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Sound absorption properties of functionally graded polyurethane foams

Olivier Doutres [a)]
GAUS, Department of Mechanical Engineering, Université de Sherbrooke, Sherbrooke, Québec, J1K 2R1, Canada
Noureddine Atalla [b)]
GAUS, Department of Mechanical Engineering, Université de Sherbrooke, Sherbrooke, Québec, J1K 2R1, Canada

Noise control over a wide frequency band is an increasingly important design criterion in the building and transport industries. Examples of well known broadband passive concepts for optimal sound absorption are multi-layering with graded properties across the thickness and optimization of the material shape (e.g., wedges). However, for typical applications, the material thickness is limited and shaping or use of different material costly. Thus, there is growing interest for developing acoustical materials having microstructure properties gradient at the micro- or meso-scale; also known as Functionally Graded Materials (FGM). Even if sophisticated models are available to predict the acoustic behavior of homogeneous and multilayered acoustical materials, there is still a need for a better understanding of FGM for sound absorption. More specifically, does a graded foam material always improve the acoustic behavior compared to a homogeneous one? This presentation proposes to answer this question by investigating numerically, using a microstructure model, the effect of varying the reticulation rate along the thickness of a highly porous polyurethane foam.

1 INTRODUCTION

Passive open-cell acoustic foams are generally used as sound absorbers to decrease the acoustic pressure in a closed acoustic medium (e.g. car interior, plane cockpit, see Fig. 1). However, acoustic absorption efficiency of such passive materials is naturally limited to medium and high frequencies. Thus, optimization and development of innovative passive open-cell sound absorbing materials still remains a subject of intensive research. These solutions are mainly based on: (i) material shape optimization[1], (ii) layering or multi-layering with graded properties across the depth[2] or the width[3], (iii) multi-layering with various interface shapes[4], (iv) macro perforations[5] or (v) addition of rigid inclusions in the porous frame[6].

[a)] email: olivier.doutres@usherbrooke.ca
[b)] email: noureddine.atalla@usherbrooke.ca

A well known application based on the reduction of the impedance mismatch between air and a reflective surface to get efficient broadband frequency sound absorbers is the acoustic lining (wedges) used in anechoic chambers. However, for typical building or transport applications, porous materials thickness is much smaller compared to anechoic chamber linings but identical passive concepts can still be used. Thus, there is growing interest for acoustical materials with microstructure gradation at the micro- or meso-scale; also known as Functionally Graded Materials (FGM). Recent developments in highly porous open-cell foams or fibrous fabrication processes allow to create porous FGMs with potential benefits for acoustic applications. For example, methods can provide density gradient for fibrous materials[7] or cell size gradient[8-10] for aluminum and polymeric foams. Another foam microstructure property that can be graded along the layer thickness is the reticulation rate, i.e. the rate of cell pores which are not closed by thin membranes, for example by using chemical treatments as proposed in reference[11].

Even if sophisticated models are available to predict the acoustic behavior of homogeneous or graded acoustical materials[12,13], there is still a need for a better understanding of the ins and outs of graded properties in porous acoustical materials and their associated potential benefits. More specifically, does a graded foam material always improve the acoustic behavior compared to a homogeneous one? This paper proposes to answer this question by investigating numerically the effect of a graded reticulation rate along the thickness of a polyurethane foam layer. A “micro/macro” semi-empirical model developed by the authors[14,15] is used to correlate the microstructure properties of the polyurethane isotropic open-cell foam, i.e. cell size C_s and reticulation rate R_w, to the non-acoustic parameters (i.e., porosity ϕ , airflow resistivity σ, tortuosity α_∞, thermal characteristic length Λ' and viscous characteristic length Λ) required in classical porous models[16]. This micro-macro model which accounts for the intercorrelation and morphology-dependency of all non-acoustic parameters is first briefly recalled. Then, the modeling of the functionally graded polyurethane foam is presented. It is based on the discrete layering of the FGM into N homogeneous sub-layers with varying microstructure property (i.e., reticulation rate) from the upstream to the downstream layers. Acoustical behaviour of this discrete FGM is finally predicted using the classical Transfer Matrix Method[10]. The influences of both the microstructure properties of the two extreme layers of the discrete FGM and of the gradient profile will be investigated.

