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Zero Inventory/Production Control Policy for Manufacturing Systems Subject to Quality Deterioration

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Abstract

In this paper, we study the production and inventory control of a manufacturing system composed of an imperfect process. After a random period operating in the 'in-control' state, the manufacturing system switches to the 'out-of-control' state and starts producing non-conforming items. The manufacturing system 'in-control' horizons have a general deterioration distribution. Restoration actions with random durations are thus planned to restore the manufacturing system to the 'in-control' state after a specific logistic period. Restoration durations have a general distribution.

Despite of numerous works published about the production and inventory control, and based on the just in time concept, this paper deals with the dynamic control of manufacturing system production rates under a zero inventory policy. In this context, two production control policies were proposed: the first one entails to stop immediately the manufacturing activity when the system transits to the 'out-of-control' state, the second policy advocates continue producing until the beginning of the restoration activity.

The objective of the paper is to determine the best control policy among the two proposed ones based on the assessment of the overall incurred cost including restoration, non-conforming items and shortage costs. Analytical models are proposed and the expressions of the overall incurred costs are derived for each control policy.

Keywords

Manufacturing systems, production and inventory control, zero inventory policy, product quality alteration.

1. Introduction

In the last few decades, production planning and control has been an active area of research to evaluate the optimality of numerous strategies. The objective was to reduce the inventory levels while ensuring the customer demand.

Akella and Kumar (1986) have introduced the Hedging Point Policy, which entails that the buffer stock is build-up with an excess production capacity, and then maintained at its maximum level in order to palliate for interruptions due to breakdowns.

Other researches have focused on integration of systematic maintenance policies in conjunction with control production policy. The objective is to determine the optimal parameters of the control law and the systematic maintenance policy minimizing the costs of corrective and preventive actions and those associated with inventory management (Cheung and Hausman 1997, Salameh and Jaber 2000, Dohi et al. 2001, Gharbi et al. 2007).

In the production planning and inventory control literature, manufacturing systems have been studied under various conditions. Most of the proposed models are based on the assumptions that the production process always produces items with perfect quality. However, in practice, the quality of the product may not be acceptable because the production process could deteriorate with time. Many authors have proposed a novel strategies integrating quality deterioration in the production and inventory control policies (Ben-Daya 2002, Radhoui et al. 2009, Dhouib et al. 2011). In order to reduce the impact of producing non-conforming items the majority of these policies consider

the construction of a security stock. However, all these researches assume that the inventory level is strictly positive and do not consider the zero inventory policy advocated by the just in time philosophy.
In this paper, and based on the zero inventory strategy, two production control policies were proposed: the first one entails to stop immediately the manufacturing activity when the system transits to the 'out-of-control' state, and the second policy advocates continue producing until the beginning of the restoration activity. For the proposed policies, we develop mathematical formulations modeling the overall incurred costs per time unit over an infinite horizon. The overall incurred cost includes shortage, defective items and restoration actions costs.
The next section presents the manufacturing system description, notations and assumptions. Section 3 discusses the dynamic and stochastic behavior of the manufacturing system. Mathematical models evaluating the overall incurred costs per time unit over an infinite horizon are developed in section 4. Section 5 exposes first a numerical example and then a sensitivity analysis to show the effectiveness of the proposed methodology. Finally, section 5 contains a summary of the paper and some concluding remarks.

