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Modelling of the acoustic radiation of a structure covered by a porous layer

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Abstract

To predict noise in enclosure containing treated vibrating panels, porous layer effect has to be taken into account. This study propose an analytical model of the effects of a porous layer on a vibrating plate by separating the acoustical and vibratory behaviours. This paper deals with acoustic coupling by means of surface impedance derived by a one-dimensional model using "Biot-Allard" theory. It is shown that this impedance is different from that calculated when the porous is backed by an impervious rigid wall. The total radiation efficiency of a circular plate characterized by the surface impedance, clamped in a rigid infinite baffle, is computed and compared with experimental measurements. This model shows good agreement with experimental datas.

1 Introduction

In many applications, porous materials such as polymers foams are used for noise control. Often attached to a structure subjected to vibration, the porous layer contributes to diminish vibration by increasing structural damping and also reduce noise level in cavities by sound absorption. The acoustic radiation of a plate covered by a porous layer can be predicted using finite element calculations including poroelastic elements. However, this method leads to significant computing time and memory usage [8]. The aim of this study is to propose an analytical model of this configuration by separating the acoustical and vibratory behaviours (Fig. 1):

- the vibratory analysis is based on a dynamic study of the two-layer system from mechanical properties of an equivalent plate. In this case, the coupling between porous skeleton and air are neglected and the porous layer can be considered as a viscoelastic layer corresponding to the skeleton in vacuum [6];
- in the acoustical analysis, a surface impedance is applied on this equivalent plate to take into account the effect of the porous layer on the fluid-structure coupling.

This paper focuses on this second step: the calculation of the surface impedance and the experimental validation of the model. The porous layer effect on the acoustic radiation of the plate is investigated by mean of the total radiation efficiency of the coupled system.
The first part deals with the radiation efficiency calculation of a plate characterized by a surface impedance, vibrating in a infinite rigid baffle. In the second part, the surface impedance is derived using a one-dimensional model based on "Biot-Allard" theory [4]. This surface impedance is then applied to a circular plate excited by a ponctual force and total radiation efficiency is calculated summing the contributions of axisymmetric modes. Finally, radiation efficiency measurements of a clamped circular plate with or without attached porous layer are carried out and compared with the model.

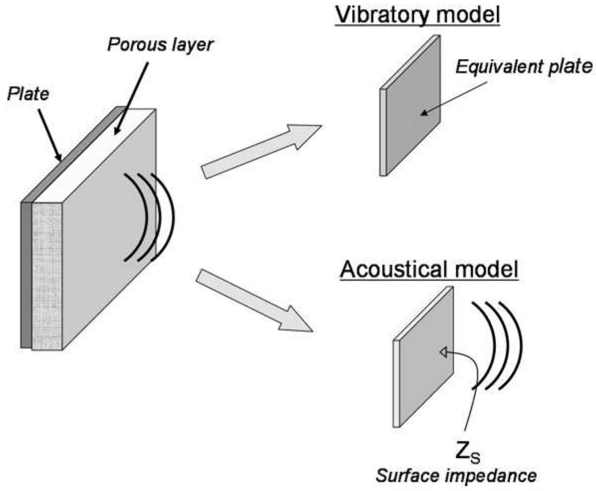

Figure 1: Schematic representation of the coupled structure modelisation

2 Theory

2.1 Acoustic radiation of the coupled structure

The radiation efficiency is defined as the ratio of the acoustic power radiated Π_a over the vibratory power Π_v. For harmonic motion at frequency ω, the total acoustic power radiated from the plate can be obtained by integrating the farfield acoustic intensity over a hemisphere of radius r to give

$$\Pi_a = \int_0^{2\pi} \int_0^{\pi/2} \frac{|p(r)|^2}{2\,\rho\,c} r^2 \sin\theta d\theta d\varphi, \tag{1}$$

where $p_(r)$ is the complex acoustic pressure at a location in space expressed in spherical co-ordinates, ρ and c are the density of air and the speed of sound in air. Considering a plate vibrating in an infinite rigid baffle and characterized by the surface impedance Z_S, the complex acoustic pressure $p(r)$ can be written in terms of the plate surface velocity using the Rayleigh integral [1],