2 MICROSTRUCTURE BASED MODEL

2.1 Homogeneous Polyurethane foam

A homogeneous open-cell foam layer is characterized by a constant cell size C_s and constant reticulation rate R_w within the porous volume. Assuming that the acoustic wavelength is much larger than the characteristic dimensions of the foam (cell size) and that the saturating fluid behaves as an incompressible fluid at the microscopic scale, the air in the porous frame can then be replaced by an equivalent fluid. This equivalent fluid is characterized by two complex frequency-dependent functions: the dynamic density $\rho(\omega)$ which takes into account the visco-inertial interaction between the frame and the saturating fluid, and the dynamic bulk modulus $K(\omega)$, which takes into account the thermal interaction. These two functions are derived here from the Johnson-Champoux-Allard model using the following non-acoustic parameters as described in reference[16]: porosity ϕ , airflow resistivity σ, tortuosity α_∞, thermal characteristic length Λ' and viscous characteristic length Λ. In this paper, the homogeneous foam layer is rather described by the following two intrinsic acoustical properties: the wave number $k(\omega)=\omega(\rho(\omega)/K(\omega))^{1/2}$ and the characteristic impedance $Z_c(\omega)=(\rho(\omega)K(\omega))^{1/2}$. For convenience, the explicit dependence on ω will be omitted. The calculation of the acoustic behavior of the polyurethane foam layer is performed by means of a transfer matrix as detailed in reference[16]. For the equivalent fluid representing the studied foam, the transfer matrix $[T]$ relates the acoustic pressure p and normal particle velocity v at the incident surface of the layer, named upstream surface $\boldsymbol{u}$, with the transmitted surface, named downstream surface $\boldsymbol{d}$. The transfer matrix of a porous "equivalent fluid" layer of thickness H excited by a normal incidence plane wave is given by

$$[T]=\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}=\begin{bmatrix} \cos(kH) & jZ_c\sin(kH)/\phi \\ j\phi\sin(kH)/Z_c & \cos(kH) \end{bmatrix} \tag{1}$$

The normal incidence sound absorption α is then derived from the reflection coefficient r such as $\alpha=1-|r|^2$, with $r=(T_{11}-\rho_0c_0T_{21})/(T_{11}+\rho_0c_0T_{21})$, c_0 being the sound speed in air and ρ_0 the fluid density.

The non-acoustic properties of isotropic and homogeneous Polyurethane open-cell foams are estimated from microstructure properties, i.e. Cell size C_s, strut length to thickness ratio $B=l/t$ and reticulation rate R_w, using the semi-empirical equations previously presented by Doutres *et al.* in reference[14] and recently simplified in reference[15] by the same authors. This model, initially developed for Polyurethane foam, consider the porous media as a packing of tetrakaidecahedra cells interconnected through pores as shown in Fig. 2. The links between microstructure and non-acoustic properties are derived using a combination of (i) geometrical calculations on one representative unit-cell, (ii) augmented scaling laws to account for the presence of closed pores and (iii) empirical observations. The main micro-macro links are:

$$\phi=1-C_t^{\rho}\frac{1}{B^2}, \tag{2}$$

$$\Lambda'=C_s\frac{C_1}{C_2-R_wC_3}=C_s\frac{8(1-(2\sqrt{3}-\pi)/B^2\sqrt{2})/3A}{(1+2\sqrt{3})-R_w(1+2\sqrt{3}-4\pi/B\sqrt{3})} \tag{3}$$