2. Description of the production system, notations and assumptions model

2.1 Description of the production system

The system under study is a manufacturing system consisting of a single machine dedicated to producing a single product type to satisfy a constant and a continuous demand. This machine can be considered as an aggregation of several machines. The manufacturing system is subject to random perturbations due to the alteration of product quality during the production phase.
At the start, the manufacturing system begins in an 'in-control' state producing conforming items of acceptable quality. After a random operating period (τ), the manufacturing process may shift from the 'in-control' state to the 'out-of-control' state and starts producing a percentage (α) of non-conforming items. The manufacturing system allows instantaneous detection of the 'out-of-control' state. The 'in-control' state period is a random variable with general probability distribution.
A restoration action is started ahead transition to the 'out-of-control' state but after a constant delay, denoted logistic delay period. The logistic delay period is required to prepare all resources necessary for restoration actions. During restoration actions, the production process is aborted. The restoration action delay is a random variable having a general probability distribution.
The manufacturing system operates at a capacity equal to demand d. In order to reduce the impact of product quality deterioration and restoration delays on the demand satisfaction, the manufacturing cell can recover the non-conformingitems by increasing its production rate during the logistic delay period.
After a restoration action, the manufacturing facility transits to the 'as-good-as-new' 'in-control' state and restarts producing conforming items. Once restorations achieved, the production process is resumed immediately. During a restoration action, an inventory shortage occurs. In this situation, quantities required during a shortage period are not delivered after the manufacturing system restoration and are considered as lost demand. The objective is to find first the expected overall cost incurred per time unit over an infinite spam and then deducing the best policy among the proposed ones.

2.2 notations

The following notations will be used:
- d: demand rate.
- U_{max}: production capacity of the manufacturing system defining its maximum production rate.
- $u(t)$: controlled production rate of the manufacturing system at time t.
- $z(t)$: production surplus at time t; if z(t) is positive, it represents inventory, while a negative value represents a shortage.
-τ: continuous time random variable identifying the distribution of the 'in-control' state periods.
- $f(\tau)$ $(F(\tau))$: density (distribution) function associated with the random variable τ.
- α: percentage of non-conforming items produced during the 'out-of-control' period.
- *LDP*: Logistic Delay Period.
- t_r: continuous time random variable identifying the distribution of the restoration action delays to bring the manufacturing system from the 'out-of-control' to the 'in-control' state.
- $h(t_r)$ $(H(t_r))$: density (distribution) function associated with the random variable t_r.
- CT_i : total cost incurred by the production-inventory control policy *i*.
- C_R : cost of restoration action.

- C_S : one-item shortage cost.
- C_{RM} : one-item raw material cost.
- C_{MSO} : manufacturing system operation cost per time unit.
- CF_i : expected total cost incurred during production phase of the policy *i*.
- CR_i: expected total cost incurred during restoration phase of the policy *i*.
- CS_i: expected total cost incurred during shortage phase of the policy *i*.
- LCi: expected value of the production/restoration cycle length for policy i.
- E(.) : expected value of a random variable.
- CNCi : expected cost of non-conforming items manufactured during production phase of the policy i.

2.3 Assumptions

Other assumptions are made as follows:

- The demands which cannot be satisfied are lost.
- The produced items are imperishable with time
- The non-conforming items are not reinserted in the production process.
- All costs related to restoration activities, quality of manufactured products and inventory management are known and constant.
- The "out of control" state is detected instantly.
- The logistic delay period is known and constant.

3. Manufacturing system production control under zero inventory Policy

The majority of control policies proposed in the literature are based on the built up of a safety stock with a strictly positive level (Z_{max}). The mathematical formulations assessing the overall incurred costs of such strategies do not consider the case where Z_{max} equals zero. However, and according to the just-in-time philosophy and add-value productivity concept, inventories are considered costly categories of waste that should be reduced to the minimum if not eliminated (Suzaki 1987, Dear 1988).

In this section, we propose a production control policy based on zero inventory concept. The zero inventory policy implies not to build and not to maintain a security stock during production phases. In effect, and in cases where managers adopt zero stock policy, they consent that it is better to suffer the costs of lost demands than cost due to inventory holding. In this case, two different policies can be developed depending on the decision about immediately stopping or not production when the first non-conforming items appeared.

3.1 Policy 1: description and mathematical formulation

This policy advocates stopping immediately the production if the manufacturing system switches to the "out of control" state. Therefore, this policy does not allow the production of any defective item. The transition from the 'in-control' state to the 'out-of-control' can occur at any instant of the production horizon. Consequently, for a constant demand rate, the related production control policy is given by:

$$u(t) = \begin{cases} d & \text{if the manufacturing system is in the 'in-control' state} \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

Figure 1 shows, under policy 1, the evolution of the inventory level during the production, the logistic, and the restoration phase.

