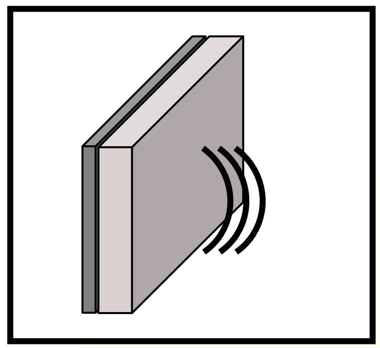
158th Meeting of the Acoustical Society of America
San Antonio, Texas

Acoustic radiation of a vibrating wall covered by a porous layer

Transfer impedance concept and effect of compression



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29 october 2009

Context

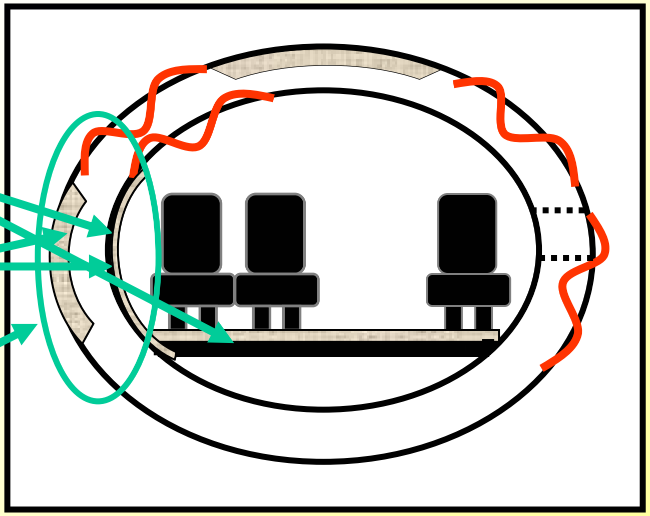
• Porous materials used in industrial applications
(*automotive, aeronautics*,…)

➡ for noise reduction

✓ sound absorption
(trim panel, floor,…)

✓ vibration damping
(fuselage)

✓ sound insulation
(fuselage)

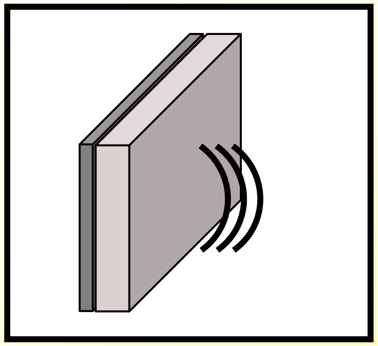


Context

➡ Porous material attached to a vibrating structure

Influence of a porous layer on the acoustic radiation of a plate ?

Method



- Analytical model using transfert impedance concept
- Experimental validation

☛ **Porous layer impedance applied to a moving wall: Application to the radiation of a covered piston,**
Doutres, Dauchez, Genevaux, J. Acoust. Soc. Am. **121**(1), 2007

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Introduction

1. Transfert impedance concept

2. Acoustic radiation efficiency

- 2.1. Infinite plate
- 2.2. Flat piston
- 2.3. Circular plate

3. Application to multilayer

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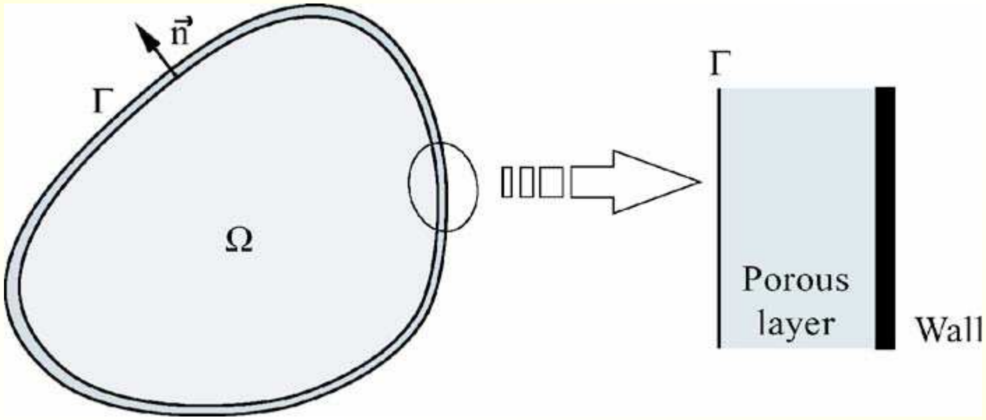
Problem to be solved



$$\text{in } \Omega, \quad \nabla^2 p + k^2 p = 0$$

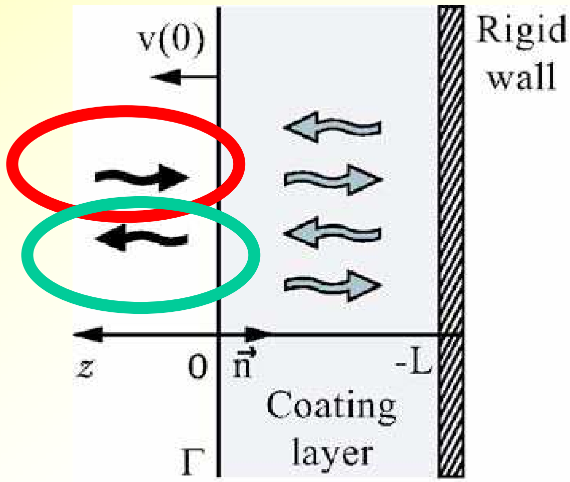
$$\text{at } \Gamma, \quad Ap + B\frac{\partial p}{\partial n} = h(r)$$

Source at boundary

Dirichlet (imposed pressure)

Neumann (imposed velocity)

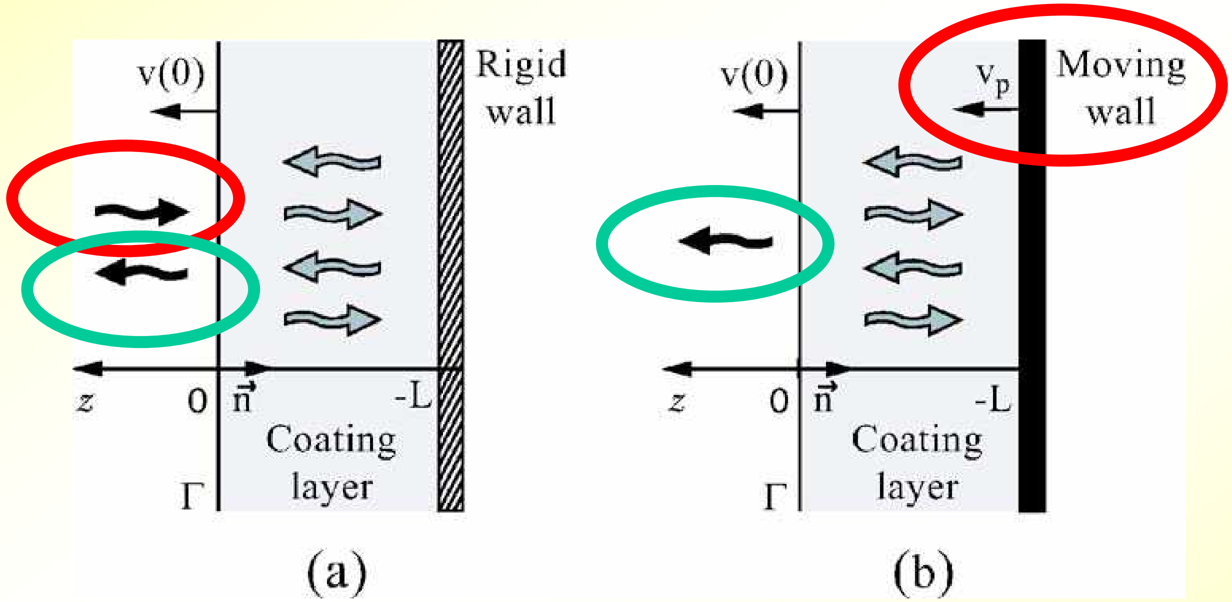
Problem divided into 2 cases :



(a)

a) Acoustic excitation : amplitude of reflected wave ?

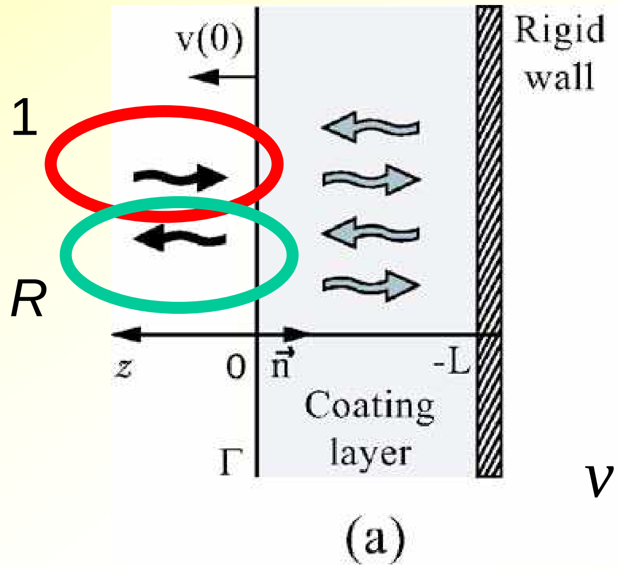
Problem divided into 2 cases :



a) Acoustic excitation : amplitude of reflected wave ?

b) Vibratory excitation : amplitude of transmitted wave ?

Surface Impedance Z_S



Porous layer characterized by

$$Z_S = \frac{p(0)}{v(0)}$$

$$p(z) = 1\,e^{jkz} + R\,e^{-jkz}$$

$$v(z) = \frac{1}{Z_0}\left(e^{jkz} - R\,e^{-jkz}\right)$$

The reflected wave is function of surface impedance Z_S:

$$R = \frac{Z_S - Z_0}{Z_S + Z_0} \quad \text{with} \quad Z_0 = \rho_0 c_0$$

→ Z_S is measured in a Kundt tube

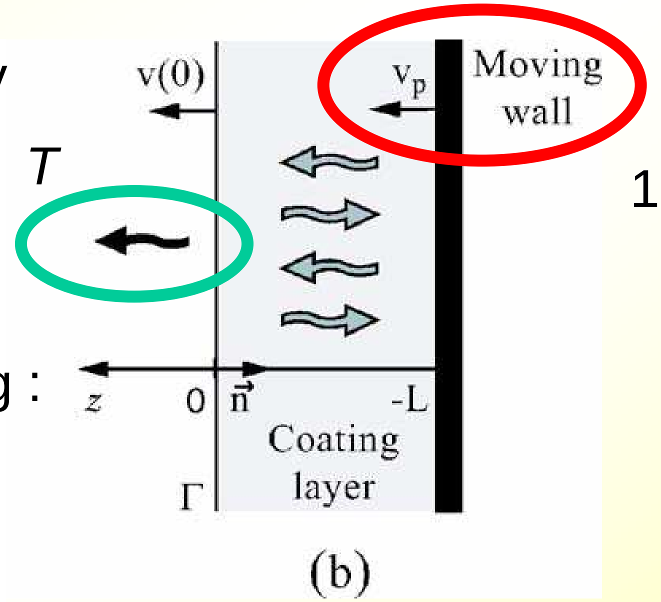
Transfert impedance Z_T

Porous layer characterized by

$$Z_T = \frac{p(0)}{v_p - v(0)}$$

Non-porous massless coating :

$$Z_T = \frac{\text{Bulk modulus}}{j\omega \, \text{Thickness}}$$



The transmitted wave is function of transfert impedance Z_T :

$$p(z) = T\, e^{-jkz} \quad \text{gives} \quad T = \frac{Z_T}{Z_T + Z_0} Z_0 v_p$$

Can Z_T be measured in a Kundt tube ?

