Synthesis of Differentially-Driven Planar Cable Parallel Manipulators

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***Abstract*—In this paper, the idea of using cable differentials in the architecture of planar cable-driven parallel robots is introduced. Cable differentials are a type of mechanisms with several outputs driven by a single input. Using them in cable parallel manipulators can decrease their cost and control complexity. However, due to their kinematic constrains, cable differentials cannot be arbitrarily used in the design of these manipulators. Thus, a synthesis method is proposed to tackle this issue. First, the general requirements and characteristics of differentially-driven planar cable mechanisms are reviewed. Then, the advantages of using these differentials instead of typically actuated cables is shown through a comparison between differentially actuated planar cable robots and fully-actuated ones. The results reveal that with the same number of actuators, using differentials leads to larger workspaces and improved kinetostatic properties. Subsequently, the systematic synthesis of differentially-driven planar cable mechanisms is presented. For this, a method to find the different arrangements of q cables in a differential is proposed. Then, valid arrangements with 2, 3, and 4 cables are investigated. Finally, several differential actuation schemes are considered and all possible differentials with $q = 2$, 3, and 4 cables are found.**

***Index Terms*—Differential mechanisms, kinematic synthesis, workspace, cable robot, parallel robot.**

I. Introduction

CABLE manipulators are a special type of parallel robots where rigid legs are replaced by cables [1]. In other words, cables are used to manipulate the moving platform (MP). This characteristic yields particular properties to these mechanisms, for instance, they inherit not only some advantages of linkage-driven parallel robots, but also gain a few additional characteristics which allow them to be the preferred solution in certain application. Examples of these beneficial properties are: a simple structure, lightness and low inertia of the moving parts, a high dexterity, typically low friction, large workspace (compared to linkage-driven parallel mechanisms), etc. [2]–[6].

On the other hand, they also suffer from some drawbacks amongst which the more common are limits in the cable tensions, poor compactness, possible interferences between cables, and vibrations [3], [4]. Also, as a result of the unilateral nature of the cables which can only produce tension forces, redundancy in the actuation is necessary. This means that to completely constrain the MP of an n-DOF cable robot, $m > n$ cables are required [2], [7], [8]. Several research initiatives have proved that using more cables results in larger workspace and generally better overall performance of the robot [6], [9].

Broadly speaking, cable robots are categorized as either incompletely or fully restrained [4], [10]. In the former, either $m \leq n$ cables are in tension or the MP is suspended from the ceiling by cables and motion relies on gravity (which is also often referred to as cable suspended robot [4], [11]). In the case of $m \leq n$ cables, the control of the system becomes more complex [8]. Yet, in cable suspended robots (where $m > n$) there are usually fewer problems with the controllability and also the volume occupied by the cables. A few robots based on this design are now commercialized and typically used for cargo transport (e.g., the NIST Robocrane [12]) or camera manipulation (e.g., Skycam [13] and Cablecam [14]). Fully restrained cable robot on the other hand can operate in any direction regardless of the direction of the wrench exerted to its MP [4].

In recent years, several aspects of cable robots such as kinematics, workspace, force distribution and cable arrangement were studied and analyzed. For instance, Shiang et al. [15] designed an incompletely restrained 3-DOF robot, derived its motion equations while the flexibility of the cables was considered, and optimized the force distribution among cables. Jiang and Kumar [16]–[18] investigated the kinematics of cable suspended mechanisms driven by a set of aerial robots for cargo transportation usage. They solved the direct and inverse kinematics of suspended cable systems with $m = 2, \ldots, 6$ cables and developed a method to analyze the stability of the mechanism in all its static equilibrium poses. Yang et al. [6] presented a 7-DOF modular cable-driven humanoid robotic arm and mainly focused on the workspace of the 3-DOF modules constituting this arm. The same authors proposed in [4] the Tension Factor (TF) index to evaluate the tension of cables and assess force-closure in a cable-driven robot in order to obtain its workspace. Rosati et al. [19] proposed a systemic methodology to optimally design a new class of cable-driven mechanisms. In this approach, they used posture dependent local performance indices instead of a global index to maximize the performance of the mechanism.

Gouttefarde and Gosselin [20] presented many theorems to characterize the wrench-closure workspace (WCW) of planar cable-driven robots. Then, these theorems were used to find the WCW of the robot within its reachable workspace. Next, Gouttefarde et al. [21] presented an interval analysis based method to investigate the wrench-feasible workspace (WFW) of a n-DOF cable robot. This method evaluates whether a given Q-dimensional box is located inside the WFW or not.

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Additionally, Bouchard and Gosselin [1] proposed a geometrical method to analyze the capability of a cable-driven robot with two to six-DOFs to generate a set of wrenches on its MP.

In all cases, the number of necessary actuators is always strictly greater than the number of DOF which increases the cost and complexity of the control equipment. This issue is even more critical since generally, the more cables are used in the structure of the robot, the better the performance in terms of the size of the WCW and WFW [6], [9]. Therefore, keeping the number of actuators at minimum while increasing the number of cables (and thus the performance of the mechanism) appears to be a dilemma.

In this paper, it is proposed to solve this conundrum by using differentials to actuate several cables of a planar cable-driven mechanism by a single actuator. To this aim, these mechanisms are first introduced and their properties are presented. Then, some examples showing the performance of planar mechanisms actuated with differentials are compared with fully-actuated cable-driven architectures. Finally, in the second part of this paper, the general methodology for the synthesis of differential cable mechanisms is presented and illustrated with examples for q = 2, 3, and 4 cables in each differential.

II. DIFFERENTIAL CABLE-DRIVEN MANIPULATOR

Using differentials in machines and mechanisms is a popular method to distribute an actuation source to several degrees of freedom [23]. A differential is a 2-DOF mechanism producing two outputs from a single input or vice-versa [22]. To drive an even greater number of outputs from a single input, these mechanisms can be connected either in serial or parallel combinations [23]. Examples of commonly found differentials are seesaw mechanisms, automotive bevel gear boxes, planetary gear differentials, and mobile pulley-cable arrangements [24].

The idea of using differentials in cable robot is to replace an actuated cable with two, three, or more cables, each connected by a mechanism but driven by the same actuator. The distribution of a driving force/torque to several degrees of freedom in a robotic system has previously been studied extensively including by the authors [23], [25]–[30] where several types of either tendon or linkage-based differentials have been used to drive phalanges of robotic fingers. In these works, the shape adaptation property of these fingers was obtained mechanically using differentials such as cables and pulleys, bevel gears, etc. and it was shown in [30] that using differentials and the spatial distribution of the generated forces were a key element in their design. In this paper, it is shown that the same principles apply when designing planar cable robots and that using differentials is again beneficial.

To use differential systems in a cable robot instead of a robotic finger, two questions should be answered: 1- which conditions should be satisfied to fully constrain the MP in this new design; 2- how can a single actuated cable be usefully replaced by a differential cable-driven mechanism.