$$p(r) = j \iint_S k\;\rho\;c\;V\;G_\omega\;dS, \tag{2}$$

where V is the complex surface normal velocity, $k = \omega/c$ is the acoustic wave number and S is the plate surface. G_ω is the Green function derived in the semi-infinite half space using image sources theory [2]. Hence, the Green function results of the superposition of the incident and reflected waves respectively generated by the primary source P_0 (at location r_0) and the image source P'_0 (at location r'_0) as shown figure (2):

$$G_\omega = \frac{e^{-jk|r-r_0|}}{4\pi\;|r-r_0|} + R\frac{e^{-jk|r-r'_0|}}{4\pi\;|r-r'_0|}, \tag{3}$$

with R the reflexion coefficient defined by

$$R = \frac{Z_S\cos(\theta) - \rho\;c}{Z_S\cos(\theta) + \rho\;c}. \tag{4}$$

2.2 Surface impedance calculation: one-dimensional plane-wave model

Calculation of the surface impedance must be studied carefully because it characterises how the porous layer bonded to a vibrating structure may affect its acoustic radiation. In the literature [3],

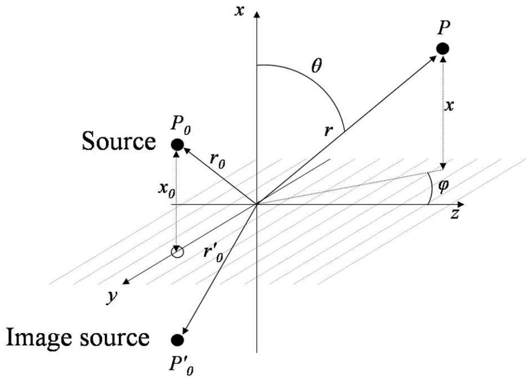

Figure 2: Schematic representation of the image sources theory

surface impedance is defined as the ratio of the surface pressure $P(d)$ (see Fig.3) over the difference between the source velocity V_0 and the surface velocity $V(d)$:

$$Z_S = \frac{P(d)}{V_0 - V(d)}. \tag{5}$$

A 1D model has been developed to calculate the surface impedance of a covered piston in a semi-infinite duct (see Fig.3). The flat piston is animated by an harmonic velocity of amplitude V_0. In the porous layer, according to "Biot-Allard" theory [4], two types of longitudinal waves can propagate forward and backward. This leads to four waves of amplitude A, B, C, D. In the fluid domain, only one wave of amplitude E propagates from the porous interface. Boundary conditions between each media are continuity of stress and displacements [5]. This system is solved numerically and surface impedance is obtained using amplitude E [9].

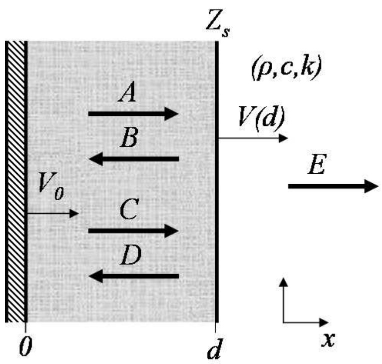

Figure 3: One-dimensional representation of the coupled structure

The surface impedance is independant of the velocity V_0 of the piston. Note that this impedance is different from the surface impedance calculated when material is bonded to an impervious rigid wall (see Fig.4). Thus, it is not possible to determine it directly with classical method such as impedance tube.

2.3 Application to a clamped circular plate

In order to compare the vibroacoustic behaviour simulated with experimental measurements, the model is applied to a circular plate fixed in a rigid baffle.