Is Z_T equivalent to Z_S ?

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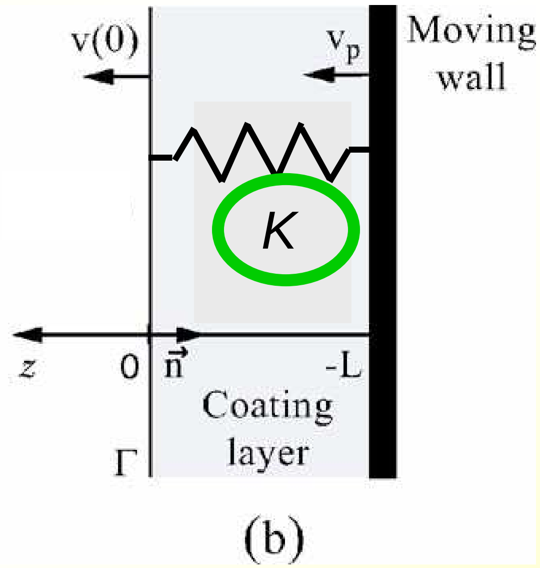
A simple coating : Spring

Static law:

$$p(0) - \frac{K}{j\omega}(v - v_p) = 0$$



Case a: $v_p = 0$

$$Z_S = \frac{p(0)}{v} = K / j\omega$$

Case b: $Z_T = \frac{p(0)}{v - v_p} = K / j\omega$

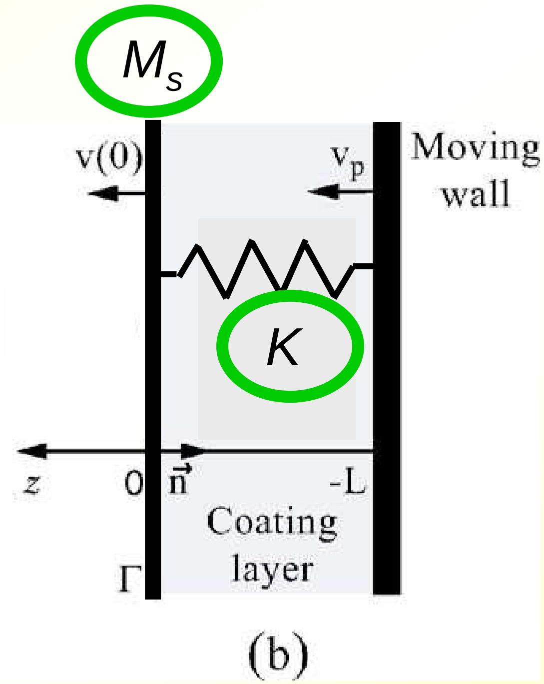
$$Z_T = Z_S = \frac{K}{j\omega} = \frac{\text{Bulk modulus}}{j\omega L}$$

Elastic layer

Adding a layer defined by its mass

Dynamic law:

$$j\omega M_S v = p(0) - \frac{K}{j\omega}(v - v_p)$$



Case a: $v_p = 0$

$$Z_S = \frac{p(0)}{v} = j\omega M_s + K / j\omega$$

Case b: $\frac{p(0)}{v} = Z_0 \quad \Rightarrow Z_T = \frac{K}{j\omega} \frac{Z_0}{Z_0 - j\omega M_s}$

$\Rightarrow$ **$Z_T \neq Z_S$ excepted when** $\omega \rightarrow 0$:

$$Z_T = Z_S = \frac{K}{j\omega} = \frac{\text{Bulk modulus}}{j\omega L}$$

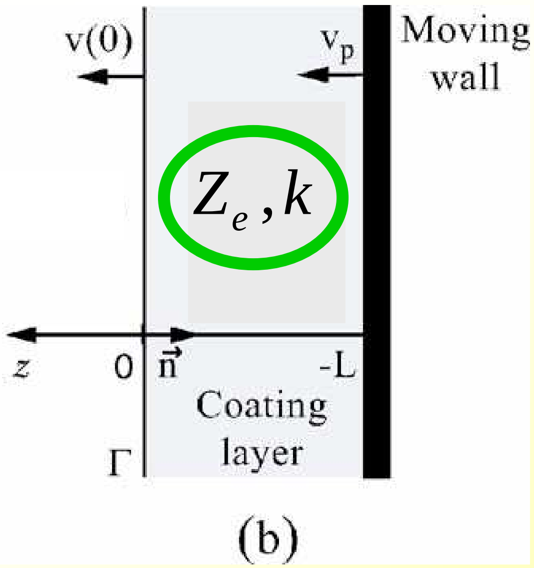
Elastic layer at low frequency

Monophasic continuous layer

Transfert matrix:

$$\begin{pmatrix} p(0) \\ v(0) \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} p(-L) \\ v(-L) \end{pmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos kd & jZ_e \sin kd \\ j/Z_e \sin kd & \cos kd \end{bmatrix}$$



Case a: $v_p = 0 \quad \Rightarrow Z_S = \dfrac{a}{c}$

Case b: $\dfrac{p(0)}{v} = Z_0 \quad \Rightarrow Z_T = \dfrac{Z_0}{a - 1 - cZ_0}$

➡ $Z_T \neq Z_S$ **excepted when** $\omega \to 0:\ a \to 1 \text{ and } c \to j\dfrac{kd}{Z_e}$

$$Z_T = Z_S = j\frac{Z_e}{kd} = \frac{\text{Bulk modulus}}{j\omega L}$$

Monophasic layer at low frequency

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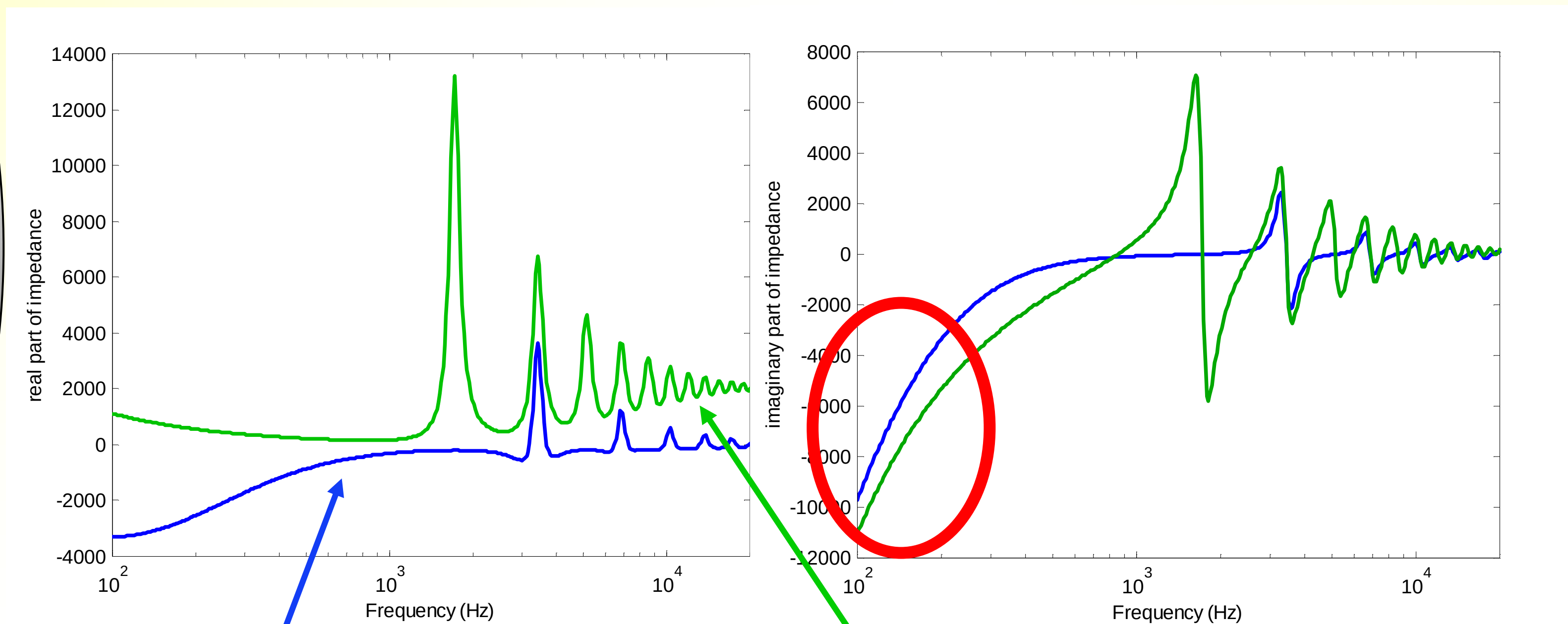
Impedance curves

Thickness	20 mm
Bulk modulus	140(1+j0.1) kPa
Density	30 kg.m^{-3}

Real part

Imaginary part



Z_T transfert impedance ≠ Z_S surface impedance

Poroelastic layer: Surface impedance z_s

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Biot-Allard theory in 1D in the porous layer :
2 waves travelling in each direction

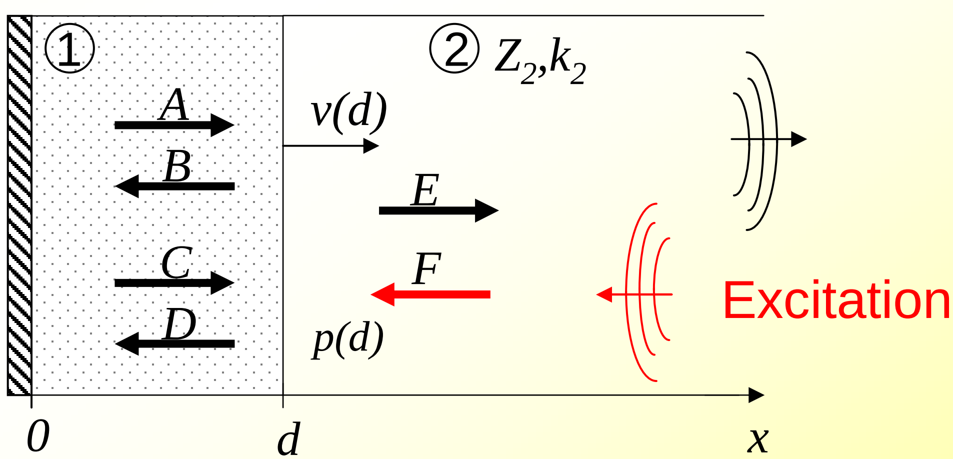
Two waves in fluid medium

Boundary conditions : Continuity of Stress and displacement

at x=0 $$v^s(0) = v^f(0) = 0$$

at x=d $$v(d) = (1-\phi)v^s(0) + \phi v^f(0)$$

$$\sigma^f(d) = -\phi p(d) \text{ and } \sigma^s(d) = -(1-\phi)\,p(d)$$



$$\Rightarrow \quad Z_S = \frac{p(d)}{v(d)}$$

Poroelastic layer: Transfert impedance Z_T

Biot-Allard theory in 1D in the porous layer :
2 waves travelling in each direction