The first question is all about the arrangement of the cables around the MP. These cables should be able to lock the pose of the MP or equivalently be able to provide full translational and rotational motions. For this, the cables should surround the MP in all directions of the task space to be able to produce any arbitrary force or torque. In other words, they should be able to satisfy the static equilibrium condition of the MP regardless of the direction of any external wrench, i.e.:

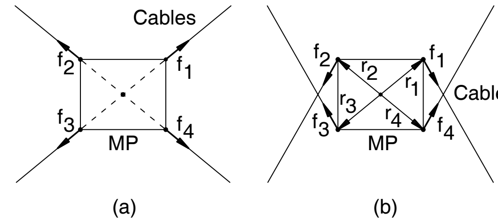

Fig. 1. MP with (a) only translational motion; (b) both rotational and translational motion.

$$\sum \mathbf{f}_i + \mathbf{f}_{ext} = 0 \quad \text{for} \quad i = 1, \ \ldots, \ m \tag{1}$$

$$\sum \mathbf{r}_i \times \mathbf{f}_i + \boldsymbol{\tau}_{ext} = 0 \quad \text{for} \quad i = 1, \ \ldots, \ m \tag{2}$$

where $\mathbf{f}_i$ and $\mathbf{r}_i$ are respectively the tension force vector applied on the MP by the ith cable and the position vector of the point of application of this force with respect to an arbitrary point on the MP. Additionally, $\mathbf{f}_{ext}$ and $\boldsymbol{\tau}_{ext}$ are respectively the external force and torque exerted to the MP. From these equations, it appears that certain geometric configuration should be avoided, for instance, if the directions of all cables intersect in one point they obviously cannot produce any torque and thus, they cannot constrain the rotational motion of the MP. Mathematically, this is because Eq. (2) is not satisfied in that case unless $\boldsymbol{\tau}_{ext} = \mathbf{0}$. This situation is shown for a planar cable robot with 4 cables in Fig. 1(a). On the other hand, in Fig. 1(b), the cable directions do not intersect and consequently, the cables can theoretically exert any torque on the MP.

The second question raised before is all about how to use a differential instead of a single actuated cable while preserving the force-closure condition. As mentioned before, m independent cables can constrain up to (m–1)-DOF. If some of these cables are connected to the same actuator through a mechanism they cannot change their lengths independently and thus, self-motion may happen, i.e., cables (and consequently the MP) might have non-zero velocities while the associated actuator is locked.

To account for this problem, instead of considering the force in each individual cable driven by a differential, the resultant force of all cables connected by the latter should be taken into account. For example, let us consider the two mechanisms shown in Fig. 2. In case (a) each cable is connected to a separate actuator generating an independent force and two independent forces in two different directions can completely constrain the single-point MP (a spring is assumed to pull the MP away from the base platform (BP)). In case (b) these two cables are replaced by a cable and pulley mechanism actuated by a single actuator. In this case, since there is just one actuator, the attachment point of the cables (i.e., the axis of the pulley if its radius is negligible) can move along an

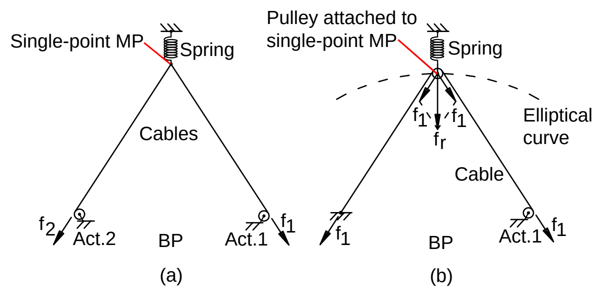

Fig. 2. Single-point MP actuated with (a) two independent cables; (b) a cable and pulley differential.

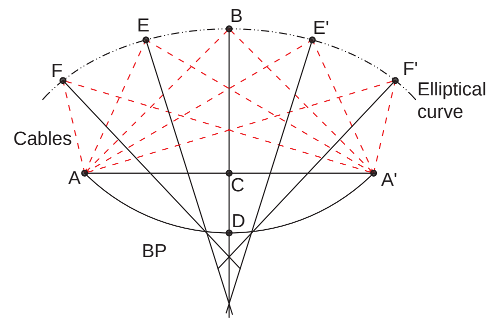

Fig. 3. Direction of the bisector of a cable differential when actuator is locked and the attachment point on MP (e.g. the pulley) moves on resultant elliptical curve.

elliptical trajectory while the actuator is locked. This happens because the resultant force of the two branches of cables always lies on their bisector (in ideal frictionless condition) and the pulley can move in the direction perpendicular to the direction of this force. Thus, considering the resultant forces of the differentials, the MP is fully constrained if and only if the twist vector reciprocal to the space of all constraint wrench vectors is zero-dimensional.

III. Comparison between Differential and Fully-Actuated Cable-driven Mechanisms

The main question in using differentials is about evaluating their advantages in comparison with the fully-actuated architectures. As mentioned before, using this novel solution, the number of actuators can be kept at minimum ($n+1$ for a n-DOF robot) while the number of cables connecting the MP to the BP is increased. Hence, a single thick cable can be replaced by several lighter cables which is beneficial with high payload cable-driven architectures where the masses of the cables are not negligible [11]. Additionally, without adding other actuators, for a given size of cables the payload of the robot can be increased. This also leads to a better distribution of the mass of the MP on the BP.

The uniqueness of using differentials on the kinematic of the mechanism yields from the direction of the resultant force of all its cables. To illustrate this, the free movement of the pulley of the mechanism introduced in Fig. 2(b) while its actuator is locked, is shown again in Fig. 3. As is apparent in this figure, for a fixed cable length, when the pulley is on the line of equidistant points from the attachment points A and A' (i.e. in point B), the bisector of the two cable directions passes through point C. In this case, the differential system is instantaneously equivalent to a single cable connecting the MP to the BP at point C. However, when the pulley moves away from point B, the intersection point between the bisector and the line AA' travels along this latter line segment. This means that the illustrated differential behaves as a single actuated cable in which the attachment point on the BP is variable. This property can be beneficial for the WCW and WFW of a cable-driven mechanism as will be shown. For this, first, the methods used to find these workspaces are briefly recalled.

The WCW of a cable-driven robot is the set of poses where the forces of the cables as well as their resultant torques (if they exist) can surround the center of the MP in all directions in the considered Q-dimensional space (the force-closure condition [4]). To find the WCW, a method similar to the one introduced in [6] is used in this paper. Namely, the unit vectors of the forces $\mathbf{e}_i$ which are in the direction of the cables in fully-actuated cable mechanisms or in the direction of the resultant forces of the differentials, are used to define the positions of a set of points constituting a convex polyhedron. Next, if the origin of these vectors is located inside this polyhedron then the mechanism can produce forces in all directions. Similarly, this situation can be investigated for the torques exerted by the MP. However, since a single-point MP is used here as an example, this second step does not apply.