The overall cost is composed of the incurred costs during the production phase and those during the shortage phase when the manufacturing system is shut down (Fig. 1). Therefore, by the renewal theorem, the overall expected cost (CT_1) is given by equation (2), where CF_1 equals 0 since no non-conforming items are produced during the production phase.

$$CT_1 = \frac{CF_1 + CS_1}{LC_1} \quad (2)$$

- Expected cycle length (LC_1)

The expected length of a production/shortage cycle includes the mean lifetime before transiting to the 'out-of-control' state, the logistic delay period required to prepare the resources for the restoration action, and the mean restoration time.

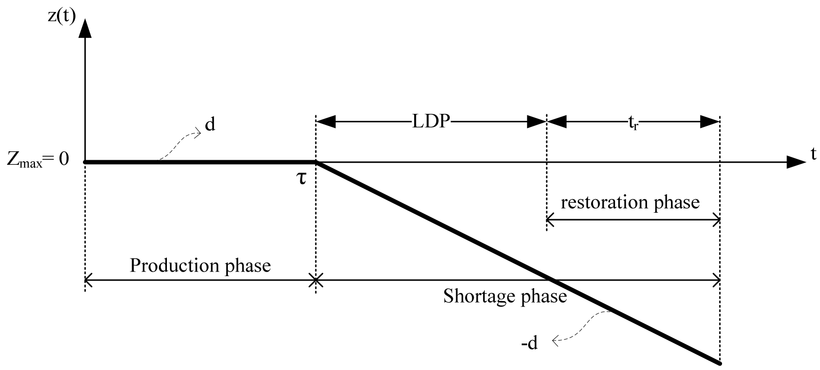

Figure1: Inventory evolution during the production/shortage phase under control policy 1

Hence, the expected production/shortage cycle length is given by equation (3), where *E(τ)* is the mean life time of the manufacturing system 'in control' state (Eq. 4), and $E(t_r)$ is the mean duration of restoration actions (Eq. 5).

$$LC_1 = E(\tau) + LDP + \mu \tag{3}$$

$$E(\tau) = \int_0^\infty \tau \cdot f(\tau) \cdot d\tau \tag{4}$$

$$E(t_r) = \int_0^\infty t_r \cdot h(t_r) \cdot dt_r \tag{5}$$

- Expected cost incurred during shortage phase (CS_1)

The expected cost incurred during restoration phase is given by equation (6).

$$CS_1 = C_R + C_S \cdot d \cdot \left(LDP + E(t_r)\right) \tag{6}$$

- Expected overall incurred cost (CT_1)

Therefore, the expected overall incurred cost under the policy 1 is:

$$CT_1 = \frac{C_R + C_S \cdot d \cdot \left(LDP + E(t_r)\right)}{LC_1} \tag{7}$$

3.2 Policy 2: description and mathematical formulation

Contrary to the policy 1, which stops immediately the manufacturing process when the production system transits to the "out of control" state, policy 2 permits the production during the logistic delay period. Under this policy, the manufacturing system produced a percentage of non-conforming items equal to α. In order to respond to the demand, the manufacturing system fills the quantity of non-conforming items by increasing its production rate. Thus the production rate of the manufacturing cell increases from d to $d \cdot (1+\alpha)$. This policy insures that less shortage items are recorded compared to those recorder under policy 1. The production control under policy 2 is dictated by equation (8).

According to the dynamic and stochastic behavior of the manufacturing system controlled by policy 2, figure 2 shows the evolution of the inventory level during the production, the logistic, and the restoration phases while the time evolving.

$$u(t)=\begin{cases} d & \text{if the manufacturing system is in the 'in-control' state} \\ d(1+\alpha) & \text{if the manufacturing system is in the 'out-of-control' state} \\ 0 & \text{elsewhere} \end{cases} \tag{8}$$