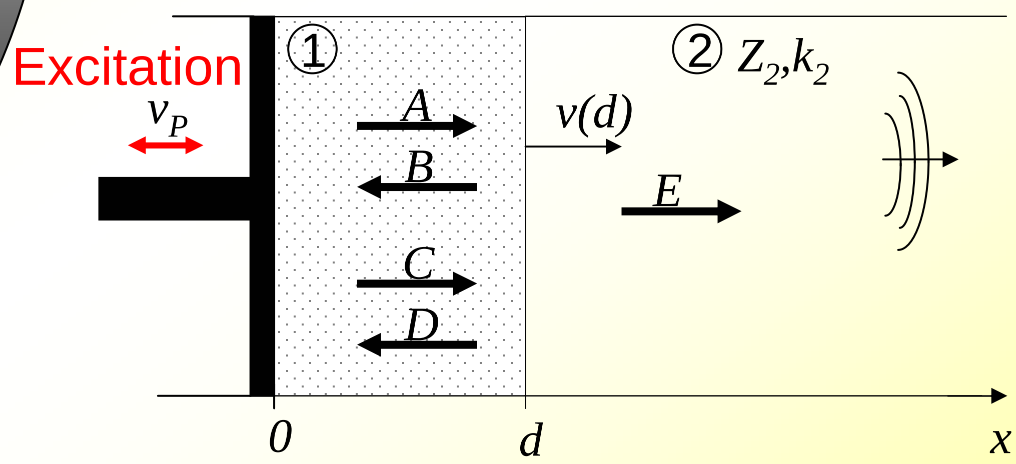
One waves in fluid medium $\Rightarrow \dfrac{p(d)}{v(d)} = Z_0$

Boundary conditions : Continuity of Stress and displacement

at x=0 $\quad v^s(0) = v^f(0) = v_p$

at x=d $\quad v(d) = (1-\phi)v^s(0) + \phi v^f(0)$

$$\sigma^f(d) = -\phi p(d) \text{ and } \sigma^s(d) = -(1-\phi)\,p(d)$$

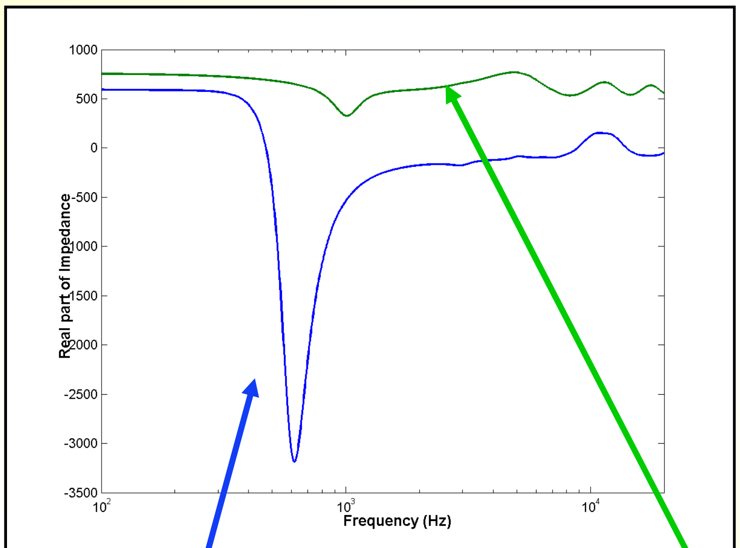


$$Z_T = \frac{p(d)}{v_p - v(d)}$$

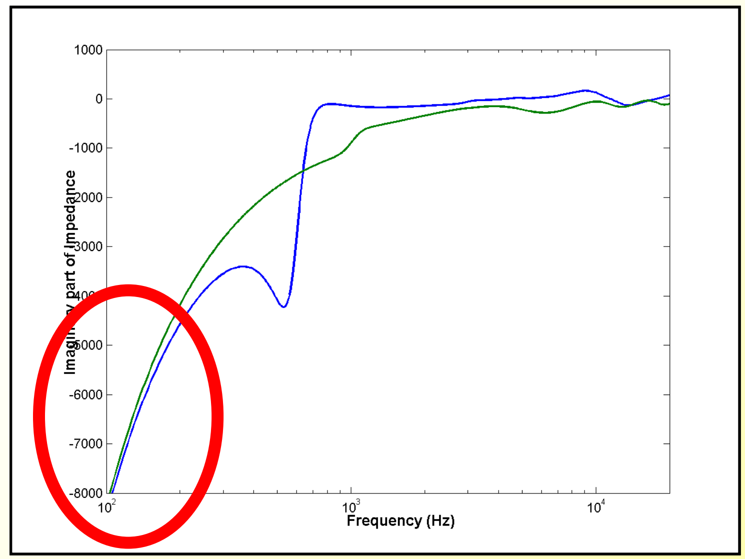
Impedance curves

Thickness (mm)	20.17
Air flow resistivity (Nsm^{-4})	75000
Skeleton Young's Modulus at 5Hz (Pa)	285000
Skeleton density (kgm^{-3})	59

Real part

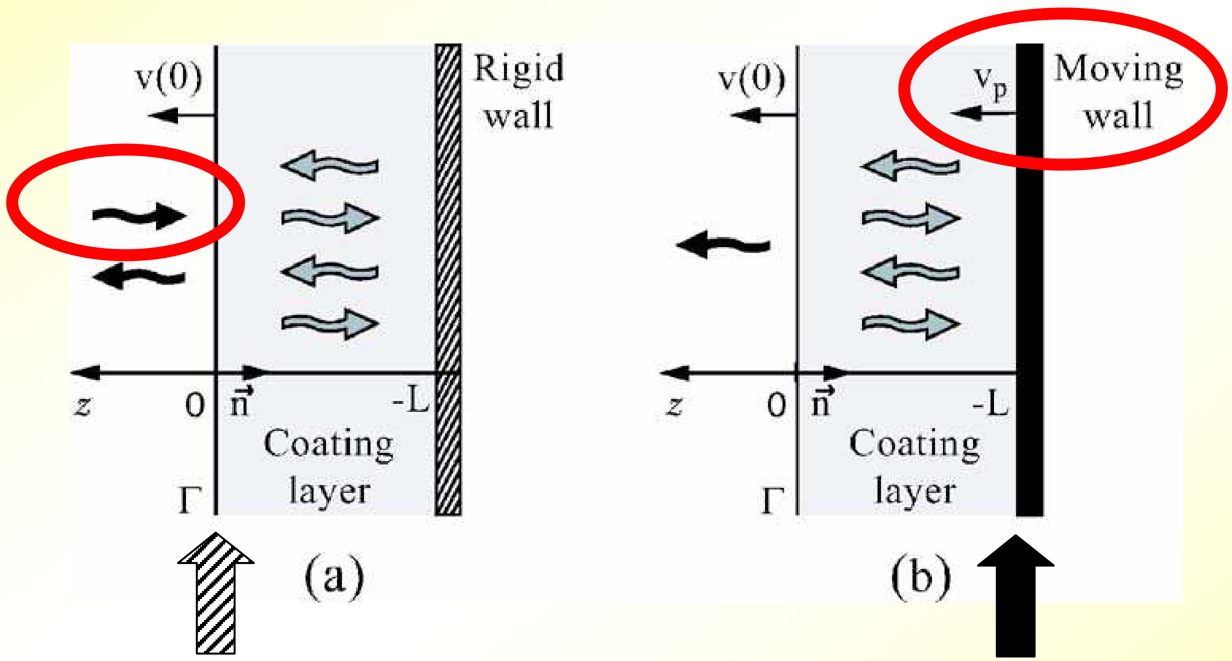


Imaginary part



Z_T transfert impedance ≠ **Z_S surface impedance**

Boundary conditions at excitation interface



Fluid-porous interface :

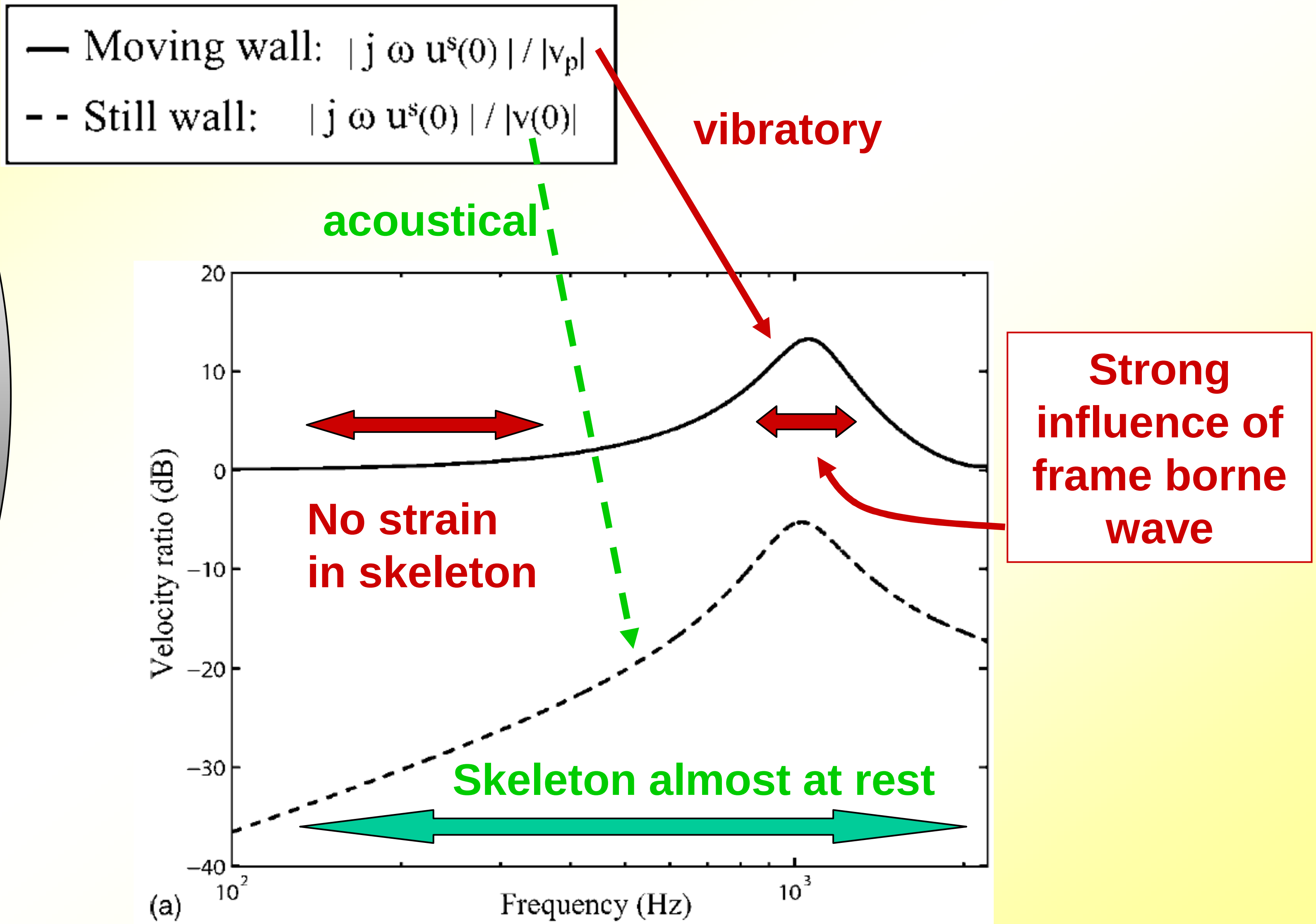
$$v(0)/\,j\omega=(1-\phi)u^s+\phi\,u^f$$

Wall-porous interface :