To obtain the WFW, a geometrical approach similar to the one proposed in [1] is used. For this, it is assumed that the tensions in all the cables should be between t_{min} and t_{max}. Then, the tension vectors $\Delta\mathbf{t}_i = (t_{max} - t_{min})\mathbf{e}_i$ can be evaluated. Next, these vectors are used as a basis to construct a zonotope (a particular class of convex polytopes) which in turn is used to calculate the WFW of the robot. For this, the vectors $\Delta\mathbf{t}_i$ are taken as a set of line segments and by calculating the Minkowski sum [1] of these lines, the initial zonotope is obtained. Afterwards, the resultant force of all the vectors with minimum values (i.e., t_{min}), is added to the Minkowski sum to modify the positions of the vertices of the zonotope and obtain its final shape. The volume of this polytope, Z, is defined as [1]:

$$Z = \alpha_1\Delta\mathbf{t}_1 \oplus \alpha_2\Delta\mathbf{t}_2 \oplus \cdots \oplus \alpha_m\Delta\mathbf{t}_m + \sum_{i=1}^{m} \mathbf{t}_{i_{min}} \qquad (3)$$

where m is total number of the independent force vectors, $\alpha_i \in [0, 1]$, and the symbol $\oplus$ represents the Minkowski sum of the line segments. Finally, the radius of the largest circle that can be completely located inside the resultant zone with its origin attached to the origin of the 2-D polytope is the magnitude of the maximum permissible force. If this force is larger than the specified minimum allowable force then that pose belongs to the WFW of the robot. In the case of the cable robots driven by differentials, instead of the tension vectors $\Delta\mathbf{t}_i$ of the cables of each differential, their resultant vector $\Delta\mathbf{t}'_j$ should be used in Eq. (3). In general, this property should be valid for both force and torque vectors which are investigated,

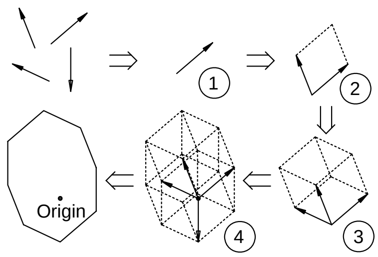

Fig. 4. Four step generation of a 2D zonotope from four coplanar vectors.

separately. In Fig. 4, an example of the generation of a 2-D zonotope from four coplanar vectors is illustrated.

Considering the above definitions, the WCWs of four planar cable mechanisms are compared for illustration purposes as illustrated in Fig. 5. All these mechanisms have a single-point MP (for the sake of simplicity) and three actuators. The attachment points of their cables on their BP are on the same circle (with identical center and radius). In all cases, the MP has the same coordinate with respect to the inertial frame attached to the center of the BP circle. The attachment points on the BP of the fully-actuated mechanism in Fig. 5(a) lies on an equilateral triangle while those of the differentially actuated mechanisms in Fig. 5(b) define a hexagonal shape, and in Figs. 5(c) and (d) they are symmetrically located on three sides of the circumcircle.

In Fig. 5(b), it is assumed that the MP is connected to three differentials on the BP respectively at points $B_1 - B_2$, $B_3 - B_4$, and $B_5 - B_6$. In the particular location of the MP shown in this figure, the bisectors of the cables attached to the differentials defined by of $B_1 - B_2$ and $B_5 - B_6$ (i.e. the direction of the resultant forces of these cables) intersect the base circle at two points further away from point S_2 than points S_1 and S_3.

As a result, in Fig. 5(b), the triangle made by the unit vectors along these bisectors includes the MP while the triangle created by unit vectors along cables in Fig. 5(a) does not. Consequently, in Fig. 5(b) the location of the MP is inside the WCW and in Fig. 5(a) it is outside the WCW. Thus, as shown in this introductory example, with a proper design and even in the same limited area for the BP, one can expect a larger WCW with the cable mechanisms using differentials.

In Figs. 5(c) and (d) differentials with respectively $q = 3$ and 4 cables are used in the architecture of this planar mechanism. Following the same rule and in ideal conditions (friction and the diameters of the pulleys are ignored) for the illustrated location of the MP, the directions of the resultant forces of the differentials are obtained. As it can be seen in these figures, in both cases the MP is again inside the aforementioned triangle but compared to case (b) the MPs are closer to one edge of the triangle (and so closer to the boundary of the WCW). Compared to the differential $B_1 - B_2$ in Fig. 5(b), in Fig. 5(c), the middle cable of the differential $B_1 - S_1 - B_2$ connected to point S_1, brings the direction of the resultant force a little closer to this point and so decreases the advantage of using differentials. Similar results are obtained for the direction of the resultant forces of differential $B_1 - D_1 - D_2 - B_2$ in Fig. 5(d).

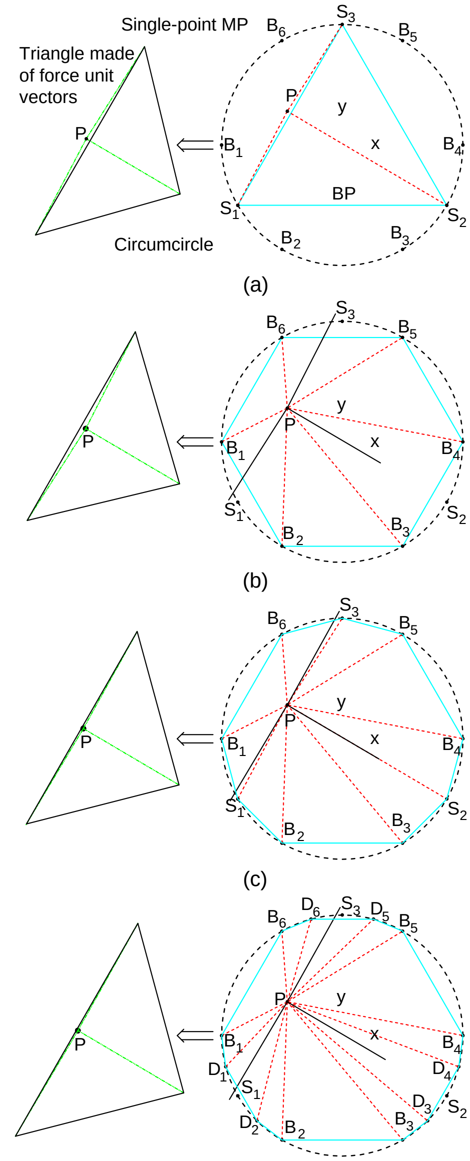

Fig. 5. Schematic of planar cable mechanisms driven by (a) three actuated cables; (b) three differentials with $q = 2$ cables; (c) three differentials with $q = 3$ cables; (d) three differentials with $q = 4$ cables.

Consequently, both the arrangement and the number of cables of differentials affect their performance. When the distance between attachment points of the cables on the BP (e.g. the distance between points B_1 and B_2 in Figs. 5(b)-(d)) becomes larger, the particular effect of the differentials (illustrated in Fig. 3) increases. On the other hand, the magnitude of the resultant force of those cables decreases. If more cables are used in the structure of the differential, the maximum magnitude of the resultant force can be improved but its direction may also be influenced (and thus, the performance of the differential can be either weakened or improved). Therefore, in Fig. 5(d), if the points D_1 and D_2 coincide respectively with points B_1 and B_2, then the WCW of cases (b) and (d) are identical. Also, if points D_1 and D_2 are located on the circumcircle beyond the boundary between B_1 and B_2,

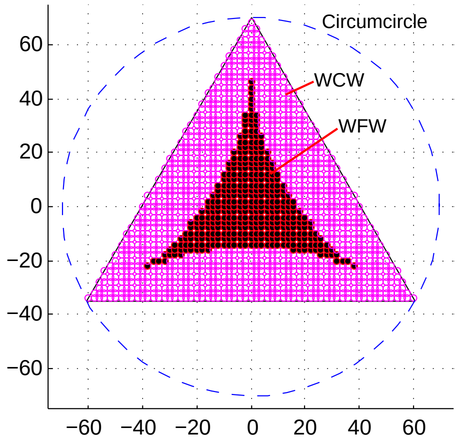

Fig. 6. WCW and WFW of the 3-3 full mechanism.