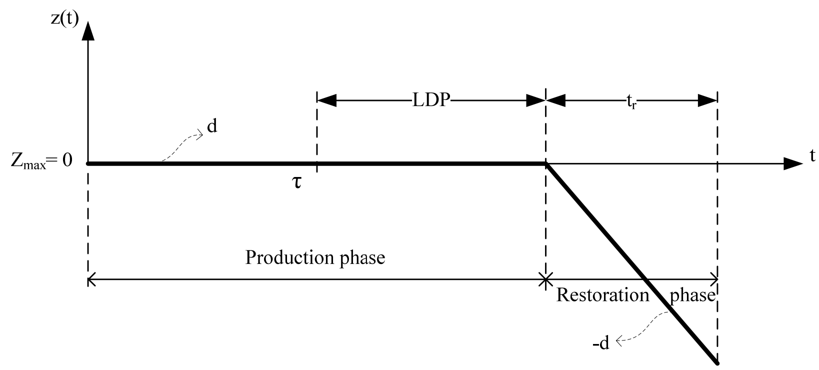

Figure2: Inventory evolution during the production/restoration phase
under control policy 2

The expected overall cost is composed of the incurred costs during the production phase and those during the restoration phase (Eq. 9).

$$CT_2=\frac{CF_2+CR_2}{LC_2} \tag{9}$$

Comparing figures 1 and 2, one can note that the expected cycle lengths (LC_1 and LC_2) under policies 1 and 2 are equals.

- Expected cost incurred during production phase (CF_2)

The expected cost incurred during the production phase is equal to the cost of non-conforming items. The cost of a non-conforming items includes the cost of the product raw material and the system operating cost. As soon as it shifts to the 'out-of-control' state, the manufacturing system starts and continues to produce non-conforming items during the logistic delay period. Thus, the expected cost of nonconforming items produced by the manufacturing system under policy 2 is given by:

$$CF_2=CNC_2=\left(C_{RM}\cdot d\cdot+C_{MSO}\right)\cdot\alpha\cdot LDP \tag{10}$$

- Expected cost incurred during restoration phase (CR_2)

The expected cost incurred during the restoration phase is the sum of a restoration activity cost and those due to non delivered items during the restoration period (Eq. 11).

$$CR_2=C_R+C_S\cdot d\cdot E\left(t_r\right) \tag{11}$$

- Expected overall incurred cost (CT_2)

Therefore, the expected overall incurred cost under the policy 2 is:

$$CT_2=\frac{CNC_2+C_R+C_S\cdot d\cdot E\left(t_r\right)}{LC_2} \tag{12}$$

4. Numerical example and sensitivity analysis

In this section, we first present a base case. Thus, we carried out a sensitivity analysis in order to show the impact of the input parameters on the determination of the best policy.

4.1 Base case

Consider a manufacturing system having an automatic machine, manufacturing a single product type, and responding to a constant and periodic demand of 20,160 units/month. The manufacturing system is subject to a random alteration of the product quality. The statistics show that the 'in-control' state periods follow a Weibull distribution with shape and scale parameters 1.5 and 1, respectively; the mean lifetime in the 'in-control' state of the manufacturing system is equal to 0.903 month.

After a random operating period in the 'in-control' state, the manufacturing process may shift to the 'out-of-control' state and starts producing non-conforming items with a percentage equal to 1% of the actual production rate.

Upon transition to the 'out-of-control' state, a preparation of the required resources for restoration with duration of 0.03 month is carried out.

Restoration actions have random durations following a Gamma distribution with shape and scale parameters 2 and 40, respectively; the mean restoration time to the 'as-good-as new' 'in-control' state is equal to 0.05 month.

The following costs are also chosen considering nevertheless realistic context:

- Shortage cost (*CS*): 300 ($/product unit)
- Restoration cost (*CR*): 10,000 ($/restoration action)
- Raw material cost (*CRM*): 500 ($/product unit)
- Manufacturing system operating cost (*CMSO*): 150,000 ($/month)

Considering the previous input data, the expected overall costs incurred by policies 1 and 2 are respectively equal to 4,202,391.02 $/month and 174,281.16 $/month. Therefore, the best zero inventory/production control policy entails to produce at the demand rate (20,160 units/month) while the manufacturing system is in the 'in-control' state. Once the manufacturing system transits to the 'out-of-control' state, it ceases producing immediately and its production rate falls to zero. According to these settings, the minimal overall expected cost incurred by the optimal policy is 174,281.16 $/month.