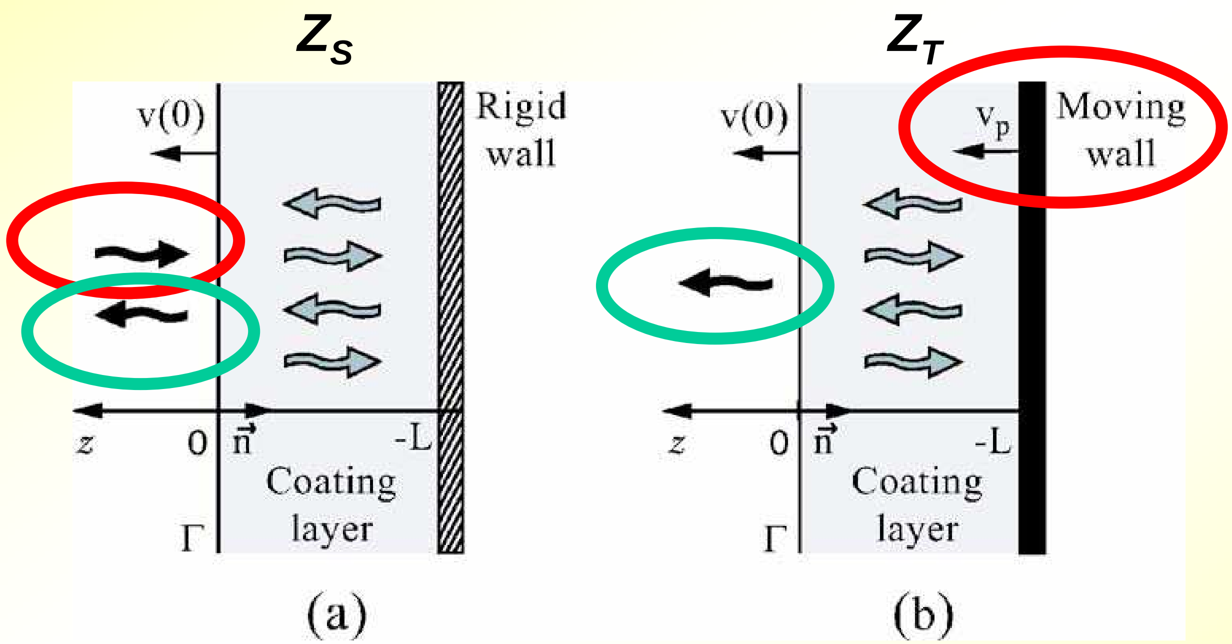
$$v_p/\,j\omega=u^s=u^f$$

$\Longrightarrow$ The skeleton is much more excited in case b)

Relative skeleton velocity for both excitation at fluid-porous interface



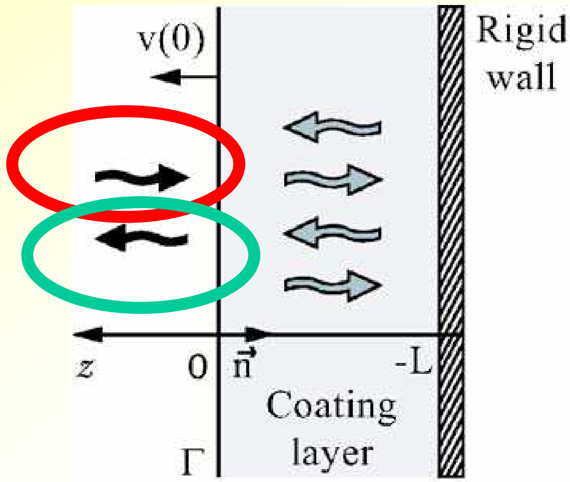
Conclusion of Part I



Excepted at low frequency, $Z_T \neq Z_S$

***Z*$_T$ can not be measured in a Kundt tube**

1 impedance for each problem



(a)

(a) $$jk\frac{Z_0}{Z_S}p+\frac{\partial p}{\partial z}=0$$

(b) $$jk\frac{Z_0}{Z_T}p-\frac{\partial p}{\partial z}=j\omega\rho_0 v_p$$

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2.1. Infinite plate

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2. Acoustic radiation efficiency

$$\sigma_R = \frac{\Pi_a}{\Pi_v}$$

Radiated acoustic power (Π_a)

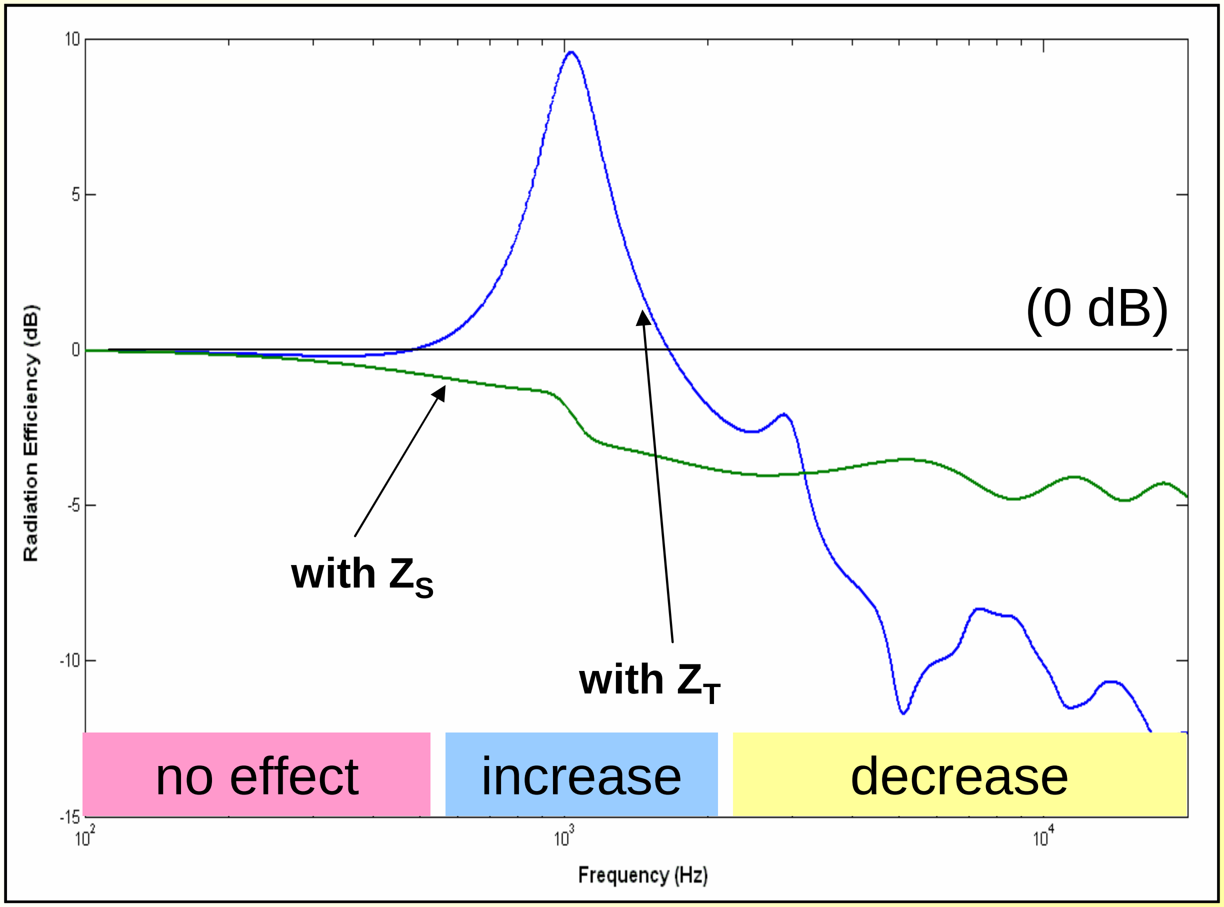
Injected vibratory power (Π_v)

$$\Pi_a = \iint_S \langle I \rangle r^2 \sin\vartheta d\vartheta d\varphi$$ with: $$\langle I \rangle = \frac{|p|^2}{2\rho_0 c_0}$$

$$\Pi_v = \rho_0 c_0 \iint_S \frac{|w(\vec{r})|^2}{2} dS$$

2.1. Acoustic radiation efficiency

Infinite plate at normal incidence (1D) : $\sigma_R = \frac{|T|^2}{Z_0 \, v_p^{\,2}} = \left|\frac{Z_T}{Z_T + Z_0}\right|^2$

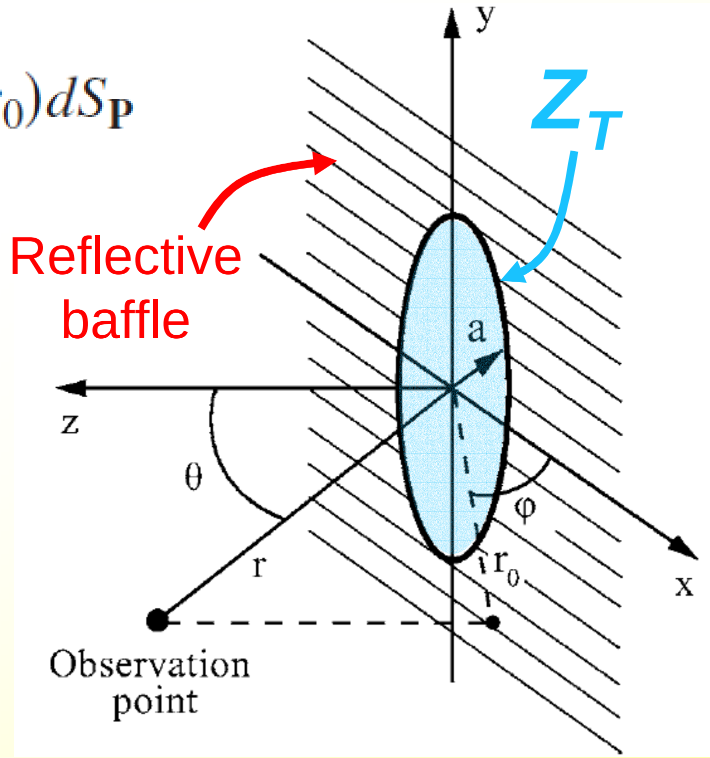


2.2. Acoustic radiation of a flat piston in semi-infinite field

Calculation of the far field pressure: Rayleigh Integral

$$p(\mathbf{r}) = j\omega\rho_0 \frac{Z_t}{Z_t + Z_0} \int_{S_P} v_p G(\mathbf{r},\mathbf{r}_0) dS_{\mathbf{P}}$$

with $G(\mathbf{r},\mathbf{r}_0) = \dfrac{e^{-jk|\mathbf{r}-\mathbf{r}_0|}}{2\pi|\mathbf{r}-\mathbf{r}_0|}$