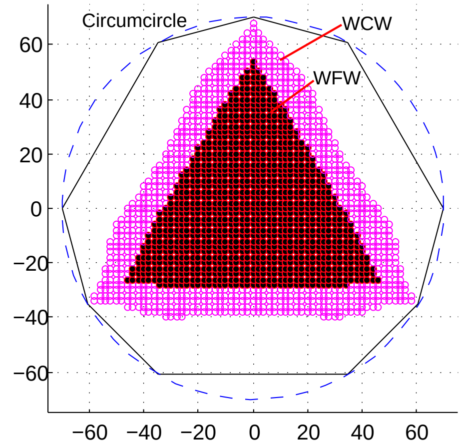

Fig. 8. WCW and WFW of the 9-3 differential mechanism ($q = 3$).

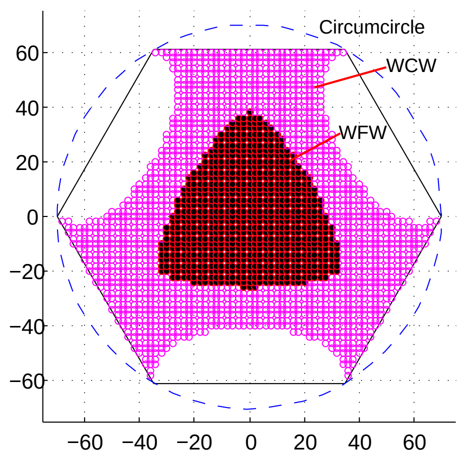

Fig. 7. WCW and WFW of the 6-3 differential mechanism ($q = 2$).

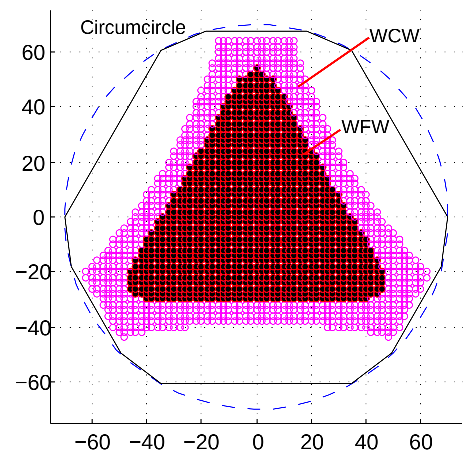

Fig. 9. WCW and WFW of the 12-3 differential mechanism ($q = 4$).

then the volume of the WCW even expands.

To illustrate the advantages and limitations of using differentials in the design of a cable robot in more details, the three planar differentially-actuated mechanisms depicted in Figs. 5(b)-(d) (each with three differentials driving respectively six, nine, and twelve apparent cables) are compared to two typical fully-actuated planar mechanisms with similar geometries. The differentially-driven cable robots shown in Figs. 5(b)-(d) are here respectively referred to as 6-3, 9-3, and 12-3 differential mechanisms. The differentials in each of these mechanisms have $q = 2, 3,$ and 4 cables. The first fully-actuated mechanism is shown in Fig. 5(a) and has three actuated cables and is called henceforth the 3-3 full mechanism. The second fully-actuated cable mechanism has six actuated cables and is similarly called the 6-6 full mechanism. It has the same geometric architecture as the 6-3 differential mechanism (c.f. Fig. 5(b)). In this paper, the performance metrics used to analyze the planar cable robots, are the sizes of the WCW and WFW.

IV. Implementation and Results

An algorithm is then used to numerically calculate the WCW and WFW of the aforementioned five planar cable architectures. For this, numerical values are chosen, e.g. it is assumed that the radius of the BP circle is $70\ cm$. Moreover, in the 12-3 mechanism, the points D_i are located on the circle at the middle of points B_i and S_j (c.f. Fig. 5(d)). Also, the maximum and minimum tension in the cables are arbitrarily chosen to be $t_{min} = 10\ N$ and $t_{max} = 100\ N$ respectively. The minimum force to be exerted to the MP of the robots in any direction is chosen at $50\ N$.

The results of the workspace calculations for these mechanisms are presented in Figs. 6-10 and Table I. Note that, WCW is a type of WFW where there is no boundary for the cable tensions and the amount of the required wrench set, thus, the WFW is a subset of the WCW [20]. The ratios of the areas of the WCW and WFW to the base circle area of the mechanisms are listed in Table I. As expected, the results reveal that both the WCW and WFW of the 6-3 differential mechanism are larger than these of the 3-3 full mechanism but smaller than those with the 6-6 full mechanism. The same results are obtained for workspaces of the 9-3 and 12-3 differential mechanisms. But when the three differential robots are compared together an interesting conclusion can be drawn. Unlike what is generally expected with the fully-actuated cable robots (i.e., using more cables one can have a

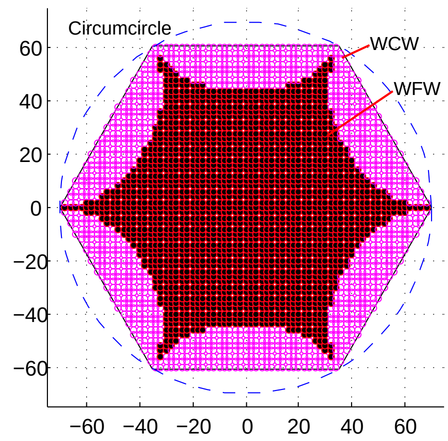

Fig. 10. WCW and WFW of the 6-6 full mechanism.

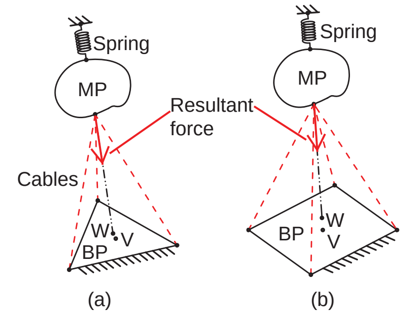

Fig. 11. Schematic of a spatial differential with (a) $q = 3$ cables; (b) $q = 4$ cables.

TABLE I
THE RATIOS OF THE AREAS OF THE WCW AND WFW TO THE BASE CIRCLE AREA OF THE MECHANISMS.

Type of workspace	3-3 full	6-3 diff.	9-3 diff.	12-3 diff.	6-6 full
WCW	0.4132	0.6019	0.4859	0.5248	0.8287
WFW	0.0937	0.1887	0.2704	0.3076	0.4965

better performance), with differential cable robots, using more cables in each differential the WCW may expand or shrink. As presented in Table I, although the WFW is grown by increasing the number of cables of each differential, the 9-3 design has a smaller WCW than the two others. Consequently, with more independent cables or differentials, larger workspaces can be expected, but the number of cables in each differential is not necessarily included in this hypothesis. Furthermore, the WCW of a cable robot generally depends on the cable configuration and the location of the attachment points of the cable on the BP and the MP [20].