$$n=\frac{\Lambda'}{\Lambda}=1.55\left(\frac{1}{R_w}\right)^{0.6763} \tag{4}$$

$$\sigma=C^{\beta}\left(C_r^{\rho}\frac{A\sqrt{2}}{B}\right)^2\left(\frac{1}{C_s}\right)^2\left(\frac{1}{R_w}\right)^{1.1166} \tag{5}$$

$$\alpha_\infty=1.05\left(\frac{1}{R_w}\right)^{0.3802} \tag{6}$$

Note that Eqns. (2) and (3) are not explicitly given in reference[15] and have been obtained by replacing the strut parameters (i.e., t and l) in Eqns. (1) and (2) of reference[15] by the cell parameters (C_s and R_w) and the two scaling factors (A and B). Since this work investigates polyurethane FGM, the two scaling factors A and B are known and are equal to 2.333 and 3.835, respectively. Note that other foams could be modeled using these links if the representative unit-cell has a tetrakaidecahedra shape (A~2) and if the scaling coefficient B is determined from porosity measurements and Eqn. (2).

2.2 Functionally graded Polyurethane foam

A functionally graded open-cell foam layer is characterized here by a gradient of its microstructure properties along its thickness. It is thus inhomogeneous at the macroscopic scale. The functionally graded open-cell polyurethane foam layer is modeled here as a discretely graded layer as shown in Fig. 3. It consists of N homogeneous discrete sub-layers of a gradually varying reticulation rate R_w. The first sub-layer, called "upstream sub-layer", is characterized by the reticulation rate R_w^1 and the last sub-layer (bonded onto a hard wall), called "downstream sub-layer", is characterized by R_w^N. Inner sub-layers are characterized by property R_w^i according to a n-power law such as

$$R_w^i = R_w^1 + \left(R_w^N - R_w^1\right)\left(\frac{x_i - h/2}{H - h}\right)^n \tag{7}$$

with x_i the coordinate of the center of the sub-layer (i=1...N), H the thickness of the graded foam, h=H/N the thickness of a sub-layer and n the parameter that governs the variation of microstructure property. Superscript n=0 corresponds to the homogeneous foam layer of thickness H described in section 2.1.

The N sub-layers constituting the open-cell porous FGM are considered homogeneous and described by a transfer matrix $[T]^i$ (i=1..N). Using the transfer matrix method, the transfer matrix of the discretely graded foam $[T]^{FGM}$ can then be computed from the transfer matrices of the N sub-layers as

$$[T]^{FGM} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{FGM} = [T]^1[T]^2[T]^3 \ldots [T]^N \tag{8}$$

The sound absorption of the porous FGM is finally derived from the expression of the reflection coefficient given in sec. 2.1 by replacing the coefficients of the transfer matrix $[T]$ by the ones of $[T]^{FGM}$.

3 RESULTS

The graded open-cell foam studied in this paper is 2 inch-thick (i.e., H=50.8 mm), with a varying reticulation rate along the thickness direction; here the direction of propagation of the acoustic waves (1D propagation). In the case of the absorption configuration presented in Fig. 3, the aim of the graded foam is to decrease the impedance mismatch between the air medium and the rigid wall. It is well known that the best results for sound absorption applications are obtained if the airflow resistivity of the graded foam increases from the upstream layer to the downstream layer in contact with the rigid backing. Thus, the upstream sub-layer (i=1) is characterized by a maximum reticulation rate R_w^1=R_w^{max}=95% (i.e., most of the cell pores are open), and the downstream sub-layer (i=N) by a minimum reticulation rate R_w^N=R_w^{min}=5% (i.e., most of the cell pores are closed by thin membranes). The cell size is set to C_s=500 μm (a value compatible with current fabrication processes for PU foams) and is kept constant within the porous volume. The modeled discrete FGM is characterized by N=15 homogeneous sub-layers (h= H/N =3.4 mm). The number of N=15 sub-layers is sufficiently low to have a minimum of six cells along the thickness of each sub-layer and at the same time is sufficiently high to correctly discretize the FGM and ensure the convergence of the solution.