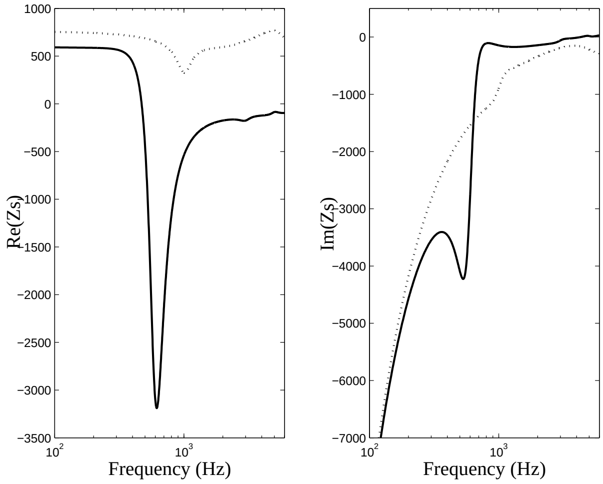

Figure 4: Real part and Imaginary part of the surface impedance: (**...**) porous layer backed by an impervious rigid wall, (**-**) porous layer backed by the vibrating piston.

We investigates the total radiation efficiency of a circular plate of radius a, clamped in a rigid baffle and excited in its center by a ponctual force. The total radiation efficiency of a plate can be obtained by summing the effect of all modes that contribute significantly in the frequency range under consideration. Here, only axisymmetric modes are taken into account to simplify the far field acoustic intensity integral (Eq. 2) and thus to reduce the computing time. It is coherent with experimental set-up which includes an excitation at the center of the plate.

To compute the far field pressure created by the moving plate, we use the Rayleigh integral (Eq. 2) in polar co-ordinates. The plate velocity V at any location **r** on the structure can be found by superposing the modal contributions from each mode of structural vibration of the plate as

$$V = \sum_n A_n \; w_n(\mathbf{r}), \tag{6}$$

where A_n is the complex velocity amplitude of the mode n, $w_n(\mathbf{r})$ is the value of the associated mode shape function at the location $\mathbf{r} \in [0, a]$ and n is the indice of the mode. A_n depends on the form of the excitation and on the frequency. Here the modal velocity amplitude is calculated considering a point force applied on the plate center. The mode shape for the clamped circular plate can be expressed as

$$w_n(\mathbf{r}) = I_0(\beta_{0n}a)J_0(\beta_{0n}\mathbf{r}) - J_0(\beta_{0n}a)I_0(\beta_{0n}\mathbf{r}), \tag{7}$$

where $\beta_{0n}^4 = \omega_n^4 \rho_1 h_1 / D_1$ is the nth structural wavenumber raised to 4th power, ω_n is the nth eigenfrequency, ρ_1, h_1 and D_1 are respectively the density, thickness and bending stiffness of the plate, J_0 and I_0 are the 0th order Bessel and modified Bessel functions.

When acoustic layer is attached to the plate, vibratory analysis must include its influence of thickness and stiffness. Characteristic of an equivalente plate has been calculated considering pure bending deformation of the structure. In this model, the porous layer is considered as a monophasic viscoelastic media thus neglecting the coupling between its solid and fluid phase [6] (porous layer under vacuum's condition). Total bending rigidity is then simply the sum of bending rigidities of the two layers related to the neutral fiber of the plate

$$D_{12} = D_1 + D_2, \tag{8}$$

with D_2 the skeleton's complex bending stiffness, and the equivalent density is given by

$$\rho_{12} = \frac{\rho_1 h_1 + \rho_2 h_2}{h_1 + h_2} \tag{9}$$

with ρ_2 and h_2 respectively the density and the thickness of the porous layer.

Substituting Eqs. (6) and (7) into Eq. (2) expressed in polar co-ordinates gives the sound pressure emitted.
The total radiation efficiency of the coupled system is presented in Fig.(5.a) for an aluminium circular plate clamped in a rigid baffle covered with or without a porous layer. Three disctinct zones (1, 2, 3) are defined. In the low frequency range (zone 1), the porous layer has no or little effect on radiation effciency. In the second zone, increased acoustic radiation is due to skeleton resonances in the thickness of the layer. Above these frequencies, radiation efficiency decreases, due to structural, viscous and thermal dissipation in the porous layer. For a better analysis of the porous layer effect on acoustic radiation, the total radiation efficiency of the coupled system is normalized by the total radiation efficiency of the not covered plate (Fig. 5.b). In this configuration, the porous layer has little effect below 170 Hz. Between 680 Hz and 1600 Hz the normalized radiation efficiency is increased up to 20 dB. Above 1600 Hz, global radiation level decreases.