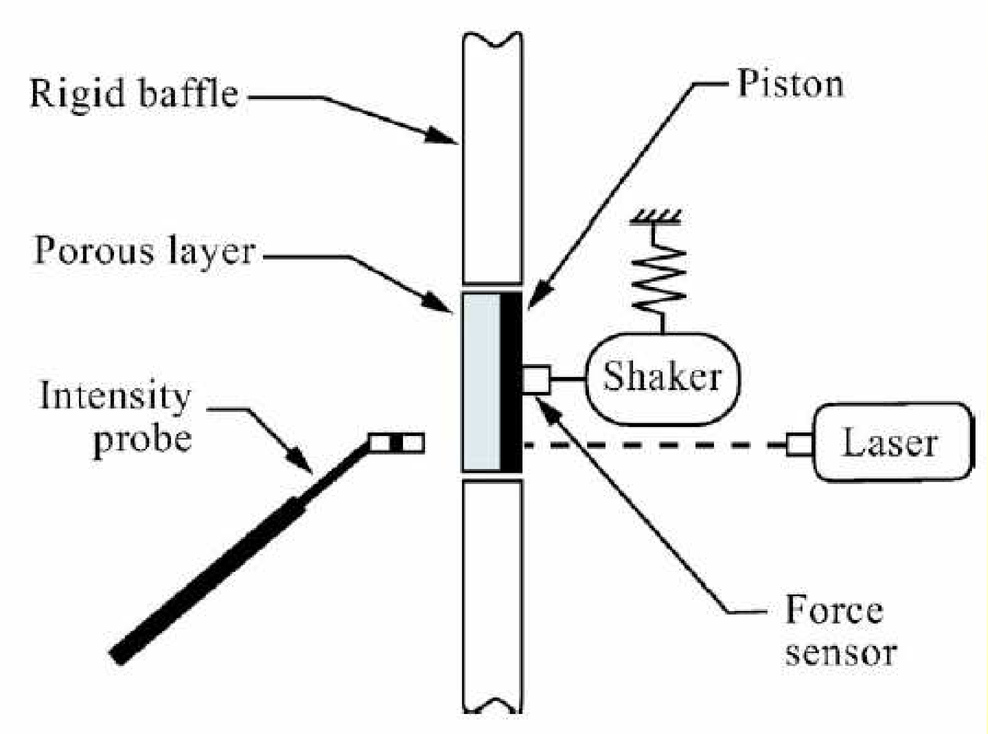
For a flat piston of radius *a*

in far field :

$$p(\mathbf{r}) = \frac{Z_t}{Z_t + Z_0} jk\rho_0 c_0 v_p \frac{e^{-jkr}}{2\pi r} \pi a^2 \frac{2J_1(ka \sin\theta)}{ka \sin\theta}$$

2.2. Acoustic radiation of a flat piston in semi-infinite field

Measurement :



2 materials :

A : polymer foam

B : soft fibrous

2.2. Acoustic radiation of a flat piston in semi-infinite field

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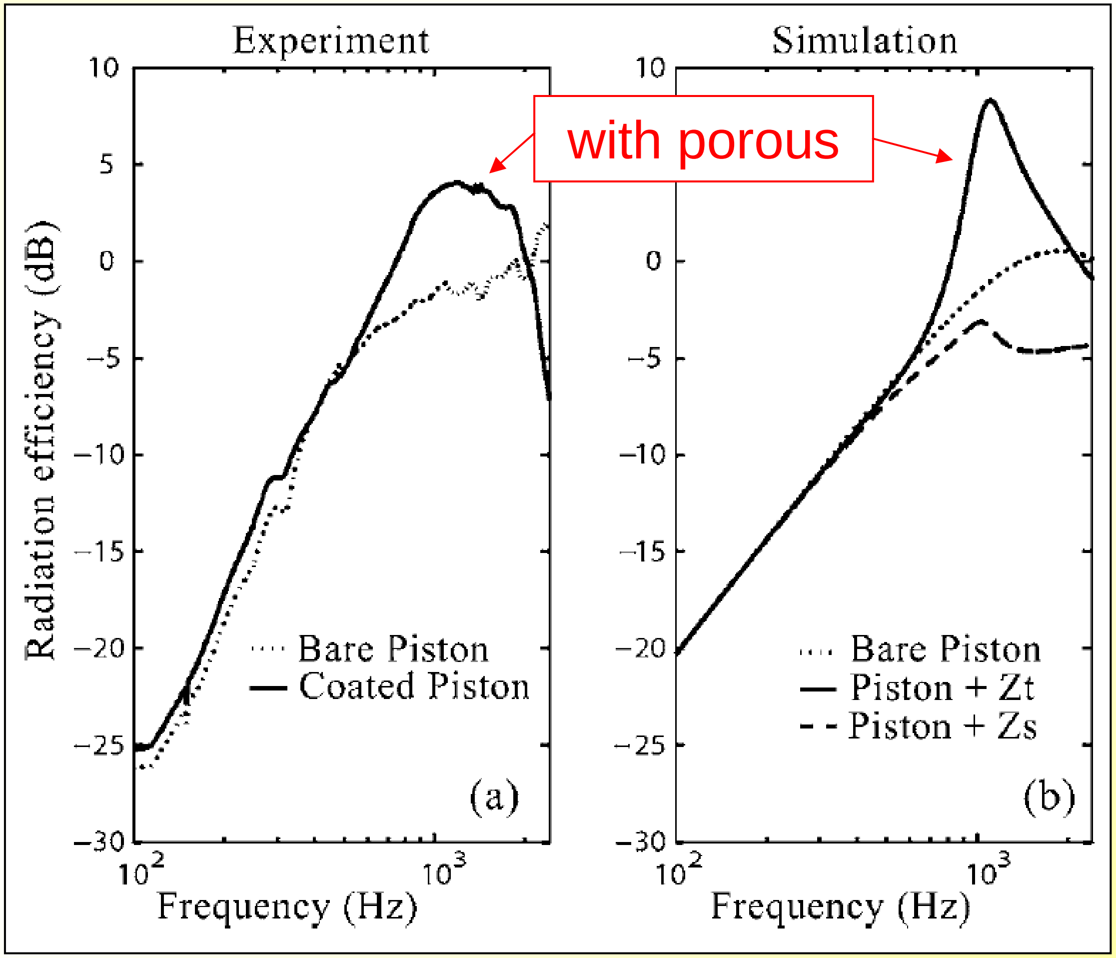
A : polymer foam



2.2. Acoustic radiation of a flat piston in semi-infinite field

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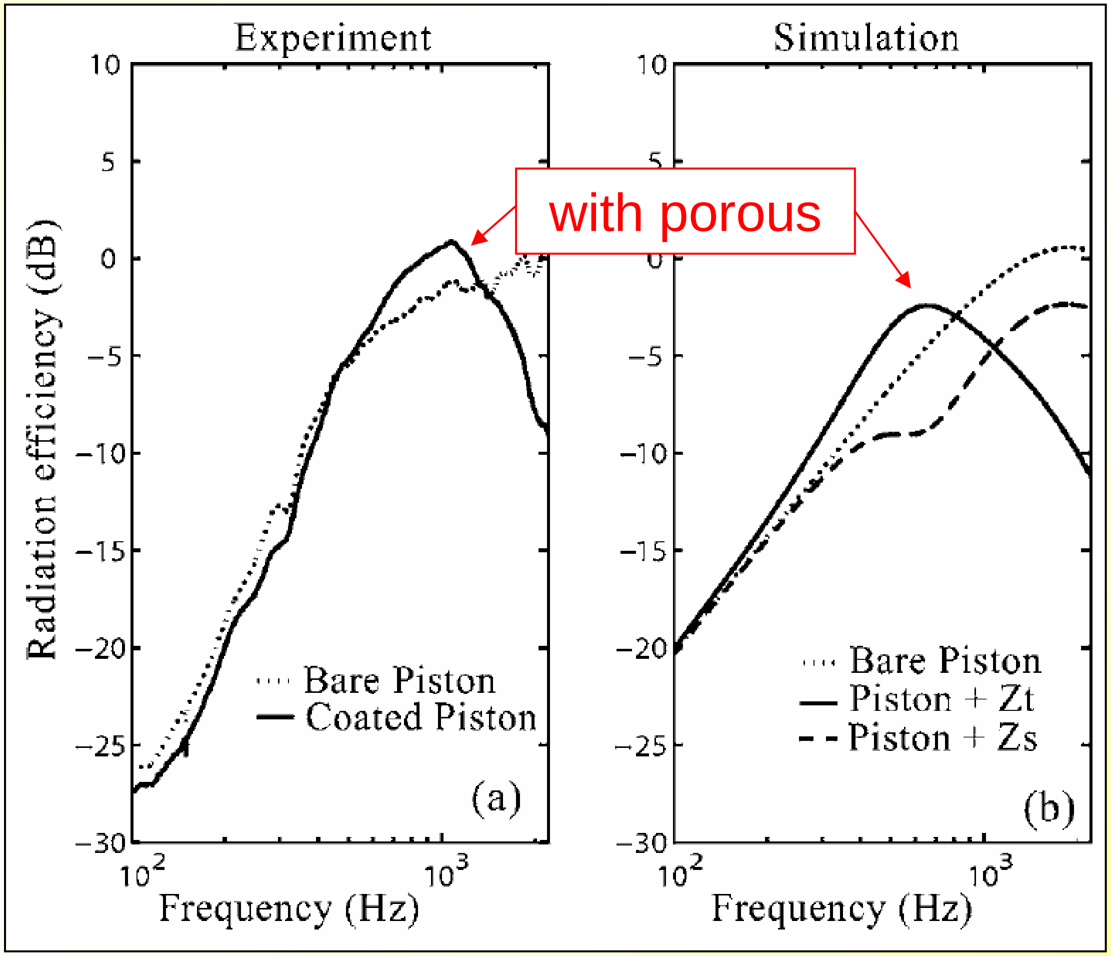
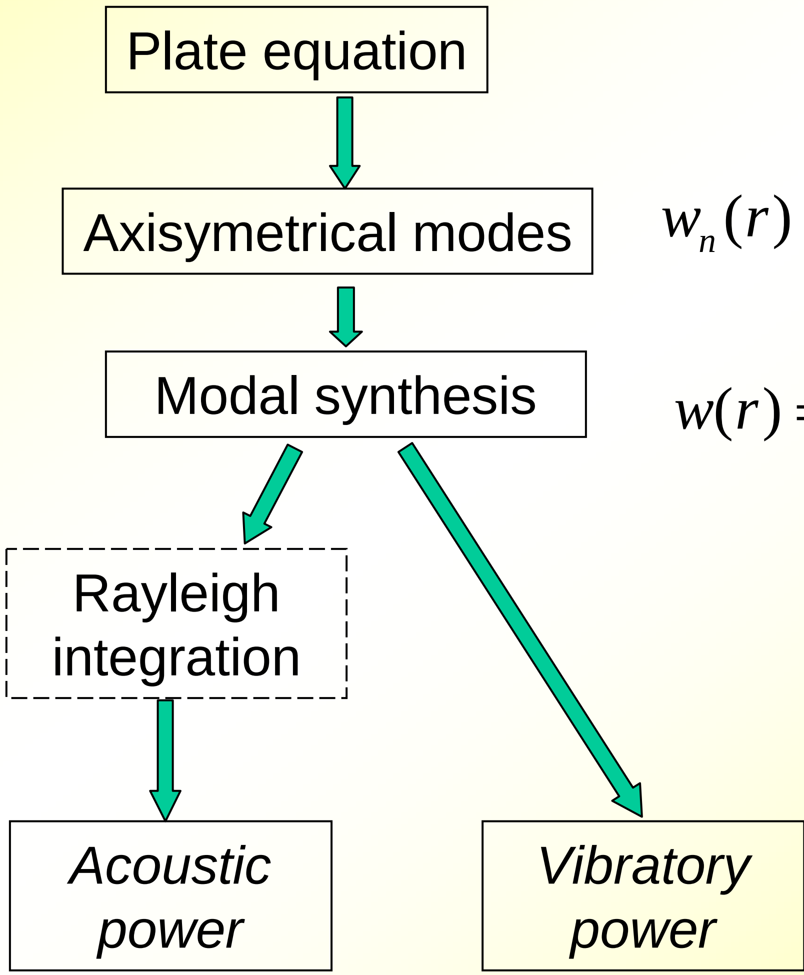
B : soft fibrous