Figure 4(a) presents the reticulation rate gradient profiles given by Eqn. (7) with n=1/4, 1/2, 1, 2 and 4. Figs. 4(b) to 4(f) present the resulting variation of the non-acoustic properties along the FGM thickness. Porosity does not depend on the reticulation rate for the open-cell polyurethane foams considered in this work[14,15] (no closed porosity) and thus remains constant along the porous thickness (such as the foam density). For this graded foam, the airflow resistivity is ranging between 5150 N.s.m^{-4} and 138 000 N.s.m^{-4}, the tortuosity between 1.07 and 3.28, the viscous characteristic length between 11 μm and 173 μm and the thermal characteristic length between 130 μm and 278 μm. It is important to note that the used micro-macro model accounts for the intercorrelation and morphology-dependency of all non-acoustic parameters: i.e., all non-acoustic parameters vary at the time by varying the microstructure properties (C_s, R_w). This in turn allows for a realistic optimisation of the foam acoustic behavior. It is important to note in Fig. 4(c) the rapid variation of the flow resistivity for low reticulation rates. That is, in this case, all power laws with n larger than one lead to large variations of the flow resistivity for the layers near the backing wall.

Figure 5 presents the normal incidence sound absorption coefficient of the 5 graded foams and of the three following homogeneous foams: (i) a 2”-thick highly reticulated homogeneous polyurethane foam layer with a maximum reticulation rate of 95% (up triangles); (ii) a 2”-thick poorly reticulated homogeneous polyurethane foam layer with the minimum reticulation rate of 5% (down triangles); and (iii) a 2”-thick homogeneous polyurethane foam layer having the optimum reticulation rate for sound absorption (squares), i.e. R_w^{opt}=30%. This optimum reticulation rate of R_w^{opt}=30% gives the maximum sound absorption coefficient in a broad frequency range in the case of a 2”-thick homogeneous foam with C_s=500 μm. This maximum is evaluated using the Noise Reduction Coefficient (NRC) computed in this work as the arithmetic average of the sound absorption at the 17 1/3 octave bands between 125 Hz and 4000 Hz. The NRC of the studied foams are given in Table 1 together with the normalized flow Resistance Per Unit Area (*nRPA*, see definition below Eqn. (9)).

As expected, the highly resistive 2”-thick homogeneous “base” material (down triangles, R_w=5%) is the most efficient sound absorbing material at low frequencies up to 260 Hz but it shows poor absorption from mid to high frequencies. The NRC of this material is only 0.60 as indicated in Table 1. On the contrary, the low flow resistance 2”-thick homogeneous and highly reticulated material (up triangles, R_w=95%) shows poor sound absorption at low frequencies (up to 800 Hz) and a relatively good efficiency at mid to high frequencies. The 2”-thick optimum homogeneous foam layer (squares, R_w=30%) with the microstructure properties {C_s, R_w^{opt}} has a good sound absorption behaviour in the whole frequency range with an NRC=0.90. It is considered here as the reference material. An improved sound absorbing behavior of the FGMs compared to this reference homogeneous foam is required in order to justify the increased complexity in FGM fabrication.

Acoustic behaviours of the graded layers are now investigated in more details. It is shown that the reticulation gradation induces an absorption increase in the mid and high frequency range compared to the poorly reticulated homogeneous foam (down triangle, R_w=5%). This improvement is due to the frequency shift of the first absorption interference maxima (around the quarter wave-length resonance frequency) toward higher frequency while increasing the coefficient n of the power law profile (see Eqn. (8)). This large frequency shift corresponds to a great decrease of the total normalized flow resistance *nRPA* of the graded foam defined by