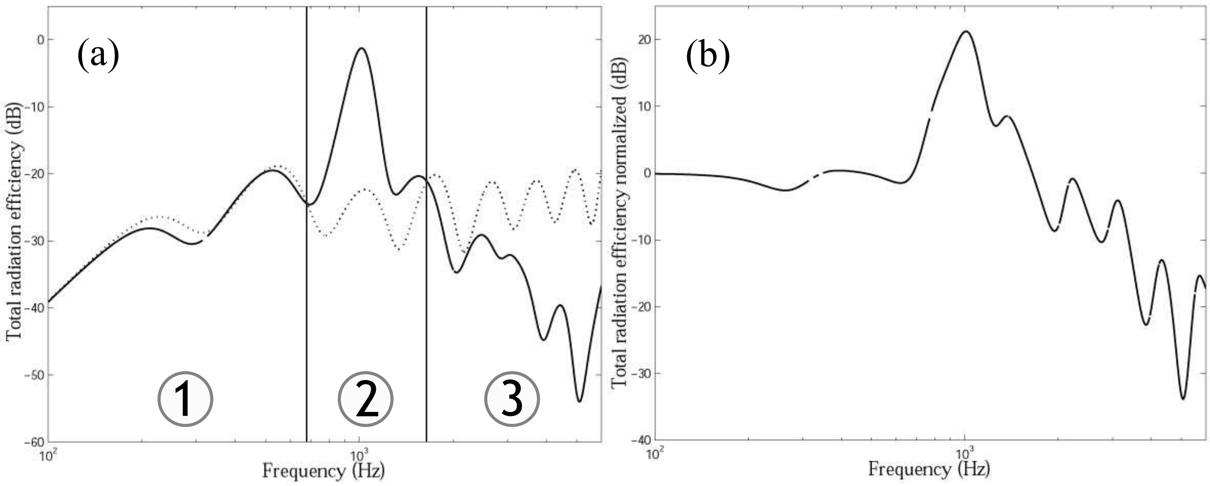

Figure 5: (a) Total radiation efficiency of an aluminium clamped circular plate fixed in a rigid baffle: ... plate, $-$ plate with porous layer; (b) Normalized total radiation efficiency.

3 Experimental validation

3.1 Experimental set-up

Measurement of the radiation efficiency of a plate with or without covering porous layer has been performed. The experimental configuration is an aluminium circular plate (Ø29 cm) clamped in a rigid baffle. Properties of the plate material are shown Tab.(1). The center of the back face is connected to a shaker to excite only axisymmetric modes(see Fig. 6).
Vibratory power of the plate is determined from the quadratic mean of normal velocities measured at 13 points by a laser vibrometer. The front face that can be covered by a porous layer is radiating in a anechoic chamber. Acoustic radiated power is determined from the average of intensity measurements carried out on a quarter of plate surface using an intensity probe. This probe is made of two $1/2$ inch microphones spaced by 12 mm. The material properties (Tab.1) are chosen so that the radiation efficiency increase can be seen.

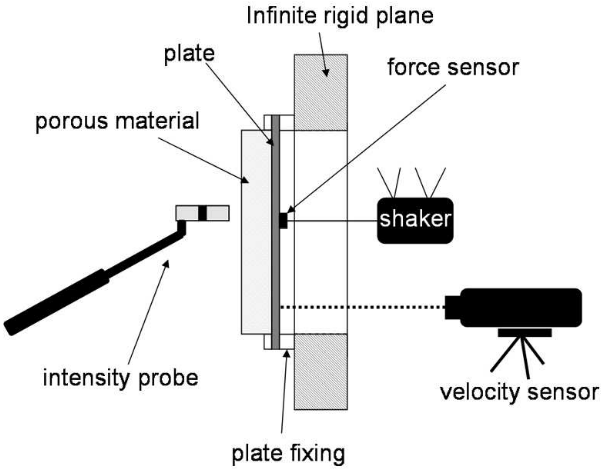

Figure 6: Experimental set-up

Plate	
Thickness	1 mm
Young's Modulus	$74\ 000$ MPa
Density	2800 kg m^{-3}
Porous layer	
Thickness	20.17 mm
Air flow resistivity	$75\ 000$ N s m^{-4}
Porosity	0.97
Viscous length	$56.9\ \mu$m
Thermal length	$170.7\ \mu$m
Skeleton Complex Young's Modulus at 5Hz	$285(1 + \jmath\ 0.128)$ kPa
Skeleton density	59 kg m^{-3}