2.3. Radiation efficiency of a circular plate



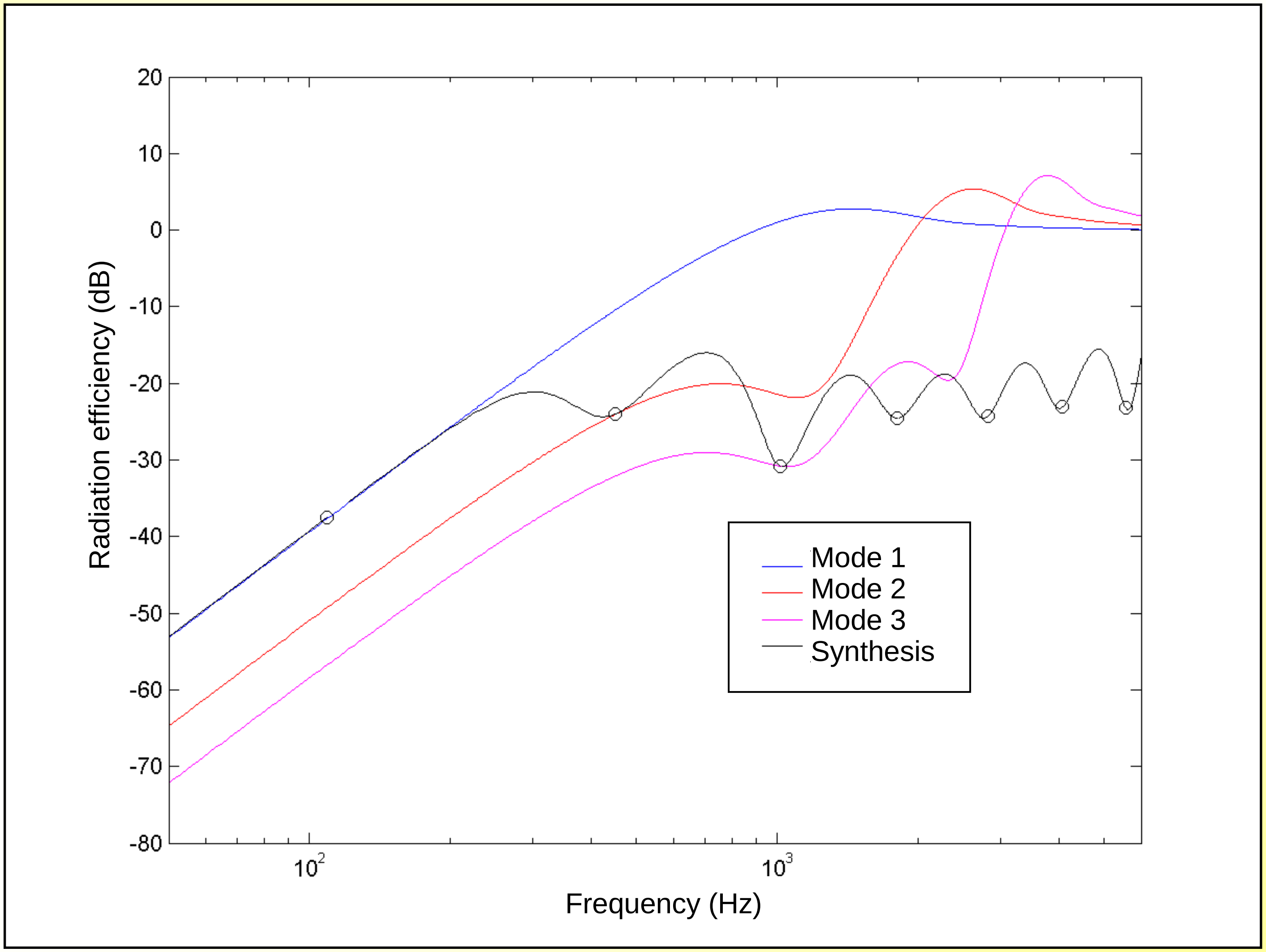
$$\nabla^4 w - \frac{\rho h \omega^2}{D} w = 0$$

$$w_n(r) = J_0(\beta_{0n} r) - \frac{J_0(\beta_{0n} a)}{I_0(\beta_{0n} a)} I_0(\beta_{0n} r)$$

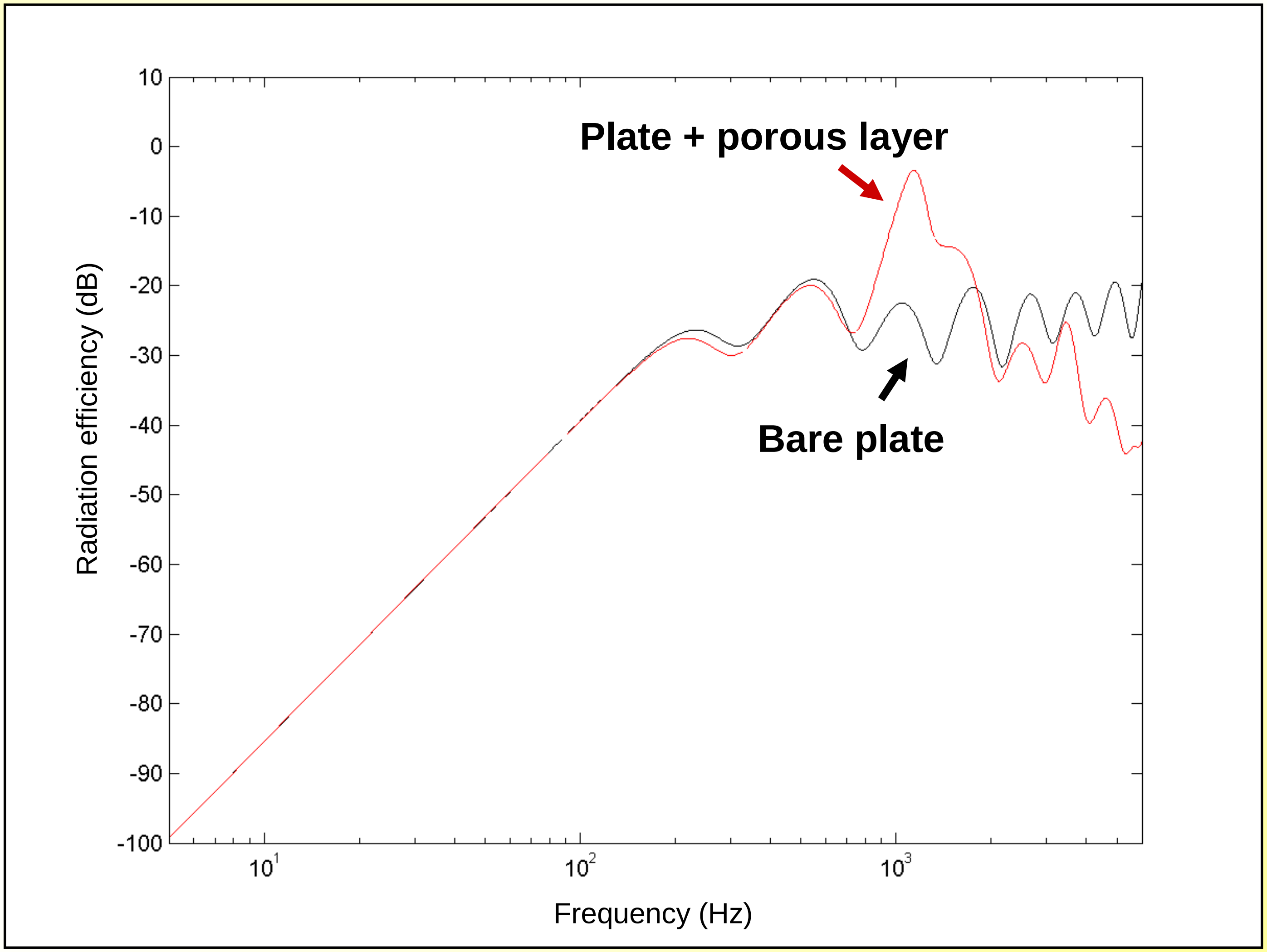
$$w(r) = \frac{F}{D} \sum_n \frac{w_n(r_s) w_n(r)}{(\beta_n^4 - \beta^4)\pi a^2 \Lambda_n}$$

