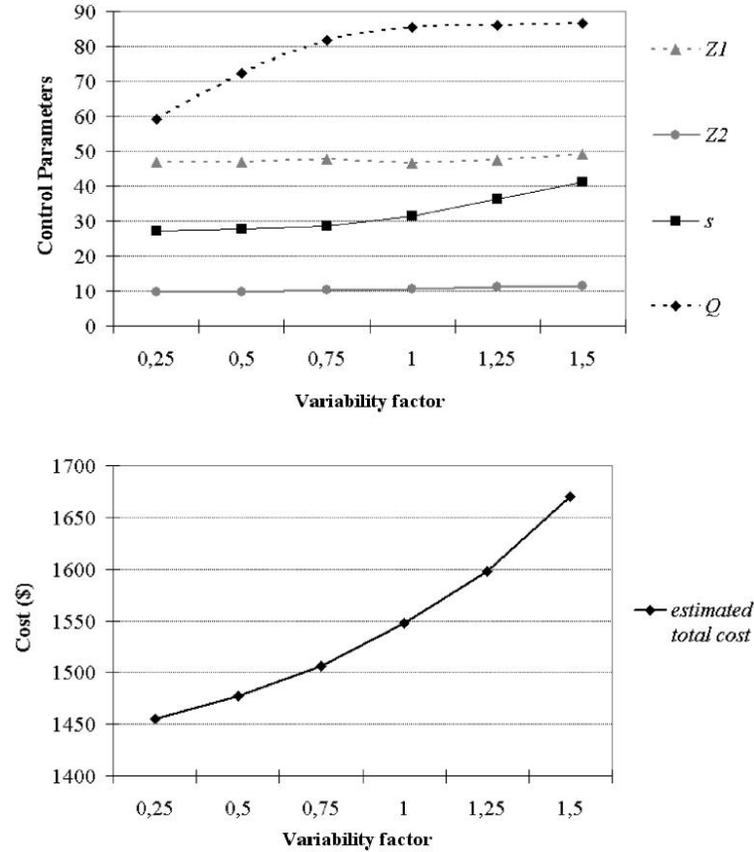


Figure 6. Effect of the lead time variability on the control policy and on the estimated cost.

5.4 Comparison with the joint HPP and (s, Q) policy

Results obtained using the joint multi-hedging point and (s, Q) policies, presented above, are compared to results of the joint single hedging point and (s, Q) policies, inspired from Hajji *et al.* (2008b). This joint policy is fully described by three parameters, namely the threshold Z , the order point s and the order quantity Q . By fitting a second order polynomial model to link the incurred total cost and the parameters of the control policy, we obtain for the basic case:

$$\begin{aligned}
 \text{Cost (case I)} = & 2101.81 + 42.68 \cdot Z + 21.09 \cdot Q + 25.16 \cdot s \\
 & + 43.74 \cdot Z^2 + 16.70 \cdot Q^2 + 33.70 \cdot s^2 \\
 & + 24.72 \cdot Z \cdot Q + 50.80 \cdot Z \cdot s + 36.59 \cdot Q \cdot s + \varepsilon
 \end{aligned} \tag{7}$$

We present in table 4 the sensitivity analysis conducted with joint HPP and (s, Q) policies. These results were obtained under the same conditions (simulation, experimental design, and response surface methodology) as those in the joint MHPP and (s, Q) policies. It is interesting to note that for each variation of cost parameter, the variation of the optimal parameters and of the incurred cost J_{HPP}^* follow the same direction as for the multi-hedging point case previously presented, but has greater amplitude.

Table 4. Sensitivity analysis for different cost for a joint HPP and (s, Q) policy with exponential lead time distribution ($\mu = \sigma = 4$).

<i>No.</i>	c_2^+	c_2^-	c_{u0}	c_{u2}	c_1^+	K	Z^*	Q^*	S^*	J_{HPP}^*
1	10	100	20	100	4	200	12.19	136.98	101.28	2088.50
2	8	100	20	100	4	200	20.36	141.00	87.85	2061.53
3	12	100	20	100	4	200	3.92	132.58	115.82	2100.86
4	10	80	20	100	4	200	8.62	128.33	97.47	2048.40
5	10	150	20	100	4	200	17.53	146.88	106.60	2149.45
6	10	100	20	80	4	200	12.24	137.00	101.20	1859.85
7	10	100	20	120	4	200	12.15	136.95	101.36	2317.14
8	10	100	20	100	3	200	5.43	140.91	122.93	1954.21
9	10	100	20	100	5	200	18.46	129.75	82.73	2200.40
10	10	100	20	100	4	100	12.05	131.15	104.21	2079.95
11	10	100	20	100	4	300	12.33	142.22	98.64	2096.71

The reorder point and quantity optimal values are considerably larger than those obtained with joint MHPP and (s, Q) policies. It is evident that since the MHPP is composed of three modes compared to the HPP composed only of two modes, the replacement mode would be more frequently executed with the latter policy and would require a more reliable replacement parts supply. For the same reason, the control policy parameters are more sensitive to a cost parameter variation with joint HPP and (s, Q) policies. Comparing these optimal cost values to those obtained by the joint MHPP and (s, Q) policies, denoted by J^* in the table 2 (previous sections), it is noted that in all cases J^* is lower by at least 20% than J_{HPP}^* . As mentioned previously in section 3.2, the joint multi-hedging point and (s, Q) policies are better in term of cost performance than the joint single hedging point and (s, Q) policies and can be used to better approximate the optimal control policy of the system.

6. Conclusion

In this paper, we have studied the production and supply problem for hybrid repair and remanufacturing systems and proposed a solution in the case of one product type. The mathematical formulation of this problem is difficult to tackle, due to the stochastic aspect of the supply, the variability of the remanufacturing processes and of the conditions of the items to be returned. In order to benefit from the flexibility of the remanufacturing execution, from repair to replacement, a near-optimal joint remanufacturing and supply control policy composed of a multi hedging point policy (MHPP) for the remanufacturing stage and of an (s, Q) policy for the replacement parts supply have been proposed. To optimize the control parameters, an experimental approach based on design of experiment, simulation modeling and response surface methodology have been used. With this approach, the influence of the lead time distributions and the effects of the lead time duration and variability on the control policy have been investigated. It has also been shown that the developed control policy is better than a combined HPP and (s, Q) policy in term of cost performances.

In conclusion, the stochastic and complex nature of remanufacturing industry problems forces managers to consider new approaches, different from those already used in traditional production management. This paper presents an easy to implement and simple structure of stock level parameters that bound the use of

predetermined repair or replacement modes, allied with a classical supply policy. Repair and replacement modes should be consciously selected by managers with an overall perspective, whereas this decision is often left up to technicians performing the work.

For hybrid repair and remanufacturing control problems where the classical control theory limits are reached, we think that adopting the experimental designs and simulation techniques presented in this paper provides satisfactory approximate solutions of the optimal control problems. The situations of multi-parts products, non-repairable returned items or multiple suppliers that differ in terms of cost, lead time and quality, are possible extensions to be investigated in future research.

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