In the proposed design, the WCW is analyzed by the direction of the resultant forces of the differentials. So, if using more cables limits the change of their directions, then the benefit of using a differential is weakened. However, with a proper design of a cable mechanism and using differentials one can improve both the WCW and WFW of a cable robot without requiring more actuators. Of course, due to the dependency of the forces in the cables of a differential, these mechanisms cannot have the same performance as the ones in which all the cables are independently actuated. However, they offer a relatively simple and inexpensive way to improve the WCW and WFW of these robots.

This paper is dedicated to the investigation of the application of differentials in the planar cable robots. However, this option can also be used in spatial cable robots and similar results can be expected for them. This issue is briefly addressed here and will be investigated and analyzed in detail in another paper. Regardless of the dimension of the space in which the mechanism manipulates (e.g., either 2-D or 3-D), the proposed differentials work with the same principle. But with spatial robots, there are more options to be used as differentials in the structure of the robot. Namely, in addition to the planar differentials, spatial types can also be used. Examples of spatial differentials with $q = 3$ and 4 cables are illustrated in Fig. 11 where points V and W are respectively representing the center of the attachment points of cables on the BP and the intersection point of the direction of the resultant force and the plane of the BP. In these examples, by moving the MP, point W moves around point V. In a spatial cable robot, when the actuator is locked, the attachment points of the cables of a differential on the MP are free to move on a free-form surface (e.g. an ellipsoid surface if $q = 2$).

Using differentials in the structure of a cable robot can also bring some difficulties. In the practical design, since there is friction among the cables and pulleys, the magnitude of the tension force of the cables may not be exactly the same. This can affect the force distribution and leads to some uncertainties in the direction of the resultant force. Also, the radii of the pulleys make the calculation of the direction of the cable force more complicated. These issues increase the difficulty of the kinematic analysis of this robot. Hence, it is preferred to minimize the use of pulleys especially if attached to the MP. However, since the total length of cables of a differential, its rate, and its acceleration are considered in the kinematic analysis, these drawbacks can be partly compensated. Nevertheless, due to using these parameters and the resultant force in this analysis, in frictionless ideal condition, the differentially actuated cable robot has a simpler kinematics comparing to when all these cables are individually actuated.

V. SYNTHESIS OF PLANAR DIFFERENTIAL CABLE-DRIVEN MECHANISMS

In this section, a method to efficiently replace a single actuated cable with a differential driving two or more cables is presented. The synthesis of differential cable-driven mechanism is divided in two parts, namely the valid arrangement of the cables and how to drive all cables of a differential using a single actuator.

A. *Synthesis of cable arrangements driven by one actuator*

The main challenge to synthesize differential cable-driven mechanisms is to find how to connect the MP to the BP via these cables (i.e., their arrangement). To solve this problem, each cable is first modeled as a line segment with two nodes at its ends (one attached to the MP and the other to the

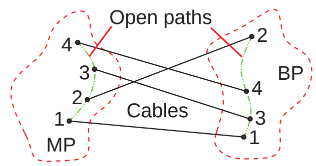

Fig. 12. Schematic of the distribution of nodes with four cables on four different coplanar locations.

TABLE II
TOTAL NUMBER OF POSSIBLE ARRANGEMENTS FOR 5 NODES.

No.	Locations & number of nodes	Solutions of each set (C_{mas} or C_{slv})
1	1,1,1,1,1	$\frac{5!}{5!}=1$
2	2,1,1,1	$\frac{4!}{1!3!}=4$
3	2,2,1	$\frac{3!}{2!1!}=3$
4	3,2	$\frac{2!}{1!1!}=2$
5	3,1,1	$\frac{3!}{1!2!}=3$
6	4,1	$\frac{2!}{1!1!}=2$
7	5	$\frac{1!}{1!}=1$
Total		16

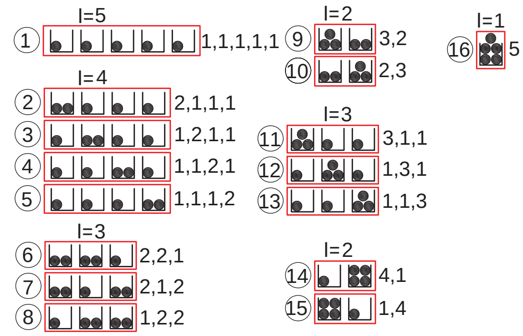

Fig. 13. All solutions for placing five balls (representing the nodes) in one to five bins ($q=5$ and $l=1,\ \ldots,\ 5$).

TABLE III
NUMBER OF SOLUTIONS FOR $q=2,\ \ldots,\ 10$ CABLES.

q	Solutions	q	Solutions
2	5	7	546193
3	33	8	9132865
4	281	9	171634161
5	2961	10	3581539973
6	37277		

BP). Also, as illustrated in Fig. 12, it is assumed that the attachment points of the cables on each body (either the MP or the BP) which are called here locations, are defined along an arbitrary planar open path. Therefore, the order of these points is important. Then, the problem is to find all possible connections between these points, referred here to as nodes, on each body. In addition, several cables can be attached at the same location on a body. In summary, the problem of finding all possible designs of an architecture with q cables can be divided in two steps:

1) Finding all possible solutions for the placement of nodes along the defined paths on each body (MP or BP).
2) Finding all possible solutions for connecting these nodes from the MP to the BP for each set of locations found in step 1.

The first step deals with finding all possible arrangements of nodes, namely the number of distinct locations, the number of nodes in each location, and the combination of all these possibilities without repetition. To do this, an algorithm is developed to find how q nodes can be located in a set of l locations where $1 \leq l \leq q$. For each l, the number of solutions can be calculated as:

$$\binom{l}{i}\binom{l-i}{j}\cdots\binom{l-i-j\cdots-k=h}{h}=\frac{l!}{i!j!\cdots h!} \quad (4)$$

where $i,\ j,\cdots,h$ are respectively the number of locations with $a,\ b,\cdots,c$ nodes while $l=i+j+\cdots+h$ and $q=a+b+\cdots+c$.

As an example, the total number of arrangements of 5 nodes is presented in Table II. As shown in this table, there are 7 different possible solutions to locate 5 nodes and considering their combinations, a total of 16 arrangements are found. To better illustrate this, the equivalent problem of placing five balls in one to five different boxes is shown in Fig. 13. All possible arrangements are then clearly apparent.

For the second step, an algorithmic solution is proposed in this paper based on using a matrix referred to as the slider. The aim is to find all possible solutions for connecting the nodes from the MP to the BP. For this, at each step of the calculation, two node solutions (one for each body) are chosen, e.g. arrangements no. 2 for the MP and no. 6 for the BP as described in Fig. 13. The node arrangement of the MP is represented by a vector called "master" and the one of the BP in another vector called "slave" defined as:

$$master=\begin{bmatrix}a_1 & a_2 & \cdots & a_{l_1}\end{bmatrix} \text{ and } slave=\begin{bmatrix}b_1 & b_2 & \cdots & b_{l_2}\end{bmatrix} \quad (5)$$

where a_i and b_i are respectively the number of nodes in the ith location on the MP and BP; l_1 and l_2 are respectively the number of these locations on the MP and BP where $a_1+a_2+\cdots+a_{l_1}=b_1+b_2+\cdots+b_{l_2}=q$. In the previous example with the arrangements no. 2 and 6, one has $master=\begin{bmatrix}2 & 1 & 1 & 1\end{bmatrix}$ and $slave=\begin{bmatrix}2 & 2 & 1\end{bmatrix}$. Next, the slider matrix is used to iteratively find all possible solutions for connecting the nodes associated with each set of master and slave vectors. In each iteration, the slider matrix has a $a_i \times l_2$ structure. In each row of this matrix, there is a non-zero component and its value is one. During the process, the column numbers of these non-zero components are changed to search for the valid solutions.