$$nRPA = \frac{\sum_{i=1}^{N} h_i \sigma_i}{\rho_0 c_0} \tag{9}$$

which varies from 5.33 for *n*=0.25 to 1.96 for *n*=4 as shown in Table 1 (note: for comparison purposes, the airflow resistance of the poorly reticulated homogeneous layer with C_s=500μm is *nRPA*=16.97). According to table 1, all FGMs give a better broadband NRC compared to the optimum homogeneous foam. This sound absorption improvement, mainly localized above 1 kHz, is due to the fact that the microstructure gradation considerably reduces the sound absorption amplitude difference between the interference minima and maxima. However, a specific gradient profile can be preferred depending on the frequency band of interest as shown in Fig. 5. Furthermore, the two FGMs with the power law profile associated to *n*=0.25 and *n*=0.5 allow at the same time; (i) to keep a good low frequency sound absorption compared to the optimum homogeneous foam and (ii) improve the high frequency sound absorption. This positive broadband effect is due to the fact that the resistive behaviour of the "base" material (i.e., in contact with the rigid backing and having the lower reticulation rate of R_w=5%) is greatly superior to the one of the optimum homogeneous layer (i.e., with a medium reticulation rate of R_w=30%). In other words, the *nRPA* of a "broadband" FGM should be greatly superior to the one of the optimum homogeneous foam layer. In the example presented above, the *nRPA* of the optimum homogeneous layer is 2.29 whereas it is equal to 5.33 and 3.90 for the FGMs associated to the power law profiles *n*=0.25 and *n*=0.5 respectively (see Table 1). This indicates that the optimum reticulation gradient profile depends on the initial reticulation rate of the base foam subjected to the reticulation process.

4 CONCLUSIONS

This paper investigated numerically the sound absorption performance of a functionally graded acoustic polyurethane foam compared to an optimum homogeneous foam layers having identical thickness, porosity and bulk density. Acoustic polyurethane foams with graded reticulation rate under the rigid frame assumption are considered. Polyurethane FGM are modeled as discretely FGM consisting of *N* homogeneous sub-layers with gradually varying microstructure property (i.e., reticulation rate) along the thickness. A "micro/macro" semi-empirical model developed by the authors is used to correlate the microstructure properties of each isotropic and homogeneous polyurethane sub-layer to the non-acoustic parameters (porosity, flow resistivity, tortuosity…) required in classical porous models. Acoustical behaviour of the discrete FGMs is finally predicted using the classical Transfer Matrix Method. It is shown that: (i) if the total airflow resistance of the FGM is high (i.e., superior to the one of the optimum homogeneous material), the microstructure gradation allows to keep a good sound absorption behavior at low frequencies and increase considerably the medium to high frequency ranges; (ii) the microstructure gradation lowers the amplitude difference between the interference maxima and minima of the absorption curves thanks to the smooth variation of the characteristic acoustic impedance along the thickness (i.e., no large impedance mismatch). Passive FGM Polyurethane porous layers are thus a potential solution for broadband sound absorption as long as the reticulation rate gradient profile along the foam thickness is controlled.

5 ACKNOWLEDGEMENTS

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Table 1 – Normalized flow resistance (nRPA) and NRC for three homogeneous and five graded foams.

Foam	*nRPA*	*NRC*
Homogeneous {C_s, R_w^{min}}	16.97	0.599
Homogeneous {C_s, R_w^{max}}	0.63	0.835
Homogeneous {C_s, R_w^{opt}}	2.29	0.900
FGM (*n*=0.25)	5.33	0.947
FGM (*n*=0.5)	3.90	0.947
FGM (*n*=1)	2.90	0.985
FGM (*n*=2)	2.29	0.983
FGM (*n*=4)	1.96	0.948

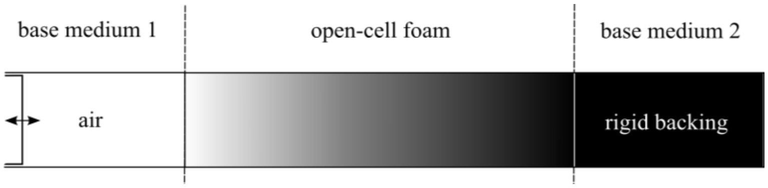

Fig. 1 - Open-cell foam layer as sound absorber.