Table 1: Properties of materials

3.2 Results and discussion

Fig. (7) presents the normalized total radiation efficiency simulated and measured. For frequencies ranging between 200 Hz and 1100 Hz (grayed zone), intensity measurements are strongly pertubated by non-axisymmetric modes influence and thus, the 'Pressure-Intensity Index δ_{pI}' defined by Fahy [7] is not satisfied.

The increase of the radiation efficiency predicted by the model is also observed experimentally. It confirms the use of the surface impedance derived when the porous is backed by the moving piston. Note that the surface impedance of the material backed by the rigid wall can not predict this behaviour. Considering the adequate surface impedance, simulation is fitted, acting on the mechanical complex Young modulus of the porous layer: it is adjusted to $685(1 + \jmath\ 0.17)$ kPa. Simulation presents good agreements with experimentation at natural frequencies of axisymmetric modes located by circles (experimental curve) and squares (analytical curve) on the figure.

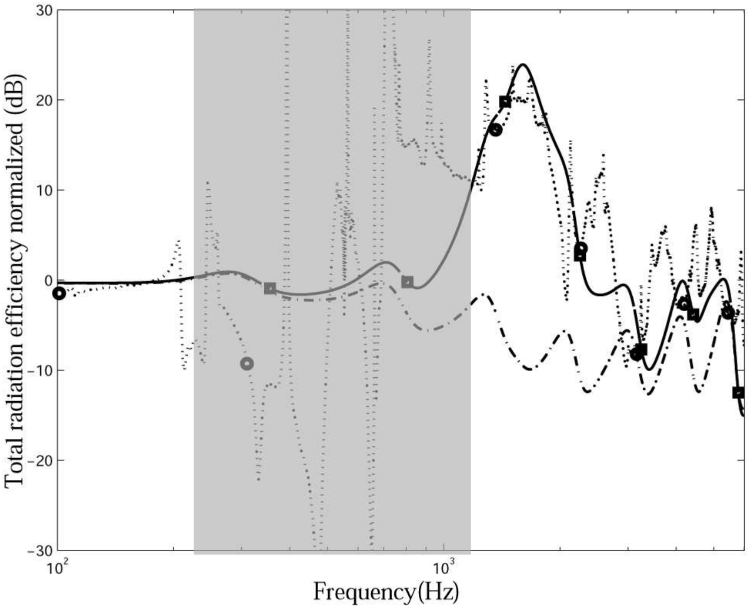

Figure 7: Normalized total radiation efficiency : (**...**) experimental, (**-**) simulation using the surface impedance of the porous backed by the vibrating piston, (**.-**) simulation using the surface impedance of the porous backed by an impervious rigid wall. Eigen-frequencies of axisymmetric modes are located by: (circle) experimental, (square) simulated.

4 Conclusion

Porous layer's effect on plate radiation has been investigated. An analytical model of the coupled system "plate/porous" has been developed, separating the vibratory and acoustical behaviors. The acoustical analysis is based on surface impedance calculation using a one-dimensional plane wave model according to "Biot-Allard" theory. The model is applied to a circular plate clamped in a rigid baffle. Total radiation efficiency is calculated by summing the effect of all the axisymmetric modes. Simulation are compared with experimental results and good agreements exist at natural frequencies of the axisymmetric modes. Note that the model must include the surface impedance of the porous backed by a moving surface and not the surface impedance when the porous is backed by an impervious rigid wall.
Further work are validation of the model using other materials such as fibrous materials and extension of the model when the structure is coupled with a cavity.

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