Using Eq. (4), the algorithm calculates all possible combinations for each of these node arrangements (e.g., third column of Table II for $q=5$) and considers them as coefficients C_{mas} and C_{slv} respectively for the master and slave vectors. For s different possibilities of locating q nodes, there are $\binom{s+1}{2}$ possible sets of two node arrangements (as presented in Table II, for five nodes one has $s=7$) and for each set, all possible connection patterns between q nodes on two bodies

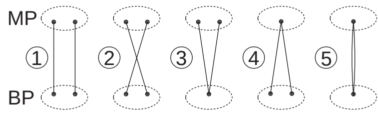

Fig. 14. All arrangements of $q = 2$ cables connecting BP to MP.

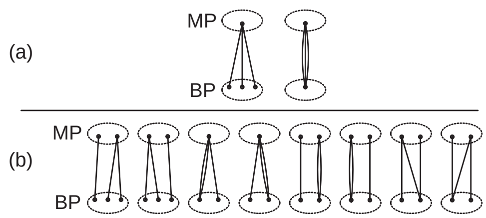

Fig. 15. Valid $q = 3$ cables (a) symmetric, (b) non-symmetric solutions.

are calculated iteratively as:

$$Sum_j = Sum_{j-1} + \begin{cases} C_{mas}C_{slv} & \text{if } master = slave \\ 2C_{mas}C_{slv} & \text{if } master \neq slave \end{cases} \quad (6)$$

Using this algorithm, the number of all possible non-repetitive arrangements of q cables is calculated. In Table III, these values for $q = 2, \ \ldots, \ 10$ cables are presented.

As it can be seen in this table, the number of solutions rapidly increases with the number of cables as would have been expected. In this paper, only the arrangement of differentials with 2, 3, and 4 cables are numerically investigated to limit the number of cases to consider to a reasonable number. The methodology is however general and can be extended to any number of cables.

The next step in the synthesis process is to seek out the valid arrangements amongst the myriads previously found. To this aim, manipulation requirements and physical restrictions are used as criteria to find the appropriate designs:

- the cables should not interfere;
- all cables of each differential should be driven by only one actuator which due to the design considerations should be located on the BP (to have the lightest possible MP);
- amongst all arrangements with similar properties, only the simplest is considered.

In Fig. 14, all arrangements previously found for two cables are illustrated. As shown in this figure, in case 1, the two cables can have either the same direction as in case 5 or an angle similar to case 4, while these two cases are more compact and have fewer attachment points on the MP. Also, the cables in case 2 intersect each other and in case 3 there are more attachment points on the MP than on the BP. Thus, based on the previously listed criteria, only cases 4 and 5 are considered valid.

Using the same technique, amongst all 33 arrangements found for three cables many cases can actually be discarded. Then, as shown in Fig. 15, the remaining solutions are categorized as either symmetric or non-symmetric. Since these differentials are assumed to be used instead of a single actuated cable, in this paper, the symmetry means that the number of

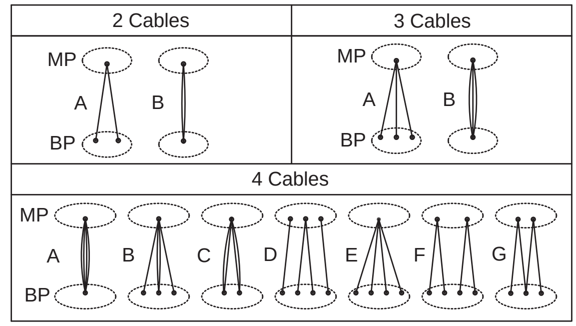

Fig. 16. All valid symmetric arrangements with for a $q = 2$, 3, and 4 cables.

nodes in each location and the number of the locations on each body (i.e., either the BP or the MP) on two sides of this imaginary single actuated cable, are the same. This symmetry is not based on physical dimensions. Using non-symmetric arrangements usually results in asymmetrical force directions. Thus, the shape of the workspace of the robot and also its control become more complex. Therefore, although they are technically valid, they are not considered in this paper.

Finally, the proper arrangements of $q = 4$ cables are also found and the final valid symmetric designs for 2, 3, and 4 cable systems are illustrated in Fig. 16. It should be again noted that this procedure can be followed to find the valid arrangements for five and more cables, but the number of designs found grows exponentially.

B. Actuation Synthesis

In the next step, the main issue is to equally distribute the actuation force amongst all the cables. Indeed, because of practical limits in their tensions, when the actuation force is equally distributed amongst all the cables of a differential, then, that mechanism can produce a larger force for the same posture. To drive the cables, three solutions are considered here:

1) The cables are branches of a single tendon connecting the MP to the BP via several pulleys, and one or both of its ends are connected to a actuated winch system. An example of this was presented in Fig. 2(b).
2) The cables are connected to the actuator using a differential (e.g., a bevel gear differential).
3) A combination of the two previous cases.

Since in the mechanical design of the systems different types of mechanisms are able to do the same task, it is difficult to determine exactly the number of systems that could be used in a particular application. This paper focuses on the types which are already used in other systems [24]. However, they may not be the only options. To drive a cable arrangement, four types of differential systems are therefore considered here, either individually or combined. They are the cable and pulley system (Fig. 2(b)), the single winch double cable system (i.e., the two ends of a cable are connected to the same winch), the double winch bevel gear system (i.e. two winches connected to the two outputs of a bevel gear system to drive two separated cables), and the triple winch planetary & bevel gear system

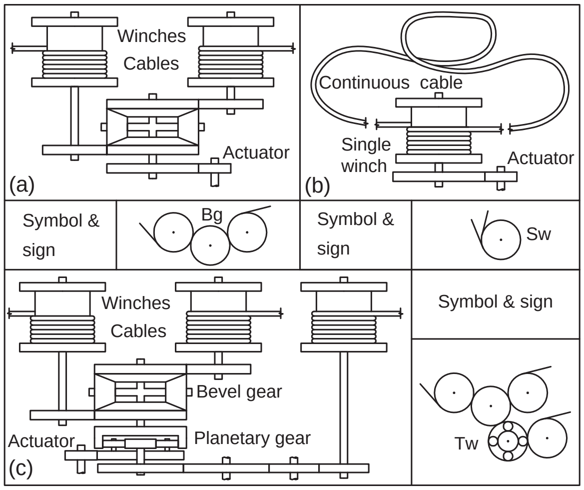

Fig. 17. Schematics of (a) the double winch bevel gear system; (b) the single winch double-cable system; and (c) the triple winch planetary & bevel gear system.