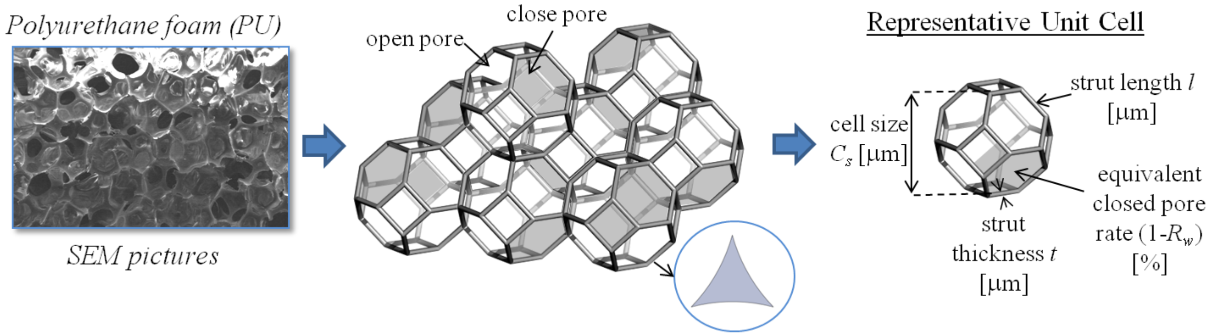

Fig. 2 - Schematic representation of a PU foam as a packing of tetrakaidecahedra unit-cells.

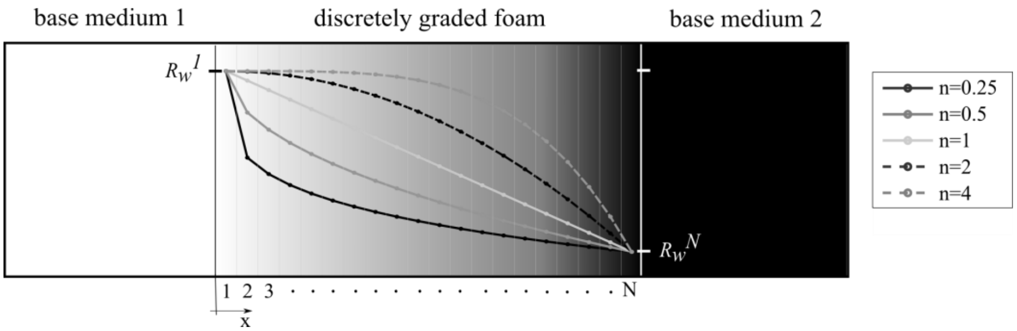

Fig. 3 - Discretely graded open-cell foam with various gradient profiles (see Eqn. (8)).