with: $\beta_{0n}{}^4 = \frac{\rho h}{D} \omega_n^2$

Modal contribution to the acoustic radiation



Influence of the porous material on the radiation of the plate



Relative radiation efficiency

$$\sigma_{Rn} = \frac{\sigma_{R\ plate+porous}}{\sigma_{R\ plate}}$$

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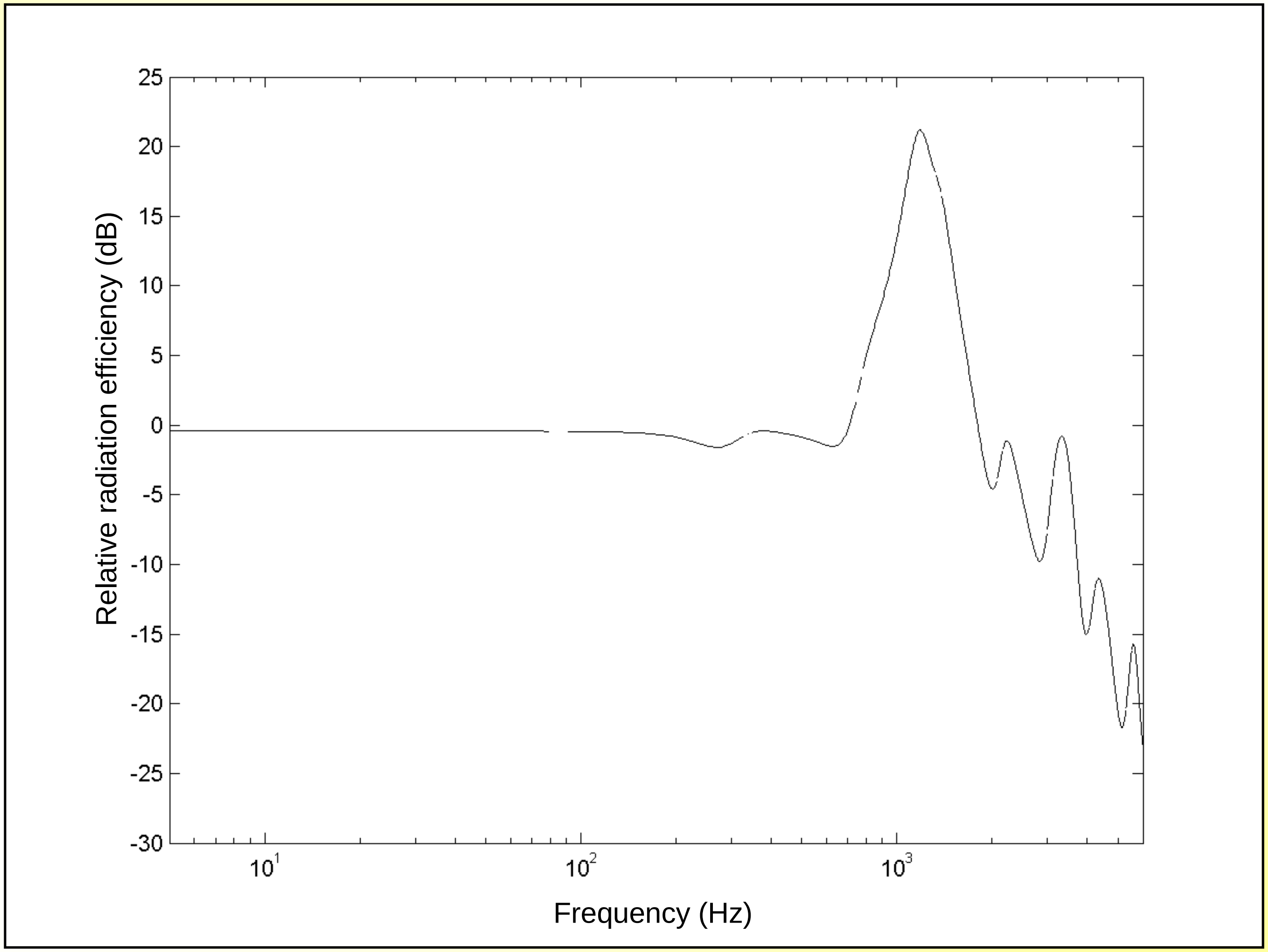
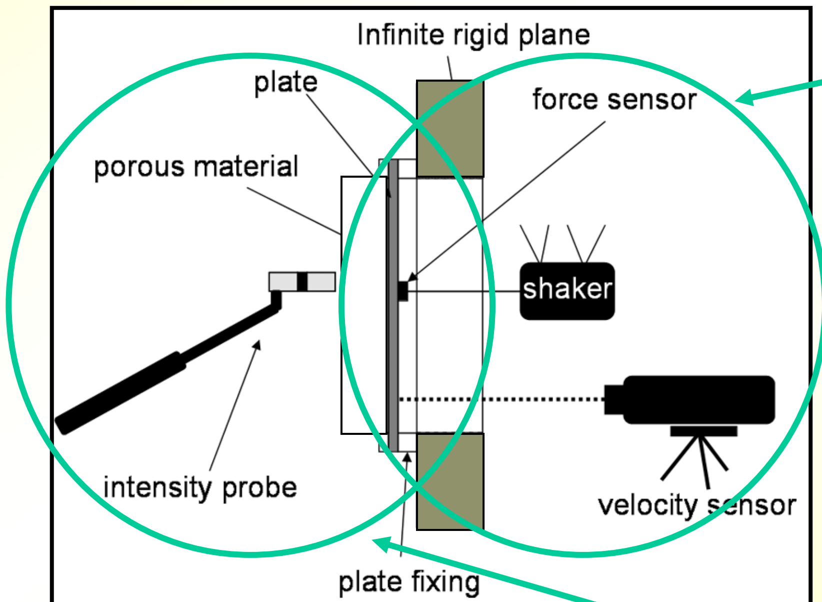
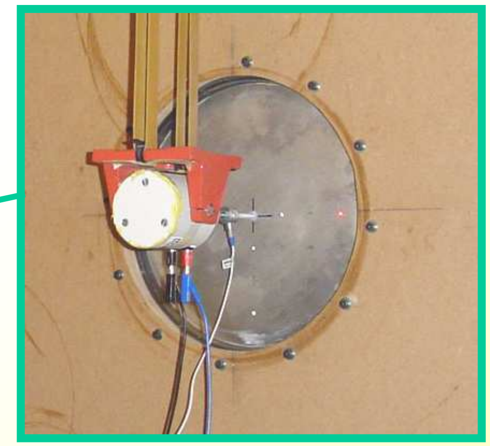
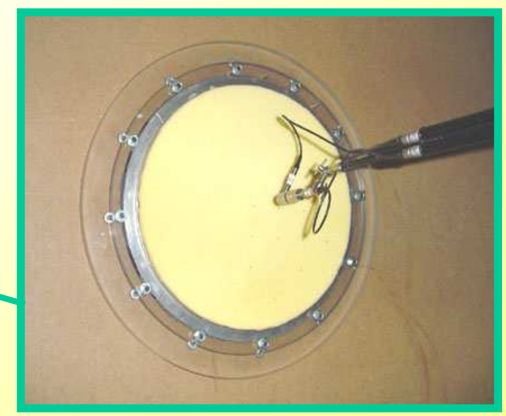


Experimental validation







Results: *Radiation efficiency of the covered plate*

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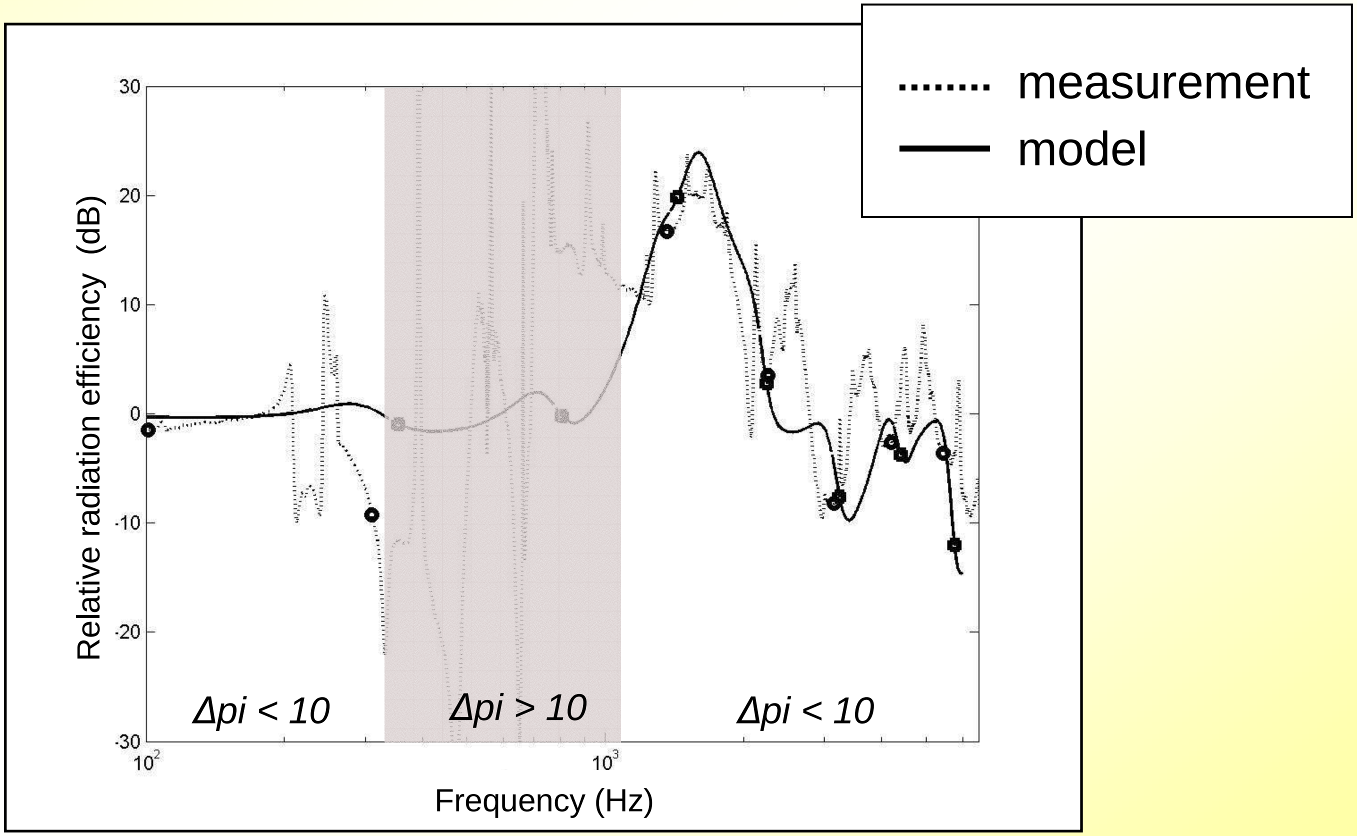

➡ **Good prediction of the acoustic radiation**

Calculation of Z_T for a multilayer

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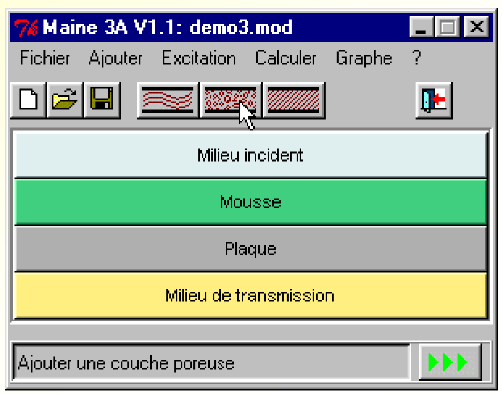
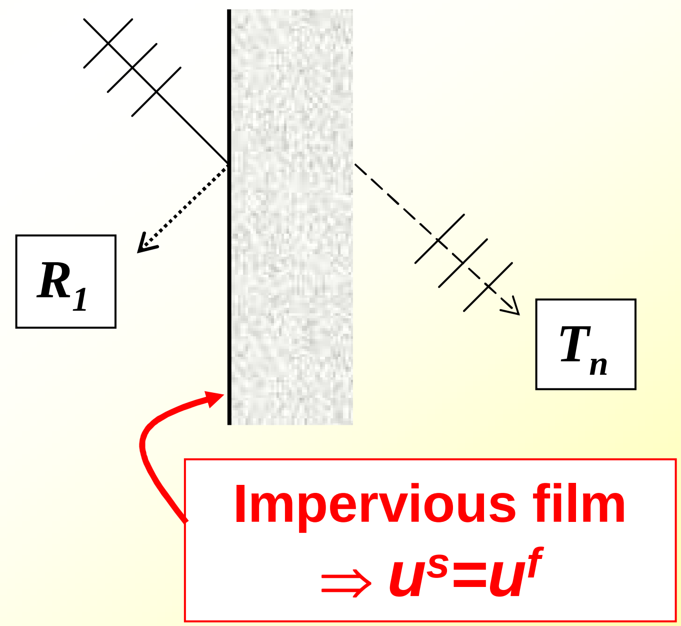
- **Use of transfert matrix method**
- **Maine3A software**