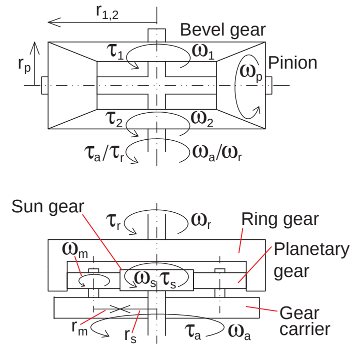

Fig. 18. Definition of the radii of the gears, input and output torques/angular velocities.

(i.e., a combination of a bevel gear and planetary differential systems connected to three winches to drive three separate cables while the actuation force/torque is equally distributed amongst them) [24], [31], [32]. These four differentials will be respectively referenced by symbols Cp, Sw, Bg, and Tw in the figures. The schematics of the last three systems are shown in Fig. 17.

Taking these four differentials into account, the relationships between inputs and outputs (i.e., respectively actuation torque/speed and winch torque/speed) can be easily obtained. To do this, it is assumed that the pulleys and gears are massless. The magnitudes of the ith velocity and force outputs in cables are respectively v_i and f_i. Also, ω_a and τ_a are respectively the angular velocity and torque of the actuator. In Cp and Sw differentials, these relationships are obtained as:

$$\omega_a = (v_1 + v_2)/r_w \quad \text{and} \quad \tau_a = f_1 r_w \quad \text{in } Cp \tag{7a}$$

$$\omega_a = (v_1 + v_2)/(2r_w) \quad \text{and} \quad \tau_a = (f_1 + f_2) r_w \quad \text{in } Sw \tag{7b}$$

where r_w is the radius of the winch. For the two latter differentials, first the radii of the gears, input torque/angular velocity (i.e., τ_a/ω_a) and output torques/angular velocities (i.e., $\tau_1, \tau_2, \tau_s/\omega_1, \omega_2, \omega_s$) are defined as illustrated in Fig. 18. Then, the relationships for the Bg and the Tw differentials are obtained as:

$$\omega_a = (\omega_1 + \omega_2)/2 \quad \text{and} \quad \tau_a = 2\tau_1 \quad \text{in } Bg \tag{8a}$$

$$\begin{cases} \omega_a = \frac{(\omega_1+\omega_2)(r_s+2r_m)/2+\omega_s r_s}{2(r_s+r_m)} \\ \tau_a = (\frac{2\tau_1}{r_s+2r_m} + \frac{\tau_s}{r_s})(r_s + r_m) \end{cases} \quad \text{in } Tw \tag{8b}$$

With Tw differentials, the output of the planetary gear system is connected to the input shaft of the bevel gear system. It is assumed that the differentials can equally divide the input force/torque between the outputs. For this, there should be a particular constraint in the gear ratios, namely r_s=$r_{1,2}$=$2r_m$. Depending on how the differentials are connected to the cables (i.e., either directly via winches or by connecting their outputs to the inputs of other differentials), Eqs. (7a)-(b) and (8a)-(b) can be combined together.

The main issue which affects the selection of the type of differential is the number of required cables. With a q cable differential mechanism, q is not necessarily the number of individual cables. It is the number of connections between the MP and the BP. For example, in a $q = 4$ system, there are four connections between the MP and the BP. Assuming that two of the connections are made by the two individual cables and two remaining are created by two branches of a single cable, then, there are three individual cables constituting this arrangement. Considering this peculiarity, from 1 to t distinct cables can be used to produce the required connections (where $1 \leq t \leq q$).

The synthesis of the actuation of such systems can be done by finding all combinations of aforementioned four differential systems able to pull t cables. For this, the followings are considered:

- If a single cable makes x connections between the MP and BP, then, at least $x - 1$ pulleys are required;
- A single winch double-cable system needs at least one pulley on the MP side because it should be connected to the two ends of the same cable;
- To drive two independent cables, one bevel gear system is used. For any additional cable, one more bevel gear system should be added. These systems can be combined in parallel or serial configurations [23], [24];
- If one bevel gear system is used to drive three or four cables, depending on the case, one or both of its winch(es) should be replaced by a single winch double-cable system;
- To actuate t cables where $t = 2k + 1$ and $k \geq 1$, triple winch planetary & bevel gear differentials are used, either individually or in combination with the other types;
- A bevel gear system should not be connected to both ends of the same cable.

Considering these rules, the possible solutions for actuating q= 2, 3, and 4 differential cable-driven systems are obtained. For this, all combinations of t separate cables needed to produce the arrangements of Fig. 16 are found while similar architectures are again discarded. Then, for each remained design, an adequate actuation system is selected.

TABLE IV
TOTAL NUMBER OF SOLUTIONS FOR THE ACTUATION OF A CABLE DIFFERENTIAL WITH $q = 2, 3$.

q	2		3		
Valid types	2-A,B		2-A,B		
t	1	2	1	2	3
No. of passes with t cables	(2)	(1,1)	(3)	(1,2)	(1,1,1)
Cp_{tj}	1,1	0,0	3,1	0,0	0,0
Sw_{tj}	1,1	0,0	0,0	0,0	0,0
Bg_{tj}	0,0	1,1	0,0	0,0	0,0
Tw_{tj}	0,0	0,0	0,0	0,0	1,1
Co_{tj}	0,0	0,0	0,0	4,2	1,1
Total	(2,2)	(1,1)	(3,1)	(4,2)	(2,2)
	6		14		

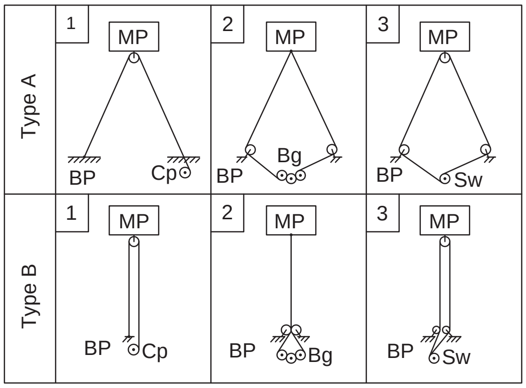

Fig. 19. Schematics of the options to drive a $q = 2$ cable system.

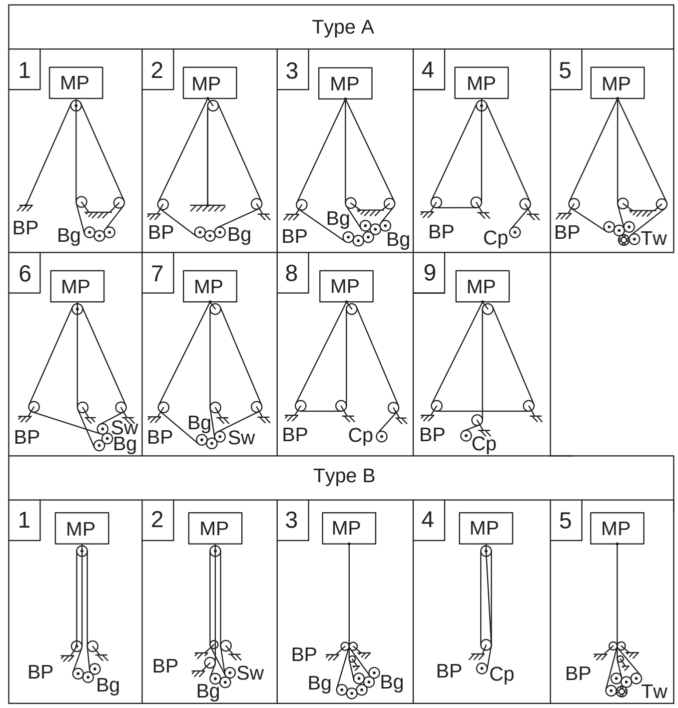

Fig. 20. Schematics of the options to drive a $q = 3$ cable system.