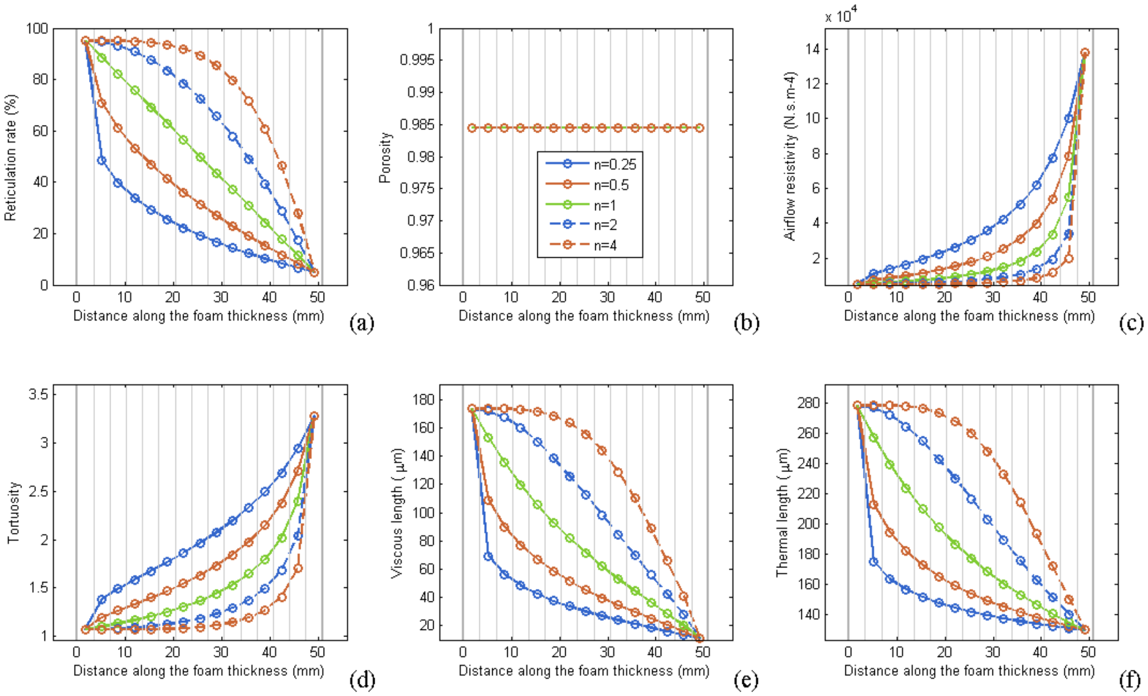

Fig. 4 - Variation of the microscopic and non-acoustic properties of a 2”-thick functionally graded polyurethane foam with varying reticulation rate along the thickness direction for various power-law orders n; Cs=500 μm: N=15 sub-layers from R_w=95% to 5%.

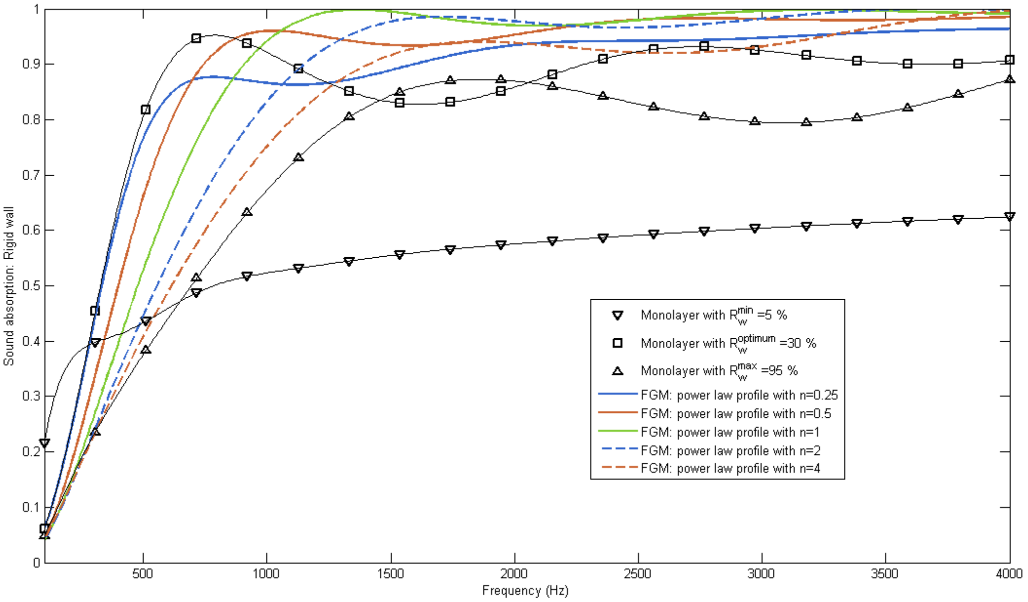

Fig. 5 - Acoustic properties of a 2”-thick functionally graded polyurethane foam with varying reticulation rate along the thickness direction; Cs=500 μm: N=15 sub-layers from R_w=95% to 5%.