4.2 Sensitivity analysis

In order to analyze the superiority of policy 1 to policy 2, expected overall incurred costs equations (2) and (9) were compared. The analysis has shown that policy 1 outperforms policy 2 if and only if the following condition is respected:

$$C_S \prec \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right) \qquad (13)$$

Equation (13) shows that the relative superiorly of a specific policy depends on the product unit shortage cost, the percentage of non conforming items, and the cost of producing a non-conforming unit which depends on the raw material cost, the manufacturing process operating cost, and the customer demand.

Considering the input parameter of the base case presented previously, equation (3) allows the assessment of the limiting value of the unit shortage cost implying no superiority of the considered production control policies. This limiting condition is obtained when C_s equals 5.0149 $/product unit.

Table 1 presents 7 configurations of cost parameters derived from the previous configuration (case 1), by changing them to higher and lower values, one at a time; the case 1 is the manufacturing system configuration considered in the previous numerical example where C_s equals 5.0149 $/product unit. Table 1 highlights the consistency between the variation of each cost parameter and the optimal decision variable and total cost.

5. Conclusions

Because of its great importance for manufacturing companies, several researches have been devoted to the problem of modeling and optimizing the production planning and control of stochastic manufacturing systems. The production control policy is generally based on building-up a security stock to palliate against random perturbations as manufacturing process failures and/or quality product degradation. However, the majority of these control policies and specifically the proposed mathematical models assume that safety stock level equals a strictly positive value. In today's industry environment, a huge effort is deployed in order to reduce to the minimum if not to eliminate inventory which are the principal source of manufacturing waste.

Table 1: Sensitivity analysis for cost parameters

Case Number	C_S	C_{RM}	C_{MCO}	$Eq.\ 14$	CT_1	CT_2	Best Policy
1	5,0149	500	30,000	$\boldsymbol{C_S = \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)}$	73,578.42	73,578.42	1 or 2
2	2	500	30,000	$C_S \prec \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	31,380.09	72,542.19	1
3	10	500	30,000	$C_S \succ \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	143,353.54	75,280.25	2
4	5,0149	200	30,000	$C_S \succ \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	73,578.68	32,612.58	2
5	5,0149	2000	30,000	$C_S \prec \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	73,578.68	278,407.89	1
6	5,0149	500	10,000	$C_S \succ \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	73,578.68	73,442.95	2
7	5,0149	500	80,000	$C_S \prec \alpha \cdot \left(C_{RM} + \frac{C_{MOS}}{d} \right)$	73,578.68	73,917.09	1

In the other, and contrary to what happens in real industrial context, most of the researches in the production planning and control field do not consider explicitly the impact of defective products and the process quality deterioration on the determination of the best production control policy of manufacturing systems.

This paper presents a production control policy of a manufacturing system including an imperfect process subject to a mono-product, constant, and continuous demand. The production control policy is based on the zero inventory strategy which entails to not building a finished product security stock during production phase. Based on the zero inventory strategy, two production control policies were proposed depending on the decision about immediately stopping or not the manufacturing process when the production system switches to the 'out-of-control' state and starts producing non-conforming items.

Analytical models have been proposed for manufacturing systems having general distributions of the residence times in the 'in-control' state and of the restoration periods. The analytical models allow assessing the expected overall costs incurred by each production control policy and which includes shortage, restoration, and non-conforming items costs. The analytical models allow determining the best zero inventory/production control policy to implement based on minimum expected overall cost.

The analysis of the analytical models shows that policy 1, which entails stopping immediately the production when the manufacturing system transits to the 'out-of-control' process, outperforms policy 2, which entails to continue producing during the logistic delay period, if the product unit shortage cost is less than a specific limit depending on the percentage of producing non-conforming units and the cost incurred by producing one non-conforming unit.

The results reported in this paper extend the theory of the production control policy of manufacturing systems including imperfect processes and subject to product quality deterioration. The proposed approach could be extended to take into account not only product quality deterioration but also equipment failures which forces the manufacturing process to be shut down. In this context, preventive maintenance policies could also be considered in order to attenuate the impact of catastrophic failures and the shift rate to the 'out-of-control' state on the overall performance of manufacturing systems.

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