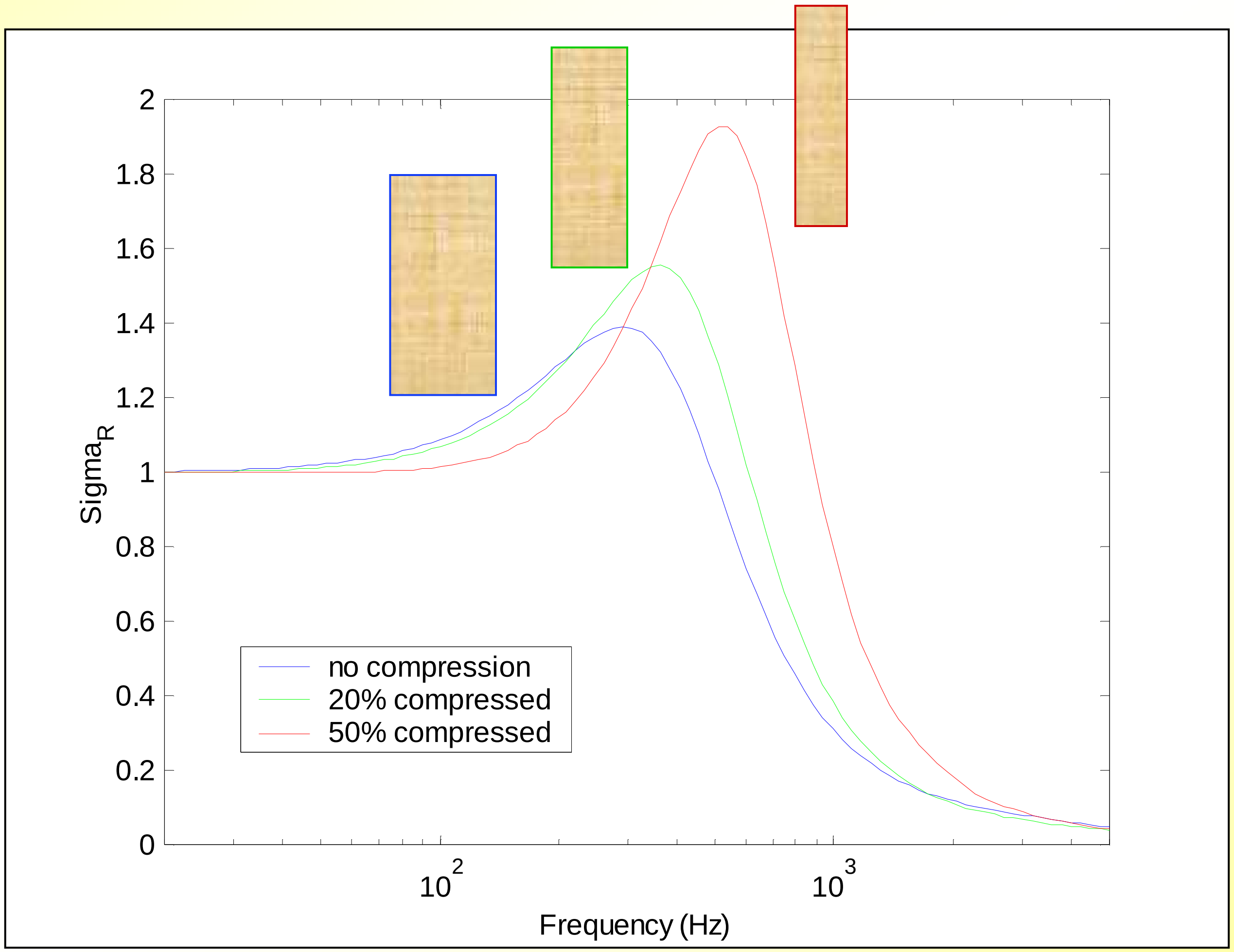




$$\Rightarrow \quad Z_T = \frac{Z_0\, T_n}{1 - R_1 - T_n}$$

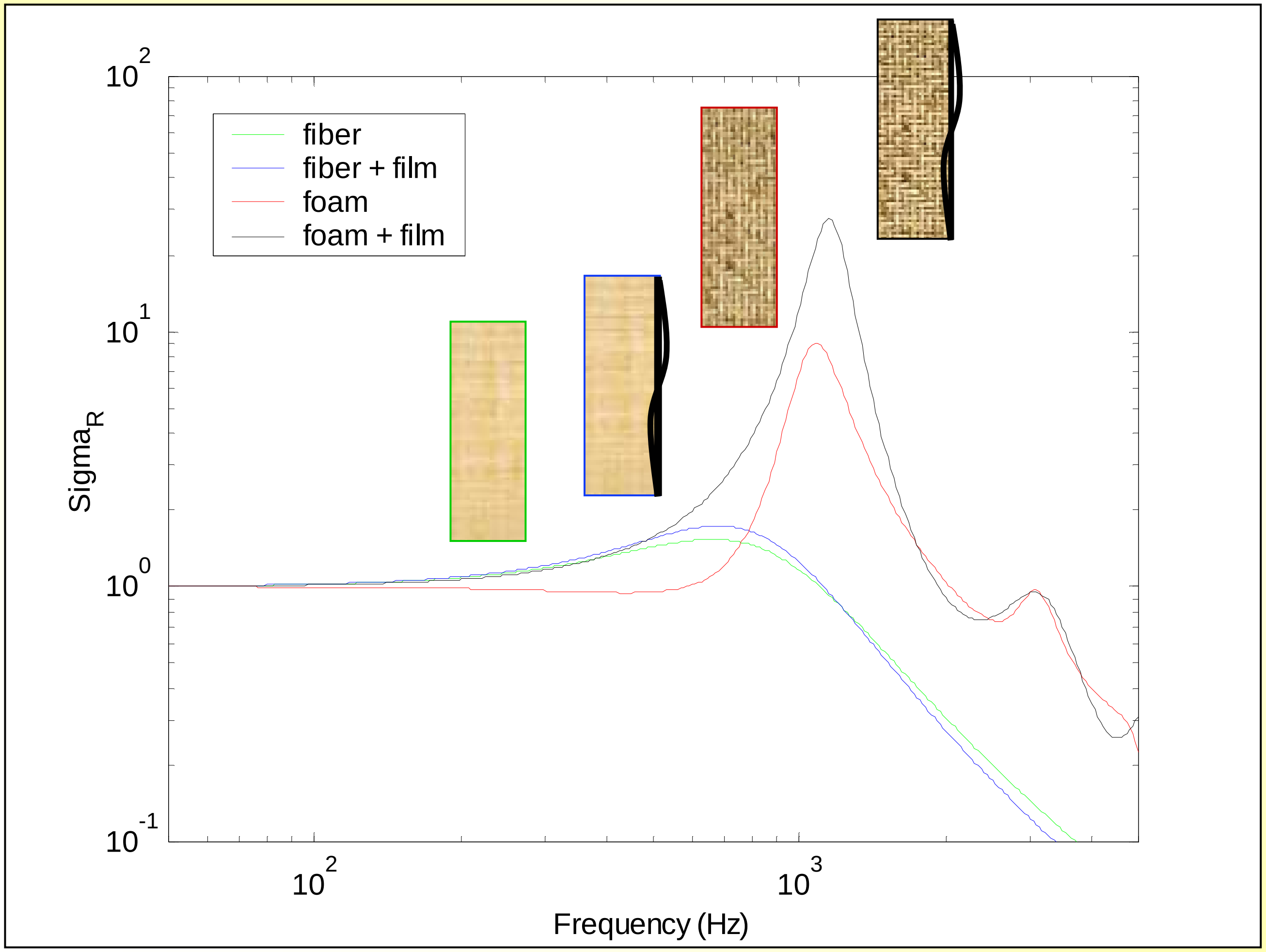
Effect of fibrous material compression on the radiation

- **Fibrous material compressed to 20% and 50%**



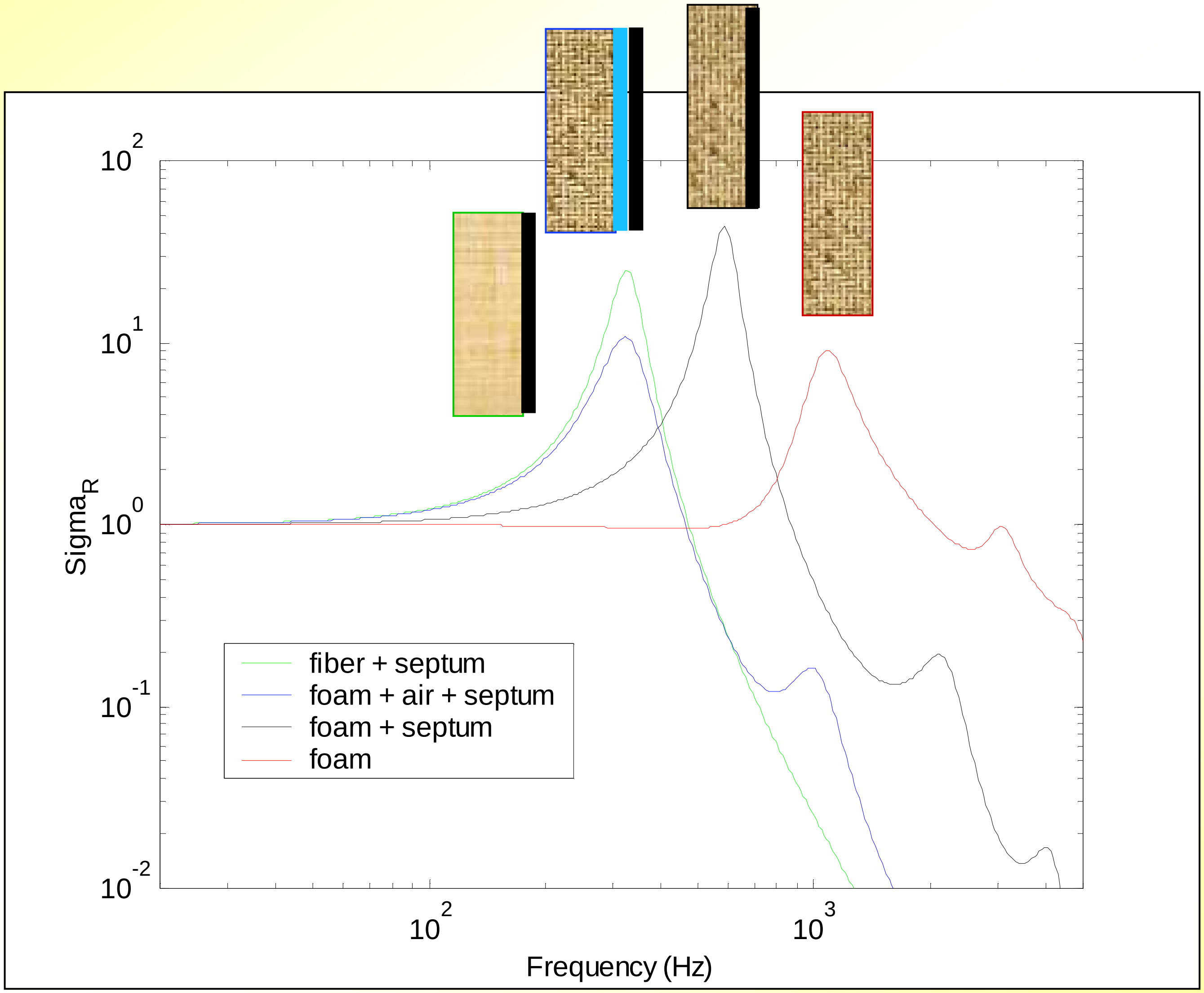
- **Increase of the radiation with compression**

Effect of a light film on the radiation



- **Increase of the radiation for the foam**
- **Almost no effect for fibrous**

Effect of a septum on the radiation



- **Decrease of the frequency**
- **Effect of air layer for the foam ➡ skeleton bypass**

Conclusion

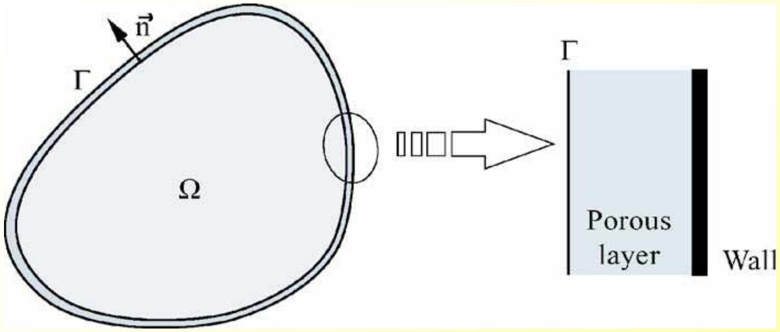
- **Acoustic radiation of a covered piston and plate**
  ➡ good agreements with measurements

- using **transfert impedance concept**
  for porous materials **(NOT SURFACE IMPEDANCE !)**

Prospects

- **Effect of mounting conditions ?**

- **Absorption versus transmission coefficient ?**



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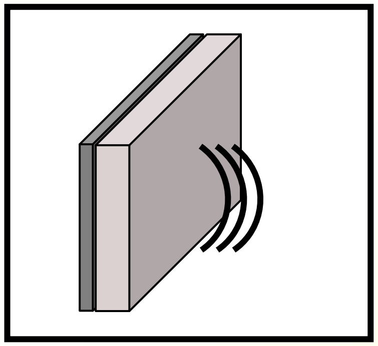

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