In Table IV, the different solutions to drive differential systems with $q = 2$ and 3 are listed. In this table, all combinations of t cables to connect the MP to the BP within q connections are presented and then the valid actuation option is selected and the total number of possibilities for each valid type of arrangement is found as:

$$T = \sum_{t=1}^{q} \sum_{j=1}^{g} (Cp_{tj} + Sw_{tj} + Bg_{tj} + Tw_{tj} + Co_{tj}) \qquad (9)$$

where g is the total number of valid arrangements for q connections (row 2 of Table IV); Cp_{tj}, Sw_{tj}, Bg_{tj}, and Tw_{tj} are respectively the number of designs, in which one can use Cp, Sw, Bg and Tw differentials for the j^{th} valid arrangement. Finally, Co_{tj} is the number of different combinations of these systems that can be used for j^{th} valid arrangement.

In Fig. 19, all possible actuation of a $q = 2$ system are presented. Amongst all of these, some are redundant or have technical problems. To select the proper architectures, a new set of criteria is again used, namely:

- If two cables have the same attachment points on both bodies they can be replaced by one thicker cable and a differential is not required except when the actuator only generates a part of the required total tension force.
- Considering the friction, in similar differential systems the one with a simpler structure and fewer pulleys or gears is preferred.
- Symmetric force distribution in the cables is preferred.

These criteria are used to select the kinematically practical designs but neither for comparing them together nor for investigating their efficiency and performance (which will remain as future work). Considering them, amongst the designs of Fig. 19, in types A, the force distribution can be affected by friction. However, in type $A1$, since just one side of the cable is pulled by the actuator, although it suffers from less friction than two other types A, the force distribution may be asymmetrical. In type $B1$, the cables are parallel, thus, the force distribution is not affected by friction. Types $B2$ and $B3$ can be replaced with a single thicker cable. Consequently, for $q = 2$, types A, and $B1$ seem to be the best options to be used.

Actuation designs of cable differentials with $q = 3$ are shown in Fig. 20. Again due to friction, the force distributions in types A may not be symmetrical. Amongst them, from a kinematic point of view, types $A1$, $A2$, $A4$, and $A6$ have simpler structure, less gears/pulleys and suffer less from asymmetrical force distribution. Thus, they are preferred to the other types. The force distributions in types $B1$ and $B4$ is not affected by friction and the actuator generates part of the total tension force while in the other types B, the differential system is useless.

A similar procedure is followed to find the designs for a $q = 4$ cable system. As presented in Table V (row 4), there are five different solutions for using t independent cables to connect the MP to the BP. The number of designs for each of the actuation systems as well as the total number of the designs are presented in the other rows of Table V. In total, for a $q = 4$ system, 462 designs exist. Finally, amongst these, using the same method, 18 options are determined as the kinematically preferred designs (c.f. Fig. 21).

TABLE V
TOTAL NUMBER OF SOLUTIONS TO DRIVE A CABLE DIFFERENTIAL WITH $q = 4$.

q	4				
Valid types	7-A,B,C,D,E,F,G				
t	1	2		3	4
No. of passes with t cables	(4)	(1,3)	(2,2)	(1,1,2)	(1,1,1,1)
Cp_{tj}	1,6,3, 14,14, 14,14	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0
Sw_{tj}	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0
Bg_{tj}	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	3,10,5,10 ,10,10,10
Tw_{tj}	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	0,0,0,0, 0,0,0	2,6,4,8, 8,8,8	0,0,0,0, 0,0,0
Co_{tj}	1,4,3,8, 8,8,8	1,6,3, 12,12, 12,12	3,10,7, 17,17, 17,17	4,14,8, 24,24, 24,24	0,0,0,0, 0,0,0
Total	2,10,6, 20,20, 20,20	1,6,3, 12,12, 12,12	3,10,7, 17,17, 17,17	6,20,12, 32,32, 32,32	3,10,5,10 ,10,10,10
	98	52	88	166	58
	462				

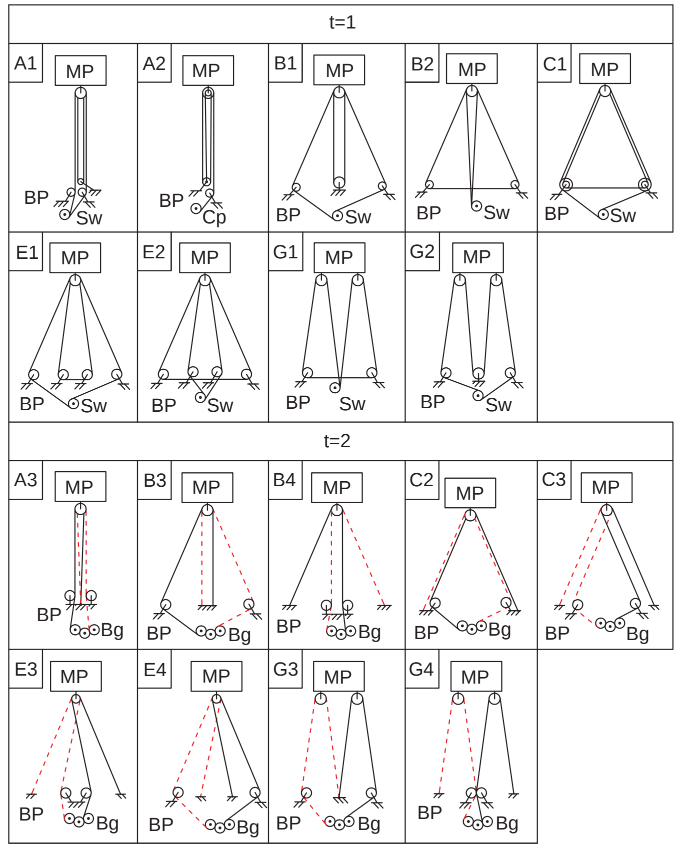

Fig. 21. Schematics of the best options to drive a $q = 4$ cable system.

VI. CONCLUSIONS

In this paper, the idea of using cable differentials in the architecture of cable-driven robots was proposed for the first time to the best of the authors' knowledge and the required properties to use such mechanisms in cable parallel robots were investigated. Next, the advantages of using differentials in the structure of these robots were discussed. Comparing three planar differential cable robots with two fully-actuated ones showed that, while the number of actuators is kept at minimum (i.e., $n + 1$ actuators for a n-DOF cable robots), using differentials one can expect larger WCW and WFW for a mechanism with the same MP and constrains on the BP. Then, the synthesis of differential cable-driven mechanisms was presented. To do this, a method was developed to find different arrangements of q cables in a differential and then, the valid arrangements with 2, 3, and 4 cables were found. Afterwards, four differential actuation systems were selected to drive the systems and finally, the valid possible mechanical architectures with q =2, 3, and 4 cables were presented.

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